Heidelberg Integration Bee

Dear challenger of the Heidelberg Integration Bee,

this document contains a collection of integrals from the previous Heidelberg Integration Bee. It serves as a resource for those preparing for this year's competition. By working through these problems, you will strengthen your integration skills and gain insight into the types of challenges you may encounter.

The integrals are categorized by competition stages, including the qualifier test, quarter-finals, semi-finals, and finals. Some problems were contributed by well-known figures in the mathematics community.

Good luck, and happy integrating!

Competition Integrals

1 Professors-Battle

Submitted by

1. $\int \frac{x^4-16}{x^2+4x+8}\,\mathrm{d}x = \frac{x^3-6x^2+24x}{3} - 40\arctan\left(\frac{2x+4}{4}\right)$ — Brian Hawthorn/BriTheMathGuy
2. $\int \sqrt{1+e^{2x}}\,\mathrm{d}x = \sqrt{1+e^{2x}} - \operatorname{artanh}(\sqrt{1+e^{2x}})$ — Tevor Cheung/Mathemaniac
3. $I(\alpha) = \int_0^\infty (1 - e^{-\frac{\alpha}{x^2}})\,\mathrm{d}x = \sqrt{\pi\alpha} : \alpha \geq 0$

2 Qualifier Test

Tie-Breakers

1. $\int_0^\infty \mathrm{d}x\ \mathrm{e}^{-\cosh^2(x)}\cosh(x) = \frac{\sqrt{\pi}}{2\mathrm{e}}$

3 Quarter-Final

QF 1

1. $\int \frac{x^3}{(x^2+a^2)^{3/2}}\,\mathrm{d}x = \frac{x^2+2a^2}{(x^2+a^2)^{1/2}}$
2. $\int_0^1 \ln^2(x^2)\,\mathrm{d}x = 8$

QF 2

3. $\int_1^4 \frac{1}{\sqrt{x\sqrt{x}-x}}\,\mathrm{d}x = 4$
4. $\int_{\ln(\tan(1))}^\infty \frac{1}{(e^x+e^{-x})\cdot\arctan(e^x)}\,\mathrm{d}x = \ln\left(\frac{\pi}{2}\right)$

QF 3

5. $\int_0^\pi \frac{\sin(x)\sin(2x)}{1-\cos(2x)}\,\mathrm{d}x = 0$

Tie-Breakers

6. $\int_e^{e^n} \left(\frac{1}{\ln(x)} + \ln(\ln(x))\right)\mathrm{d}x = e^n\ln(n)$ for $n > 1$

4 Semi-Final

SF 1

1. $\int_0^{\frac{\pi}{2}} \frac{1}{1+\tan(x)^{2024}}\,\mathrm{d}x = \frac{\pi}{4}$
2. $\int \frac{1}{x(x^2+a^2)}\,\mathrm{d}x = \frac{1}{2a^2} ln \frac{x^2}{x^2+a^2} \; : a \neq 0$

SF 2

3. $\int \frac{e^{ax}-1}{e^{ax}+1}\,\mathrm{d}x = \frac{2}{a} ln(e^{ax}+1) - x = \ln\left(\tanh\left(\frac{ax}{2}\right)\right) : a \neq 0$
4. $\int_0^1 x \arctan(x)\,\mathrm{d}x = \frac{\pi-2}{4}$
5. $\int \frac{1}{x^2(x+a)^2}\,\mathrm{d}x = \frac{2\ln(x+a)-2\ln(x)}{a^3} - \frac{1}{a^2(x+a)} - \frac{1}{a^2 x} : a \neq 0$

5 Final

Submitted by

1. $\int_0^{\pi} \ln(\sin(x))\,\mathrm{d}x = -\pi \ln(2)$ — Tevor Cheung/Mathemaniac
2. $\int_1^2 \frac{\arcsin(\ln(x))}{x}\,\mathrm{d}x = -1 + \sqrt{1-\ln^2(2)} + \ln(2)\arcsin(\ln(2))$
3. $\int \frac{3\sin(x)+4}{(3+4\sin(x))^2}\,\mathrm{d}x = -\frac{1}{\frac{1}{\cos(x)}+4\tan(x)}$ — Presh Talwaker/MindYourDecisions
4. $\int_0^{\pi} \mathrm{e}^x \sin(x)^3\,\mathrm{d}x = \frac{3(e^{\pi}+1)}{10}$
5. $\int_0^{\infty} \frac{\ln(x)}{x^2+1}\,\mathrm{d}x = 0$ — Tevor Cheung/Mathemaniac