Heidelberg Integration Bee

This document contains a collection of integrals from this years Heidelberg Integration Bee. By working through these problems, you will strengthen your integration skills and gain insight into the types of challenges the participants encountered.

The integrals are categorized by competition stages, including the qualifier test, quarter-finals, semi-finals, and finals. Some problems were contributed by well-known figures in the mathematics community.

Happy integrating!

Competition Integrals

1 Professors-Duel

Submitted by

1. $$\int_0^1 \left(\sum_{k=1}^{999} \frac{1}{x - e^{2\pi i k/1000}}\right)\left(\prod_{n=1}^{999} (x - e^{2\pi i n/1000})\right) \mathrm{dx} = 999$$ Noahexplainsphysics
2. $$q := e^{2\pi i z}, \int_0^1 e^{-2\pi i z}\, q \prod_{n=1}^{\infty} (1 - q^n)^{24}\, \mathrm{dx} = 0$$
3. $$\int_{-\infty}^{\infty} \frac{\cos x}{1 + x^2}\, \mathrm{dx} = \frac{\pi}{e}$$
4. $$\int e^{ik\cdot r} \frac{4\pi}{k^2 + (\alpha m)^2}\, \mathrm{d}^3\mathrm{k} = \frac{4\pi^3}{r} e^{-\alpha m r} \quad \text{mit} \quad r = |r|$$
5. $$\int_{-2}^{2} \frac{1}{x^6 + 1}\, \mathrm{dx} = \frac{\ln\left(5 + 2\sqrt{3}\right) - \ln\left(5 - 2\sqrt{3}\right)}{2\sqrt{3}} + \frac{\arctan\left(\sqrt{3} + 4\right) - \arctan\left(\sqrt{3} - 4\right) + 2\arctan(2)}{3}$$

2 Audience integral

1. $$\int_{-\arctan \sqrt[3]{3B}}^{\arctan \sqrt[3]{3B}} iH \frac{\tan^2 x}{\cos^2 x}\, \mathrm{dx} = 2\ \mathrm{HiB}$$

3 Qualifier-Test

1. $$\int_0^{\infty} x^n \cdot e^{-x}\, \mathrm{dx} = n!$$
2. $$\int \frac{3x}{(1 + 9x^2)^4}\, \mathrm{dx} = -\frac{1}{18} \frac{1}{(1 + 9x^2)^3}$$
3. $$\int x^{\frac{x}{\ln x}}\, \mathrm{dx} = e^x$$

4. $\int (8x-12)(4x^2-12x)^4 \,\mathrm{dx} = \frac{1}{5}(4x^2-12x)^5$

5. $\int 3x^{-4}(2+4x^{-3})^{-7} \,\mathrm{dx} = \frac{1}{24}(2+4x^{-3})^{-6}$

6. $\int (3-4x)(4x^2-6x+7)^{10} \,\mathrm{dx} = -\frac{1}{22}(4x^2-6x+7)^{11}$

7. $\int \frac{3x}{1+9x^2} \,\mathrm{dx} = \frac{1}{6}\ln\left(\left|1+9x^2\right|\right)$

8. $\int_{-1}^{1} \frac{1}{x}\sqrt{\frac{1+x}{1-x}} \ln\left(\frac{2x^2+2x+1}{2x^2-2x+1}\right) \mathrm{dx} = 4\pi \operatorname{arccot}(\sqrt{\phi})$

9. $\int_{0}^{\infty} \frac{\ln x}{1+x^2} \,\mathrm{dx} = 0$

10. $\int_{\frac{\pi}{2}}^{0} \frac{1+\cos(2x)}{2} \,\mathrm{dx} = -\frac{\pi}{4}$

11. $\int_{\frac{1}{2}}^{1} \left(\frac{1}{x^3} - \frac{1}{x^4}\right) \mathrm{dx} = -\frac{5}{6}$

12. $\int_{9}^{4} \frac{1-\sqrt{x}}{\sqrt{x}} \,\mathrm{dx} = 3$

13. $\int_{0}^{\ln \pi} e^{2x} \cos\left(e^x\right) \mathrm{dx} = -\sin(1) - \cos(1) - 1$

14. $\int_{-1}^{1} \sqrt{4-x^2} \,\mathrm{dx} = \sqrt{3} + \frac{2\pi}{3}$

15. $\int \frac{3}{1+9x^2} \,\mathrm{dx} = \arctan(3x)$

Tie-Breakers

16. $\int x^2 e^x \,\mathrm{dx} = \left(x^2-2x+2\right) \mathrm{e}^x$

17. $\int e^x \sin(x) \,\mathrm{dx} = \frac{e^x(\sin(x)-\cos(x))}{2}$

4 Quarter-Final

Submitted by

QF 1

1. $\displaystyle\int \tan\left(\arcsin(x)\right)\,\mathrm{dx} = -\sqrt{1-x^2}$

2. $\displaystyle\lim_{n\to 0}\int_1^2 \frac{x^n-1}{n}\,\mathrm{dx} = \ln(4)-1$

3. $\displaystyle\int_0^{2\pi} -\frac{1}{2}\sin(\omega t)\sin\left(\frac{\pi}{2}-\frac{\pi}{4}\sin(\omega t)\right) - \frac{3\pi}{8}\cos(\omega t)\cos\left(\frac{\pi}{2}-\frac{\pi}{4}\sin(\omega t)\right) - \frac{\pi}{8}\cos^2(\omega t)\cos\left(\frac{\pi}{2}-\frac{\pi}{4}\sin(\omega t)\right)\,\mathrm{dt} = 0$

Tie-Breakers

4. $\displaystyle a<0,\ b^2-ac>0,\ \int \frac{1}{\sqrt{ax^2+2bx+c}}\,\mathrm{dx} = -\frac{1}{\sqrt{-a}}\arcsin\left(\frac{ax+b}{\sqrt{b^2-ac}}\right)$

5. $\displaystyle b^2>4ac,\ \int \frac{1}{a+bx+cx^2}\,\mathrm{dx} = \frac{1}{\sqrt{b^2-4ac}}\left[\ln\left(x-\frac{-b+\sqrt{b^2-4ac}}{2c}\right)-\ln\left(x-\frac{-b-\sqrt{b^2-4ac}}{2c}\right)\right]$

6. $\displaystyle\int \frac{x^2+2025}{x^3+2x}\,\mathrm{dx} = \frac{2025\ln(x)}{2}-\frac{2023}{4}\ln(x^2+2)$

QF 2

7. $\displaystyle\int_0^b (e^x-e)\,\mathrm{dx} = -1,\ b=1$ — Magda liebt Mathe

8. $\displaystyle\int_0^\infty x^5e^{-2x}\,\mathrm{dx} = \frac{5!}{2^5} = \frac{15}{8}$

9. $\displaystyle\lim_{n\to\infty}\int_n^{n+1} \frac{x^{\frac{1}{x}}}{x} - x^{\frac{1}{x}}\,\mathrm{dx} = -1$

QF 3

10. $\displaystyle\int_0^1 x^2\left(1-x\right)^{2024}\,\mathrm{dx} = \frac{2}{2025\cdot 2026\cdot 2027}$

11. $\displaystyle\left\lceil 2\cdot\int_0^1 \frac{\tan(x)}{\Gamma(x+1)}\,\mathrm{dx}\right\rceil = 2$

12. $\displaystyle\int_0^1 \frac{x^2}{1+x^2}\,\mathrm{dx} = 1 - \frac{\pi}{4}$

5 Semi-Final

Submitted by

SF 1

1. $\displaystyle\int_0^{2\pi} \frac{1}{2+\cos(x)}\,\mathrm{dx} = \frac{2\pi}{\sqrt{3}}$

2. $\displaystyle\int_0^{\pi} \sin\left(\int_1^{\frac{1+\sqrt{5}}{2}} 2x-1\,\mathrm{dx}\right)(x)\,\mathrm{dx} = 2$

3. $\displaystyle\int_1^2 x^4(\ln x)^2\,\mathrm{dx} = \frac{2}{5}\left(16\ln^2(2) - \frac{32}{5}\ln(2) + \frac{31}{25}\right)$

4. $\displaystyle\int_{-2}^{2} \left(x^3\cos\left(\frac{x}{2}\right)+\frac{1}{2}\right)\sqrt{4-x^2}\,\mathrm{dx} = \pi$

5. $\displaystyle\int_{-1}^{1} \ln\left(x+\sqrt{1+x^2}\right)\mathrm{dx} = 0$

Tie-Breakers

6. $\displaystyle\int \frac{1}{\sqrt{\cosh x - 1}}\,\mathrm{dx} = \sqrt{2}\ln\left(\tanh\left(\frac{x}{4}\right)\right)$

7. $\displaystyle\int \frac{x^2}{(x^2+1)^2}\,\mathrm{dx} = \frac{\arctan(x)}{2} - \frac{x}{2x^2+2}$

SF 2

8. $\displaystyle\int_0^{\frac{\pi}{2}} \ln(\sin(x))\,\mathrm{dx} = -\frac{\pi}{2}\ln(2)$ Dr. Peyam

9. $\displaystyle\int_0^{\infty} \exp\left\{\left(\frac{1-x}{2}\right)\right\}\sqrt{x}\,\mathrm{dx} = \sqrt{2\pi e}$

10. $\displaystyle a>0,\ \int_0^{a} \cos\left(\frac{x}{5}\right)\sin\left(\frac{x}{21}\right)\sin\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)\mathrm{dx} = 0,\ a = 210\,\pi$

11. $\int (\sec x)^3 \, \mathrm{dx} = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x|$

6 Final

Submitted by

1. $\int\limits_{-\infty}^{\infty} \frac{e^{-2\pi i x y}}{\cosh(\pi x)} \, \mathrm{dx} = \frac{1}{\cosh \pi y}$ — Richard Borcherds

2. $n >= 0, \; \int\limits_{0}^{\infty} \sum_{m=0}^{\infty} \sum_{p=0}^{\infty} \frac{i^{p+2n} \pi^p}{n!m!p!} \; x^{n+m} \; (n+m)^p \; \mathrm{d}x = 1$ — Richard Behiel

3. $\int \cos\left(2 \tan^{-1}\left(\frac{3}{x+1}\right)\right) \mathrm{dx} = x + 6 \tan^{-1}\left(\frac{3}{x+1}\right)$ — Blackpenredpen

4. $\int \frac{x \sec^2(x^2)}{\sqrt{\tan^3(x^2)}} \, \mathrm{dx} = -\frac{1}{\sqrt{\tan(x^2)}}$

5. $\int \sqrt{\frac{1}{1+\tan x} + \frac{1}{1+\sin x} + \frac{1}{1+\cos x} + \frac{1}{1+\csc x} + \frac{1}{1+\sec x} + \frac{1}{1+\cot x}} \mathrm{dx} = \sqrt{3} x$