Problem 1

Magnet in a conducting tube

Philipp Heinen – *Ruprecht-Karls-Universität Heidelberg*

Background A setup that is often used in high school and undergraduate university classes for demonstrating the effect of electromagnetic induction is the "magnet in a tube" experiment: A permanent magnet falls through a thin tube made of a conducting but not ferromagnetic material (typically copper). The fall of the magnet turns out to be surprisingly slowed down, as if it were falling through water instead of air. While this effect is usually explained qualitatively as a consequence of Lenz's law, this problem aims at a precise quantitative description of the phenomenon.

Consider a vertically aligned tube (i.e. aligned along the z-direction) made of a conducting but not ferromagnetic material with specific conductance σ. The radius of the tube be given by R and the thickness of its wall by ϵ with $\epsilon \ll R$. The length of the tube may be much larger than R such that it can be assumed to be infinitely extended.

We now place a permanent magnet of mass m exactly in the center of the tube. Let us assume that we can model the permanent magnet as a perfect magnetic dipole with magnetic moment $\boldsymbol{\mu}$. We take $\boldsymbol{\mu}$ to be aligned along the z-direction, $\boldsymbol{\mu} = \mu\,\mathbf{e}_z$, i.e. parallel to the tube. Upon the magnet acts a gravitational force $\mathbf{F}_{\mathrm{g}} = -mg\,\mathbf{e}_z$.

a) [*2 points*] What kind of motion do you expect the magnet to perform if it is let loose inside the tube? Why?

Can you guess by purely heuristic reasoning and/or dimensional analysis the functional dependence of the magnet's terminal velocity v_{term} on the parameters σ, ϵ, R, m, μ, g and the magnetic field constant μ_0?

b) [*8 points*] Now perform a rigorous calculation. Determine the position of the magnet in the tube at time t if its initial velocity is zero. Give an exact expression for the terminal velocity v_{term}. Neglect the self-inductance of the tube, as well as the effect of air resistance and the earth's magnetic field.

Useful formulas:

- *Magnetic field of a magnetic dipole placed at the origin:*

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{3\mathbf{r}(\boldsymbol{\mu}\cdot\mathbf{r}) - \boldsymbol{\mu}|\mathbf{r}|^2}{|\mathbf{r}|^5} \tag{1.1}$$

- *Force on a magnetic dipole in an external magnetic field* $\mathbf{B}_{\text{ext}}$*:*

$$\mathbf{F} = \nabla(\boldsymbol{\mu}\cdot\mathbf{B}_{\text{ext}}) \tag{1.2}$$

- *Ohm's law:*

$$\mathbf{j} = \sigma\mathbf{E} \tag{1.3}$$

- *Biot-Savart law:*

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int d^3r'\,\mathbf{j}(\mathbf{r}') \times \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} \tag{1.4}$$

- *Faraday's law*

$$U_{\text{ind}} = -\dot{\Phi} \tag{1.5}$$

- *Integral:*

$$\int_{-\infty}^{\infty} dx\, \frac{x^2}{(1+x^2)^5} = \frac{5}{128}\pi \tag{1.6}$$