Cavity optomechanics with polarizable particles: From atoms to dielectric objects

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Ultracold Quantum Matter
August 2025

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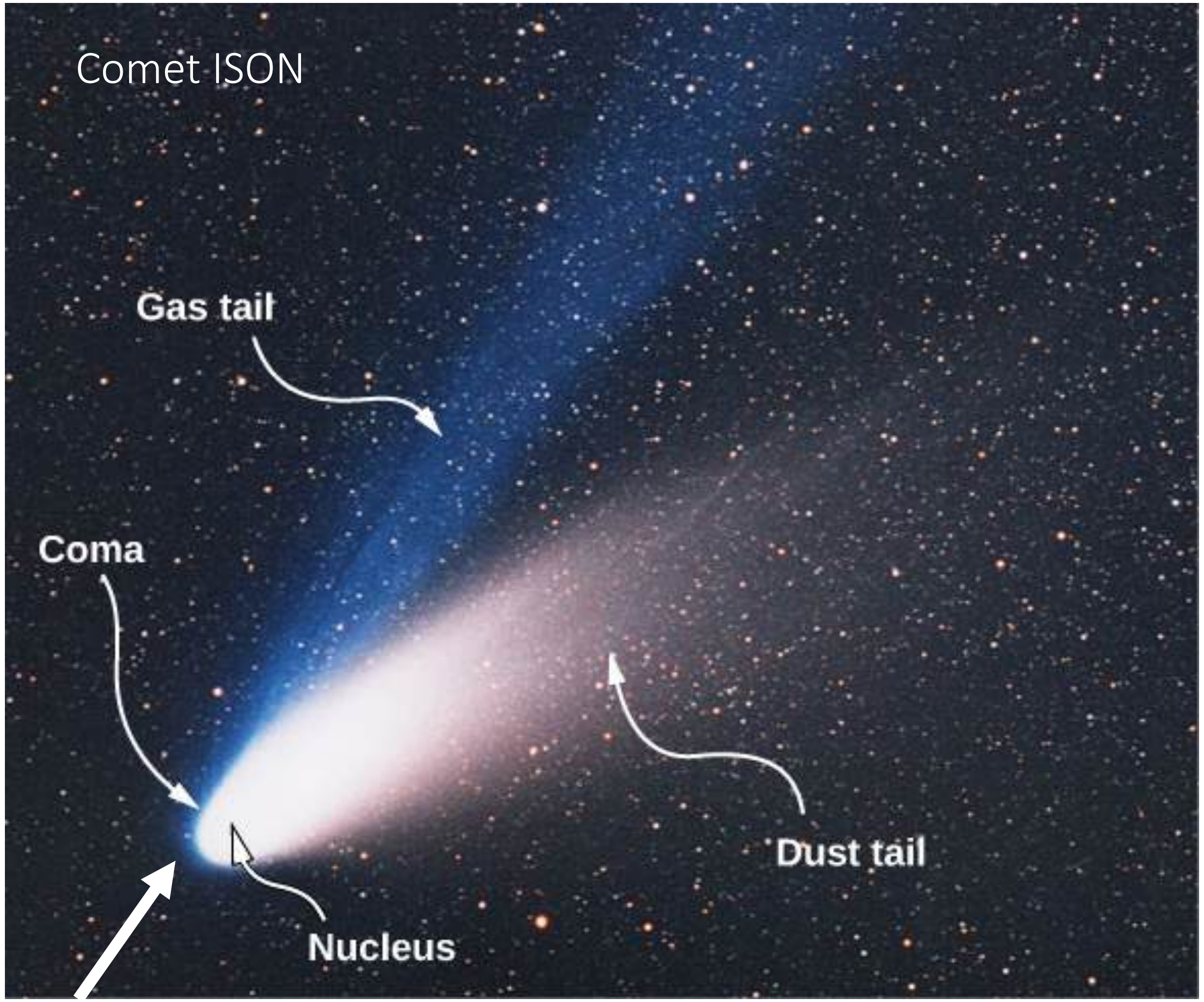
VCQ
Vienna Center for Quantum Science and Technology
quantA
wpi
Wolfgang Pauli Institute

1619: Force on the comet's tail



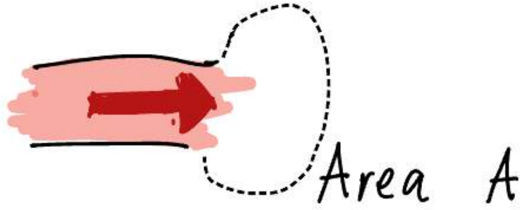
Sunlight direction

Johannes Kepler

Maxwell’s equations predict radiation pressure

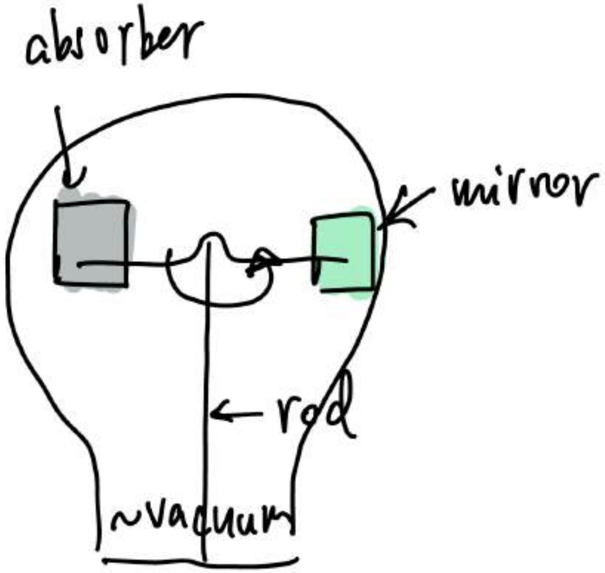
Poynting vector $\langle \vec{S} \rangle = \langle \vec{E} \times \vec{H} \rangle = \frac{\text{Power}}{\text{Area}} = \frac{\frac{d\,\text{Work}}{dt}}{A} = \frac{F}{A} \frac{dx}{dt}$, $\frac{dx}{dt} = c$

c: speed of light



Force $F = \frac{P}{c}$

Crookes radiometer



Early experiments: **It turns the wrong way!**

Issue: still bad vacuum, radiometric forces dominate

1900/1: Lebedew, then Nichols & Hull demonstrate radiation pressure force for the first time

1930s: Otto Frisch does experiments on Sodium atoms

1909

Physikalische Zeitschrift. 10. Jahrgang. No. 22. 817

A. Einstein (Zürich), Über die Entwicklung unserer Anschauungen über das Wesen und die Konstitution der Strahlung.

Als man erkannt hatte, daß das Licht die Erscheinungen der Interferenz und Beugung

den sei, und daß es auch im Innern der ponderabeln Körper im wesentlichen der Lichtäther sei, welcher die Ausbreitung des Lichtes vermittelt. Die Existenz jenes Lichtäthers schien unbezweifelbar. In dem 1902 erschienenen

“On the development of our views concerning the nature and constitution of radiation”

The only known “quantum light” at the time: Planck’s formula for the blackbody radiation spectrum

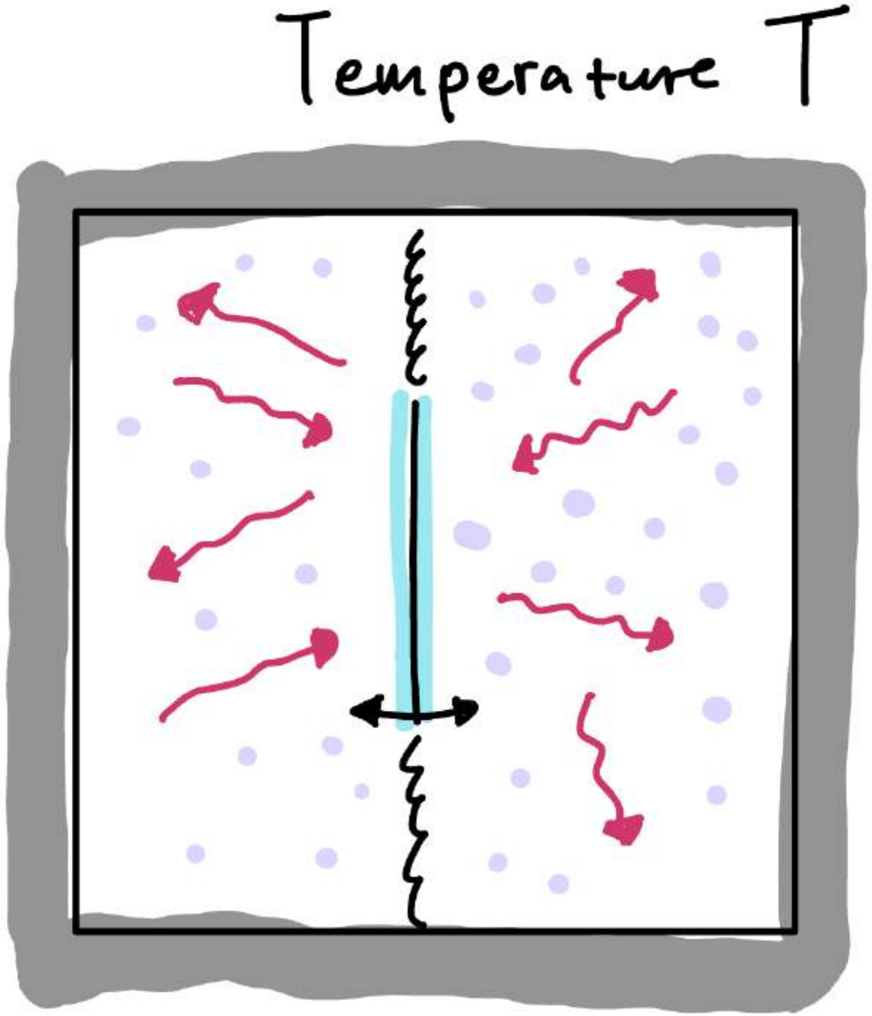
$$\varrho = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{e^{\frac{h\nu}{kT}} - 1}.$$

Energy density ρ at frequency ν

theorie. Wir sehen die Plancksche Strahlungsformel als richtig an und fragen uns, ob aus ihr etwas gefolgert werden kann bezüglich der Konstitution der Strahlung. Von zwei Betrach-

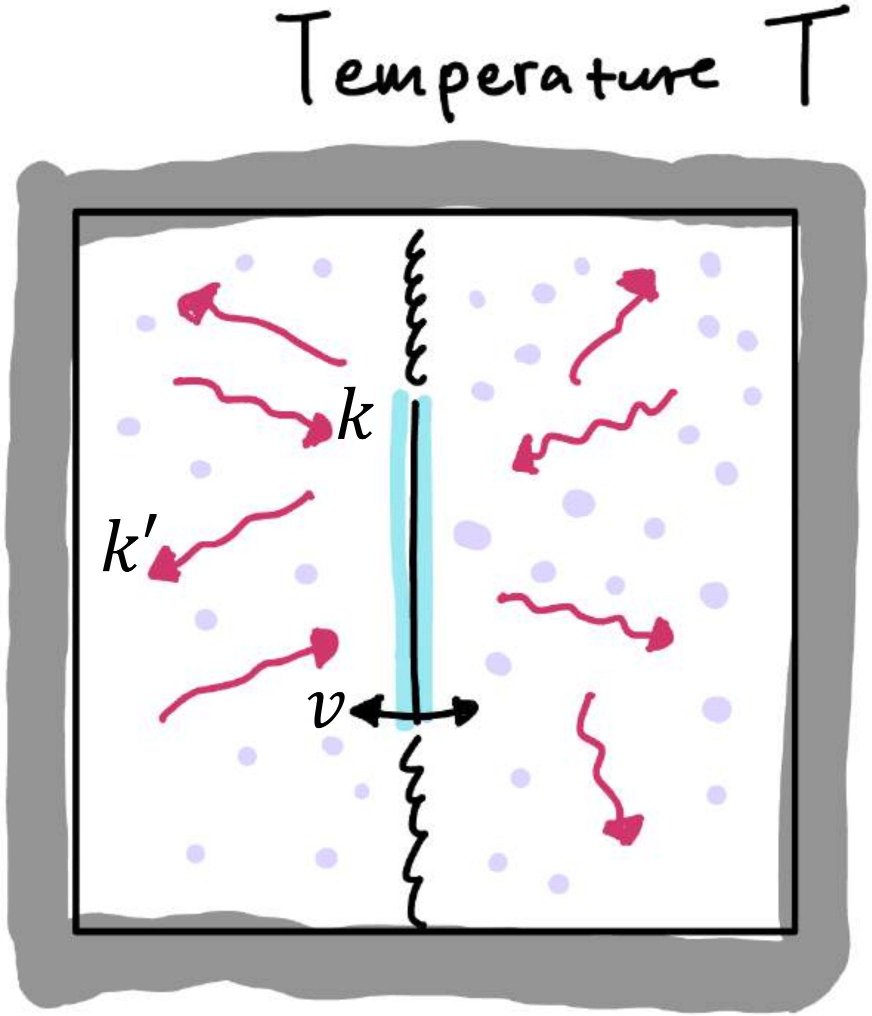
“We assume Planck‘s theory to be correct and ask ourselves if we can deduce something about the constitution of radiation.”

Gedankenexperiment



$$\varrho = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{e^{\frac{h\nu}{kT}} - 1} .$$

Gedankenexperiment



Doppler shift of the reflected wave:

$$k' = -k\left(1 - \frac{2v}{c}\right)$$

"Radiation friction" due to momentum transfer $k - k'$:

$$F = -\frac{2P}{c}\left(1 - \frac{2v}{c}\right)$$

with damping $\gamma = \frac{2P}{mc^2}$

"Doppler cooling" of mirror via radiation pressure

Thermal equilibrium?

Fluctuation-dissipation theorem: friction forces come with a related fluctuating force

→ Radiation pressure force fluctuates according to Planck's formula, causing momentum diffusion of the mirror

$$\overline{\Delta^2} = \frac{1}{c}\left[h\varrho\nu + \frac{c^3}{8\pi}\frac{\varrho^2}{\nu^2}\right] d\nu\, f\, \tau$$

"Quanta", or "atoms of radiation"

Interference of classical waves

Einstein's conclusions:

- Light has both wave and particle properties
- At low energy densities, the quantum part dominates!

Planck's comment:

"In any case, I think that first of all one should attempt to transfer the whole problem of the quantum theory to the area of interaction between matter and radiation energy."

Today: Quantum fluctuations in gravitational wave detectors

PHYSICAL REVIEW LETTERS

VOLUME 45 | 14 JULY 1980 | NUMBER 2

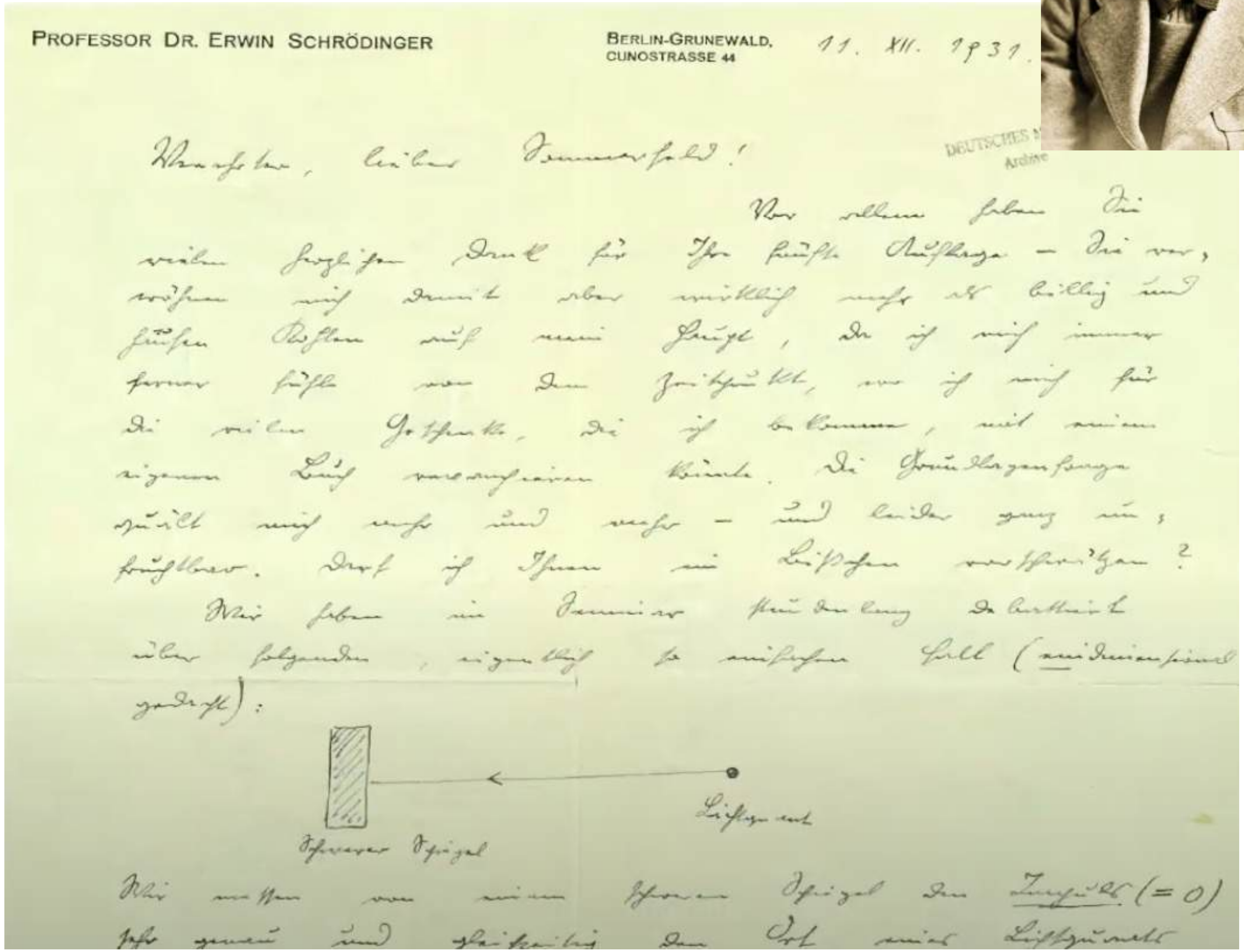
Quantum-Mechanical Radiation-Pressure Fluctuations in an Interferometer

Carlton M. Caves
W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125
(Received 29 January 1980)

The interferometers now being developed to detect gravitational vaves work by measuring small changes in the positions of free masses. There has been a controversy whether quantum-mechanical radiation-pressure fluctuations disturb this measurement. This Letter resolves the controversy: They do.

Quantum light ✓; Quantum motion?

"Our mirror is a universal measurement tool […] momentum and position of the photon are imprinted on the mirror, namely both are registered with accuracies, the product of which can be pushed way below the limit of h"


PROFESSOR DR. ERWIN SCHRÖDINGER

BERLIN-GRUNEWALD,
CUNOSTRASSE 44

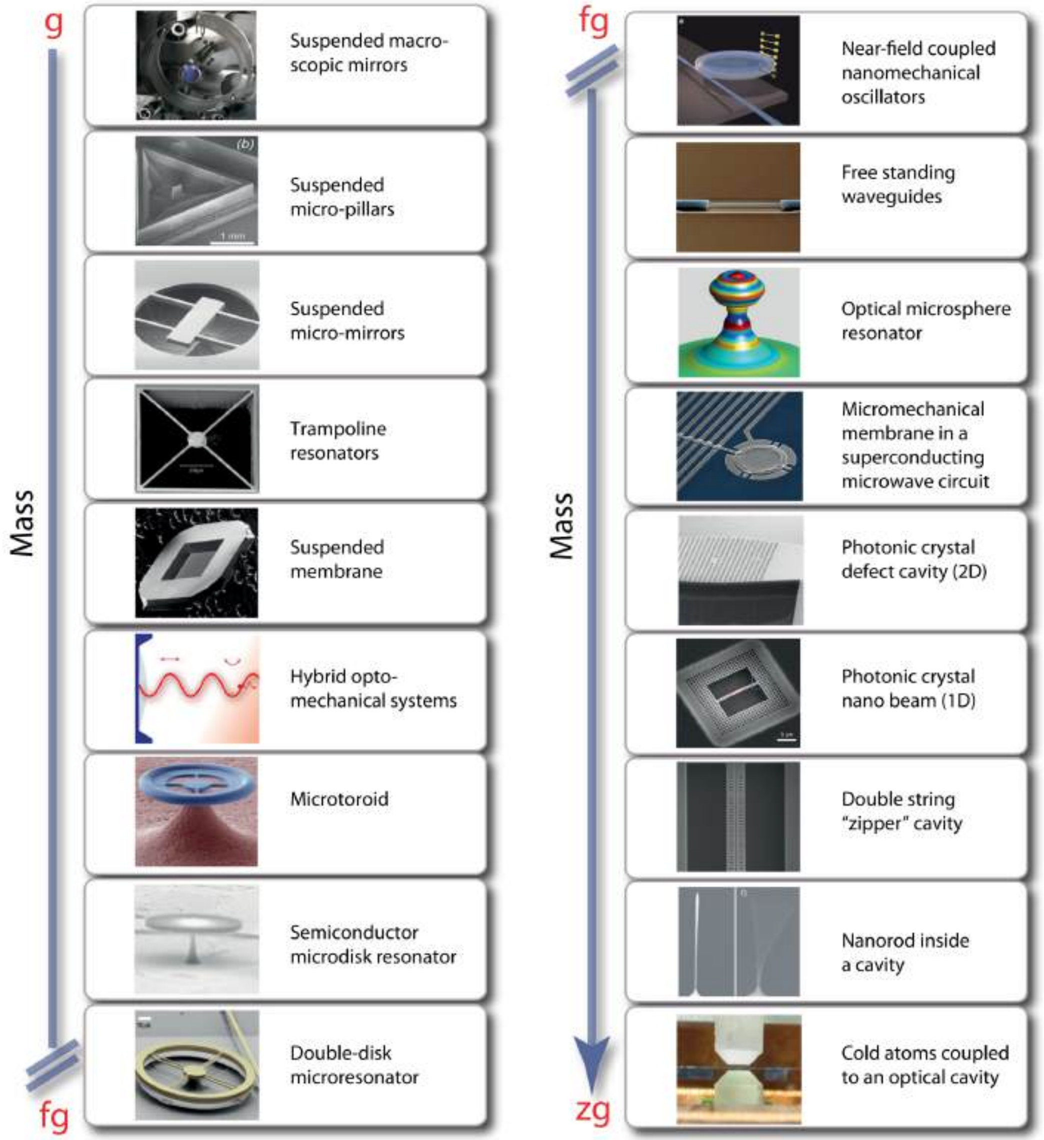
11. XII. 1931.

Optomechanical entanglement!

"Putting mechanics into quantum mechanics"

Keith Schwab and Michael Roukes, Physics Today 58, 36 (2005)

Everything moves! In a world dominated by electronic devices and instruments it is easy to forget that all measurements involve motion, whether it be the motion of electrons through a transistor, Cooper pairs or quasiparticles through a superconducting quantum interference device (SQUID), photons through an optical interferometer—or the simple displacement of a mechanical element. Nanoscience today is driving a resurgence of interest in mechanical devices, which have long been used as front ends for sensitive force detectors. Among prominent historical examples are Coulomb's mechanical torsion balance, which allowed him in 1785 to establish the inverse-square force law of electric charges, and Cavendish's mechanical instrument that allowed him in 1798 to measure the gravitational force between two lead spheres.



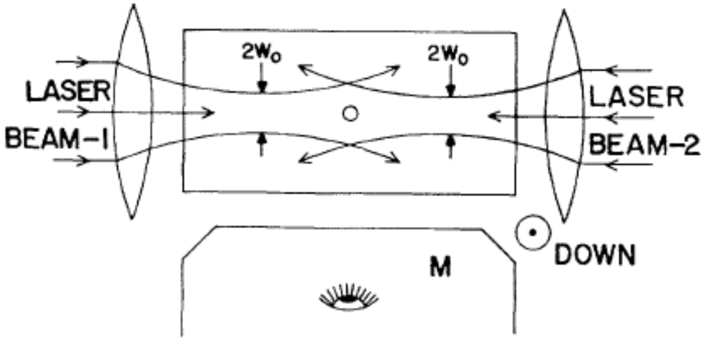
Aspelmeyer, Kippenberg, Marquardt, RMP 86, 1391 (2014)

VOLUME 24, NUMBER 4 PHYSICAL REVIEW LETTERS 26 JANUARY 1970

ACCELERATION AND TRAPPING OF PARTICLES BY RADIATION PRESSURE

A. Ashkin
Bell Telephone Laboratories, Holmdel, New Jersey 07733
(Received 3 December 1969)

This Letter reports the first observation of acceleration of freely suspended particles by the forces of radiation pressure from cw visible laser light. The experiments, performed on micron-sized particles in liquids and gas, have yielded new insights into the nature of radiation pressure and have led to the discovery of stable optical potential wells in which particles were trapped by radiation pressure alone. The ideas can be extended to atoms and molecules where one can predict that radiation pressure from tunable lasers will selectively accelerate, trap, or separate the atoms or molecules of gases because of their large effective cross sections at specific resonances. The author's interest in



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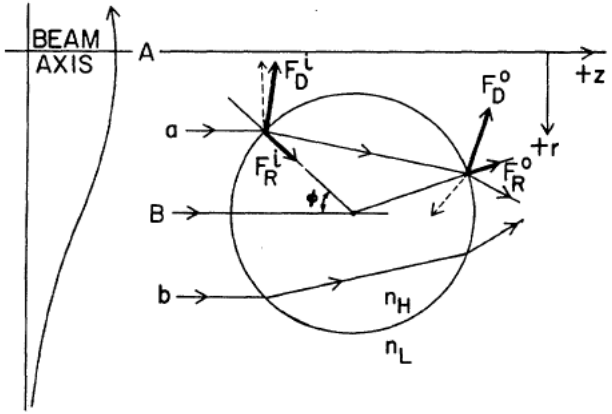

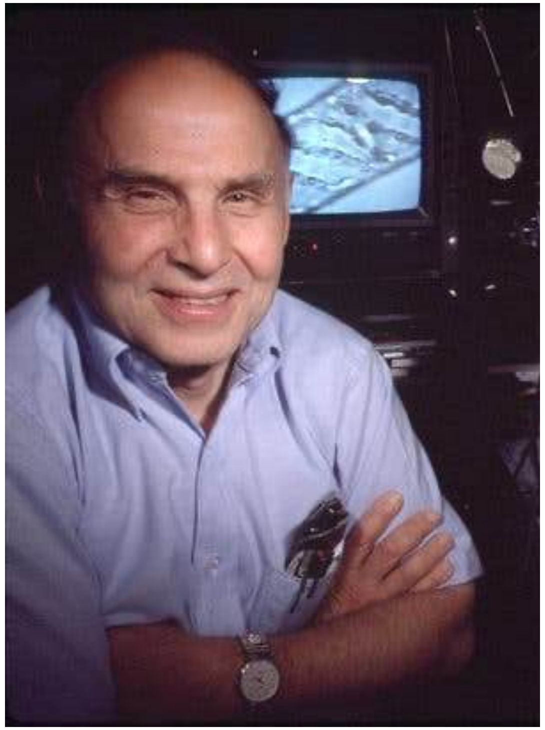

Volume 24, Number 4 PHYSICAL REVIEW LETTERS 26 January 1970

ACCELERATION AND TRAPPING OF PARTICLES BY RADIATION PRESSURE

A. Ashkin
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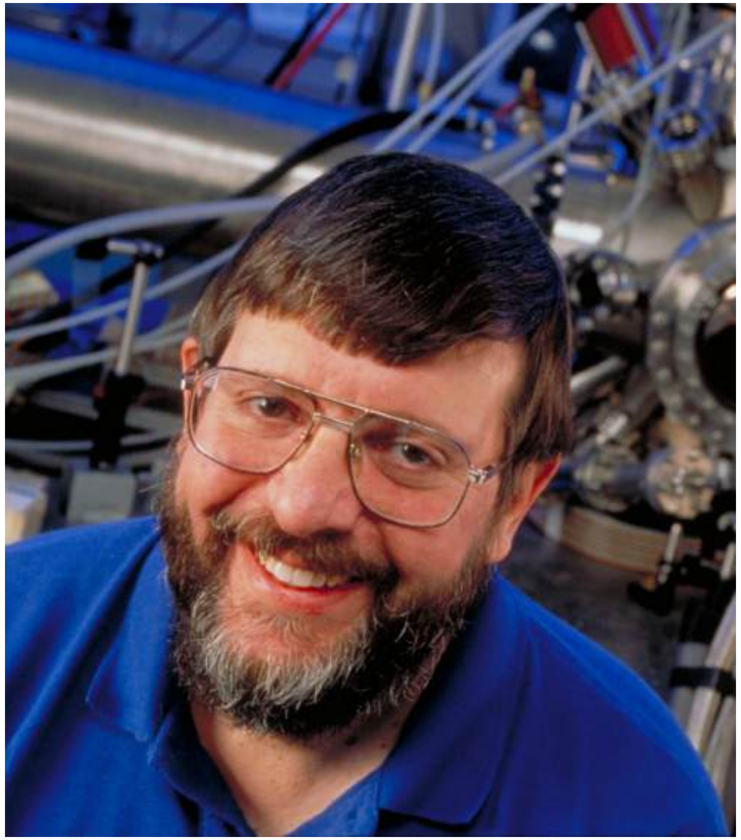
(Received 3 December 1969)

In 1978, while I was a postdoctoral fellow at MIT, I read a paper [1] by Art Ashkin in which he described how one might slow down an atomic beam of sodium using the radiation pressure of a laser beam tuned to an atomic resonance. After being slowed, the atoms would be captured in a trap consisting of focused laser beams, with the atomic motion being damped until the temperature of the atoms reached the microkelvin range. That paper was my first introduction to laser cooling, although the idea of laser cooling (the reduction of random thermal velocities using radiative forces) had been proposed three years earlier in independent papers by Hänsch and Schawlow [2] and Wineland and Dehmelt [3]. Although the treatment in Ashkin's paper was necessarily over-simplified, it provided one of the important inspirations for what I tried to accomplish for about the next decade. Another inspiration ap-



William D. Phillips

Optical dipole trap

Chu, Bjorkholm, Ashkin, Cable
Phys. Rev. Lett. 57, 314

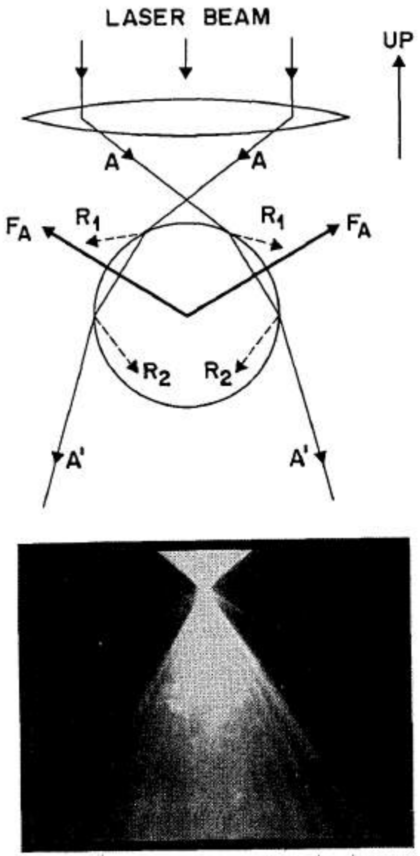
Ashkin, Dziedzic, Bjorkholm, Chu,
Optics Letters 11 (5), 288

1986: optical dipole trap for ∽500 Sodium atoms and micro-(10 µm) to nanoparticles (∽ 25nm)

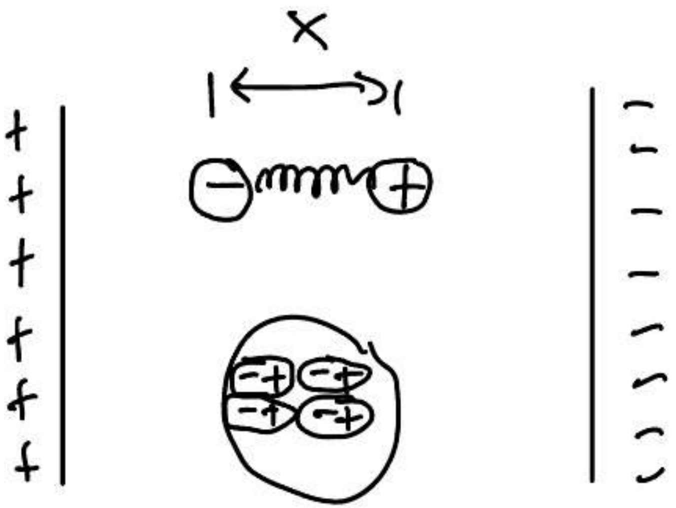
$$U_a = \frac{3\pi c^2}{2\omega_0^2}\frac{\Gamma}{\Delta} I(r)$$

$$U_p = -\frac{\alpha}{\varepsilon_0 c} I(r)$$

$$\alpha = 3\varepsilon_0 V \frac{n^2 - 1}{n^2 + 2}$$



Optical trapping of polarizable objects



Permanent dipole in a static electric field: $W = -\vec{p} \cdot \vec{E}$

Induced dipole in an **AC** electric field: $W_{die} = -\dfrac{\vec{p} \cdot \vec{E}}{2}$

with $\vec{p} = \alpha \cdot \vec{E}$

α: polarizability

→. Interaction energy depends on the light intensity

$$W_{die} = -\frac{\alpha}{2} |\vec{E}|^2 \qquad I_{light} = \frac{\varepsilon_0 \cdot c |\vec{E}|^2}{2}$$

$$E_{light} = E_0 e^{-i\omega t} \qquad \omega: \text{light frequency}$$

What is the polarizability of an atom?

Classical oscillator model (Lorentz model)

$$\ddot{x} + \kappa \dot{x} + \omega_0^2 x = \frac{eE(t)}{m}$$

- $\ddot{x}$: acceleration
- κ: damping; $\dot{x}$: velocity
- ω_0: frequency; x: position
- m: mass

e: electron charge

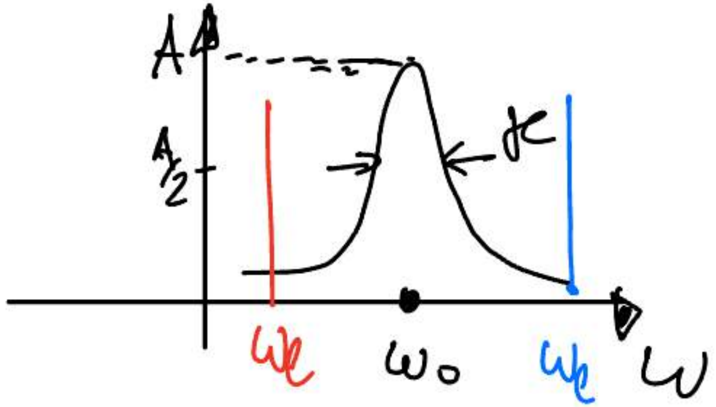
$$E(t) = E_0 e^{-i\omega_L t}$$

$$\Rightarrow x_0 = -\frac{e}{m}\frac{E_0}{\delta \cdot 2\omega_0 + i\kappa \cdot \omega_0} = -\frac{e}{2m\omega_0}\frac{E_0}{\delta + i\kappa/2}$$

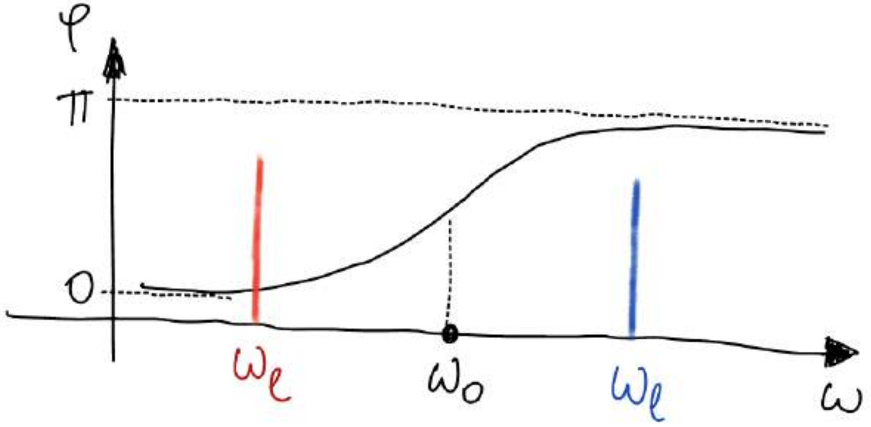
$\underbrace{\omega_L - \omega_0}_{\text{detuning}} = \delta$

Absorption profile

$$|x_0|^2 = \left(\frac{e}{2m\omega_0}\right)^2 \frac{E_0^2}{\delta^2 + (\kappa/2)^2} \leftarrow \text{Lorentzian}$$



Phase delay (also in the radiated field!)



What is the polarizability of an atom?

Classical oscillator model (Lorentz model)

$$\ddot{x} + \gamma_e \dot{x} + \omega_0^2 x = \frac{eE(t)}{m}$$

damping: γ_e; frequency: ω_0

$\ddot{x}$: acceleration; $\dot{x}$: velocity; x: position; m: mass

e: electron charge

$E(t) = E_0 e^{-i\omega_L t}$

$$\Rightarrow x_0 = -\frac{e}{m}\frac{E_0}{\delta \cdot 2\omega_0 + i\gamma_e \cdot \omega_0} = -\frac{e}{2m\omega_0}\frac{E_0}{\delta + i\gamma_e/2}$$

$$p = e\cdot x = e\cdot A\cdot e^{i\varphi}\cdot e^{-i\omega_L t}$$

Polarizability: $\alpha = \frac{p}{E} = \frac{e\cdot A}{E_0}\cdot e^{i\varphi} = \mathrm{Re}(\alpha) + i\,\mathrm{Im}(\alpha)$

Coherent scattering

$\mathrm{Re}(\alpha) \propto \frac{1}{\delta}$

Preserves the properties of the drive

Absorption, incoherent scattering

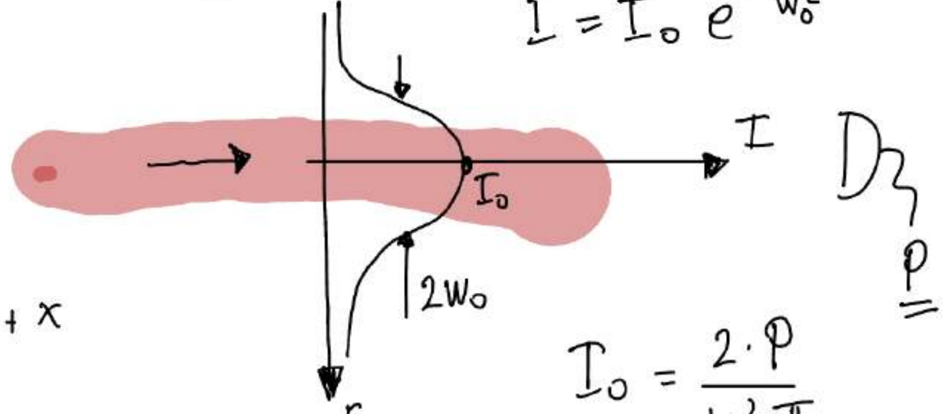
$\Gamma_{scatt} \propto \mathrm{Im}(\alpha) \propto \frac{1}{\delta^2}$

Trapping of atoms and nanoparticles

$$W = -\frac{1}{2}\,\vec{p}\cdot\vec{E} = -\frac{1}{2}\,\alpha\cdot\vec{E}^2$$

$$I = \frac{\varepsilon_0 c E^2}{2}$$

$$I = I_0\, e^{-\frac{2r^2}{w_0^2}}$$



$$I_0 = \frac{2\cdot P}{w_0^2\cdot\pi}$$

$$W(r) = -\frac{1}{2}\,\alpha(\delta)\,\frac{2\cdot I_0}{\varepsilon_0 c}\cdot e^{-\frac{2r^2}{w_0^2}}$$

$$\approx -\frac{1}{2}\,\alpha(\delta)\,\frac{2I_0}{\varepsilon_0 c} + \underbrace{\frac{1}{2}\,\alpha(\delta)\cdot\frac{2I_0}{\varepsilon_0 c}\cdot\frac{2r^2}{w_0^2}}_{\text{Potential energy for a harmonic oscillator}} \quad \Big|\; e^x \overset{x\ll 1}{\sim} 1 + x$$

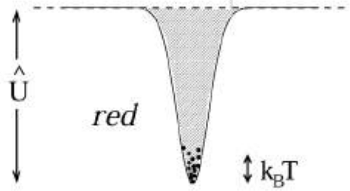
Potential energy for a harmonic oscillator

$$\Rightarrow \alpha(\delta)\,\frac{2I_0}{\varepsilon_0 c w_0^2}\,r^2 \equiv \frac{m\cdot\Omega^2\cdot r^2}{2}$$

Spring constant $k = m\cdot\Omega^2 = \frac{4I_0}{\varepsilon_0 c w_0^2}\cdot\mathrm{Re}(\alpha)$

Case #1: $\delta < 0$ (red-detuning)

$\Omega^2 > 0$

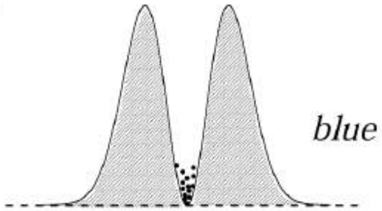


Case #2: $\delta > 0$ (blue-detuning)

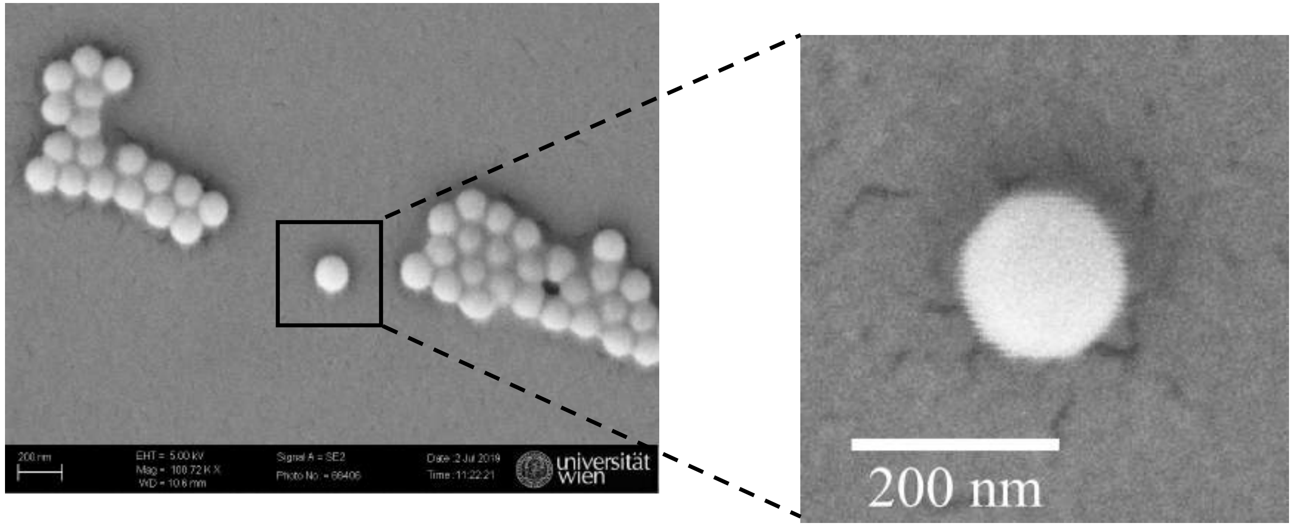
$\Omega^2 < 0$

$\Omega \in \mathbb{C}$

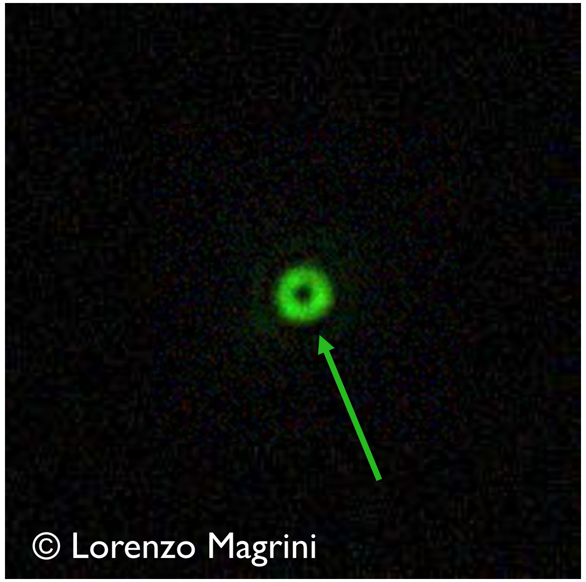
"anti-damping"



Dielectric nanoparticles: induced dipoles



2.8 fg – 10^9 amu



Radius ≪ wavelength: induced dipoles

Induced dipole moment → $\vec{p} = \alpha\,\vec{E}$ ← Electric field

↑ Polarizability $\alpha = 3 \cdot \varepsilon_0 \cdot V \cdot \frac{n^2-1}{n^2+2}$

$V = \frac{4}{3}\pi r^3$

$Re(\alpha) = 3\varepsilon_0 V\, Re\left(\frac{n^2-1}{n^2+2}\right)$

Note:

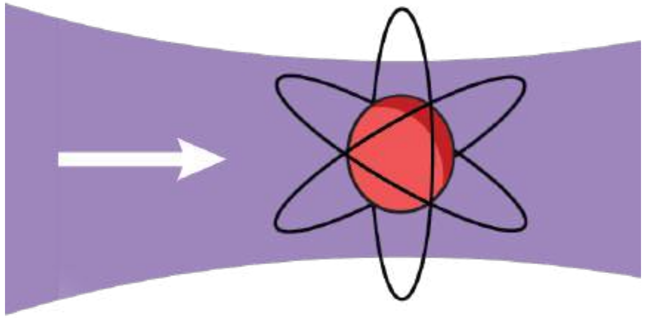
- Internal transitions are not explored
- Mechanical degrees of freedom
- Scattering up to 10^{14} photons/s

polarizability of a 71.5 nm radius particle = polarizability of a Cesium atom 50 MHz off-resonance

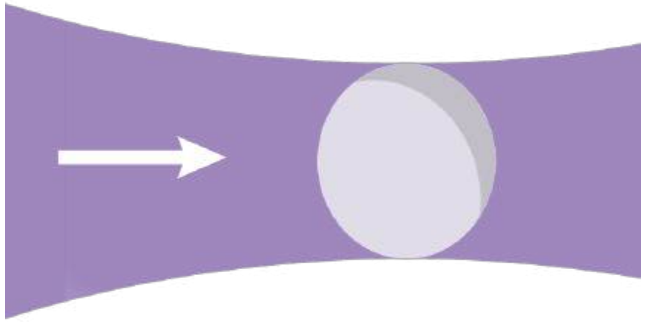
Proposals for cavity optomechanics with nanoparticles:

Chang et al., PNAS 107:1005–1010 (2010), Romero-Isart et al., NJP 12(3):033015 (2010), Barker, Shneider, PRA 81(2):023826 (2010)

Trapping of atoms vs. nanoparticles



Cs atom
D2 line: 852 nm



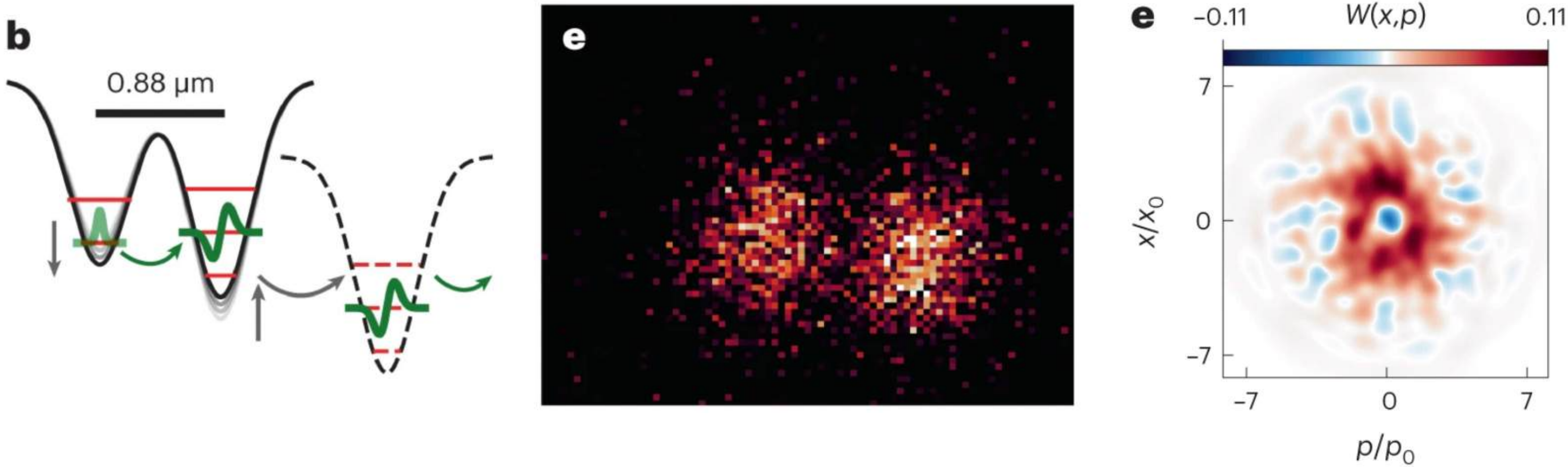
Silica nanoparticle
Radius $r = 71.5$ nm

Common parameters: trap wavelength 1064 nm and mechanical frequency of 100 kHz

Detuning to the D2 line: 1 GHz
Trap power: 1 mW
Scattering rate: 10^4 photons/s
Trap depth: 100 mK
Occupation in trap: <10 phonons
Ground-state wavefunction size: 100 nm
Cooling to the ground state, e.g., via Raman sideband cooling (Regal, Lukin, ...)

Detuning: -
Trap power: 100 mW
Scattering rate: 10^{14} photons/s
Trap depth: 1000s of K
Occupation in trap: 10^8 phonons
Ground-state wavefunction size: 10 pm
Cooling to the ground state?

Non-Gaussian states of atoms without transitions



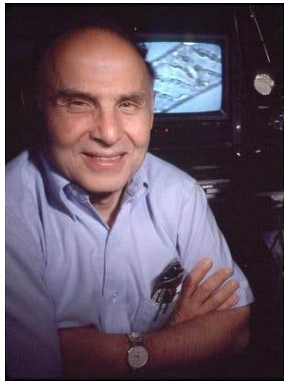
Ground-state wavefunction size of the **atom**: 100 nm
Ground-state wavefunction size of the **nanoparticle**: 10 pm

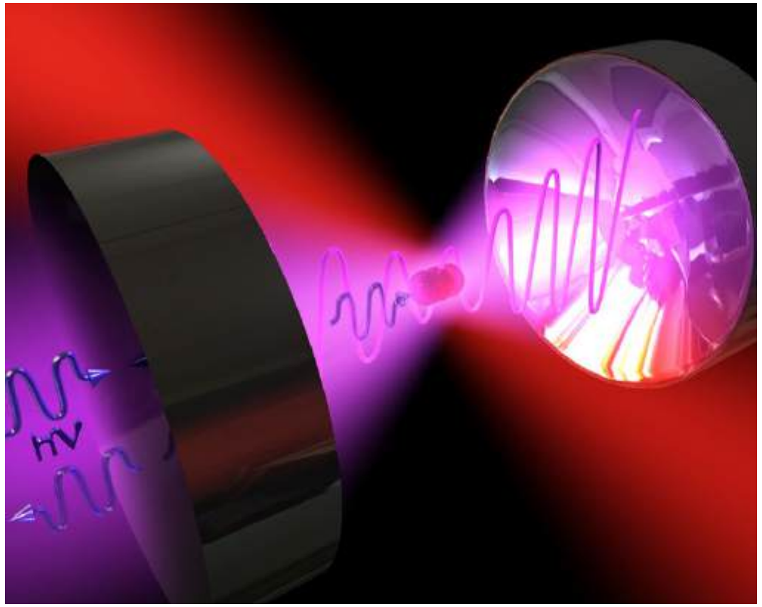
How do we create non-Gaussian states of motion of trapped nanoparticles?

Brown et al. (Regal group), Nature Physics 19, 569 (2023)

Quantum optomechanics with nanoparticles

The extension to vacuum of the present experiments on particle trapping in potential wells would be of interest since then any motions are frictionless.





Cavity opto-mechanics using an optically levitated nanosphere

PNAS, 107, 1005

D. E. Chang[a], C. A. Regal[b], S. B. Papp[b], D. J. Wilson[b], J. Ye[b,c], O. Painter[d], H. J. Kimble[b,1], and P. Zoller[b,e]

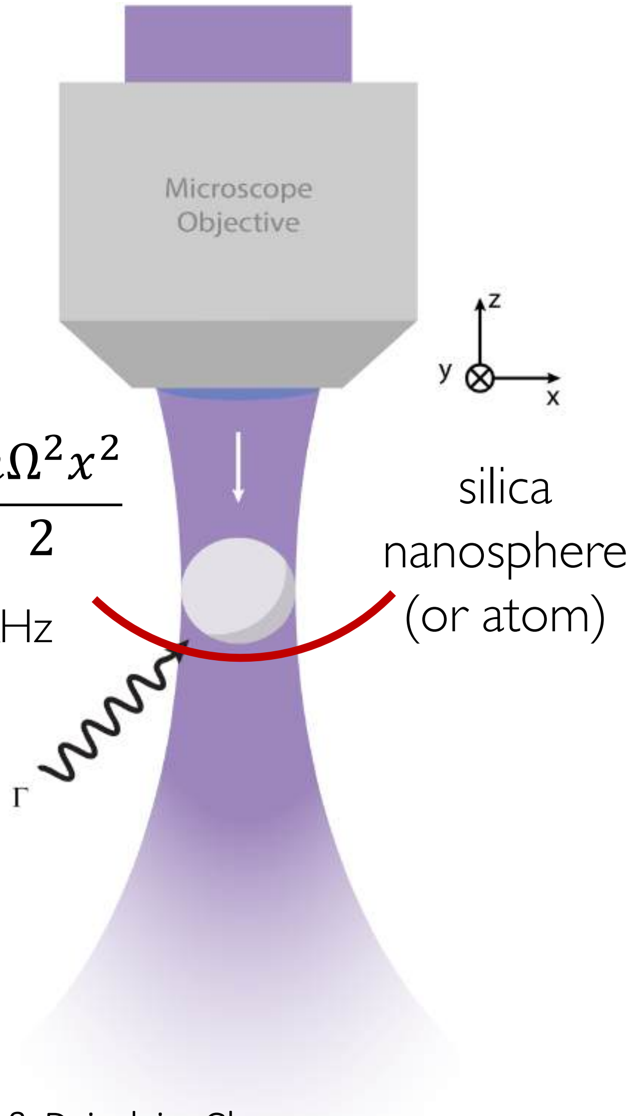
Toward quantum superposition of living organisms

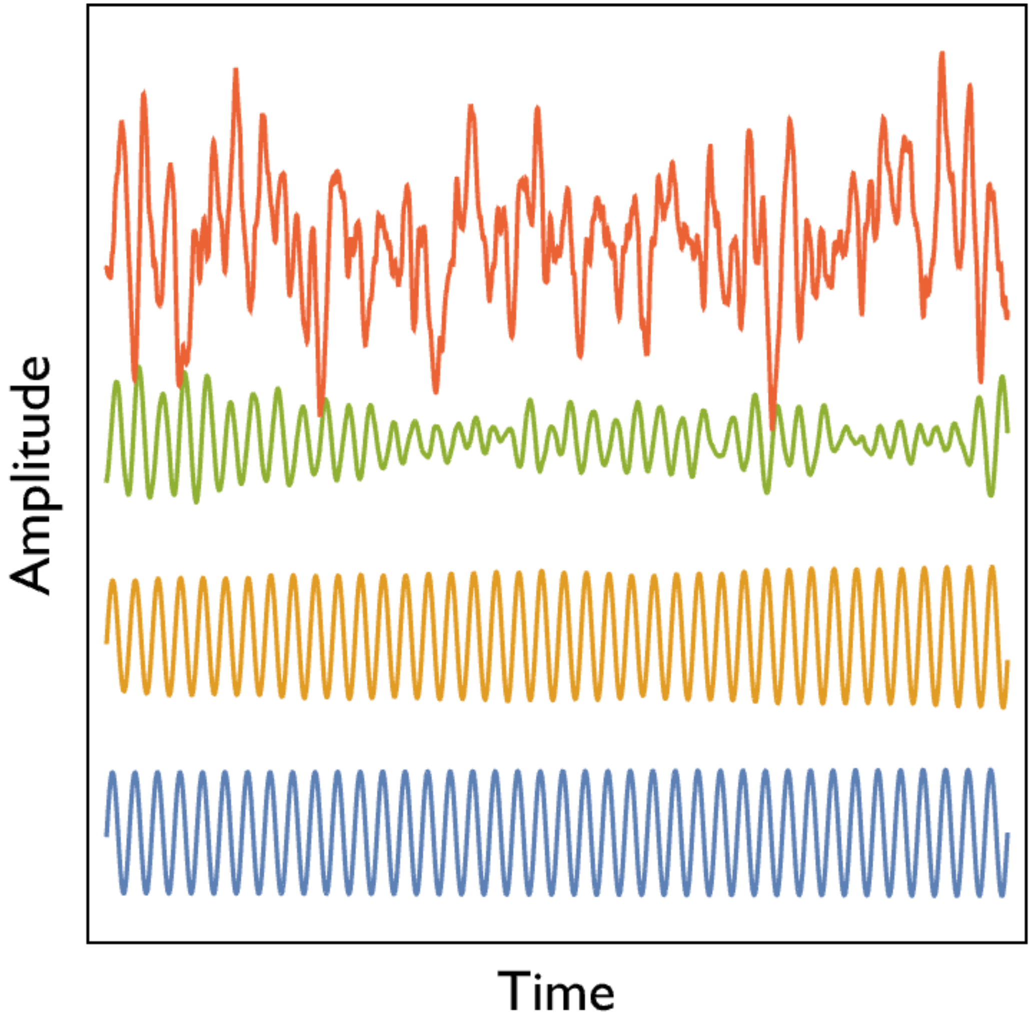
Oriol Romero-Isart[1,4], Mathieu L Juan[2], Romain Quidant[2,3] and J Ignacio Cirac[1]
New Journal of Physics 12, 033015

Optical trapping: High quality oscillator

$$H = -\frac{\alpha}{2} E_{tw}^2 \rightarrow \frac{m\Omega^2 x^2}{2}$$

$\Omega \sim 100$ kHz





$\sim 1, \gamma \sim 100$ kHz

Quality factor Ω/γ

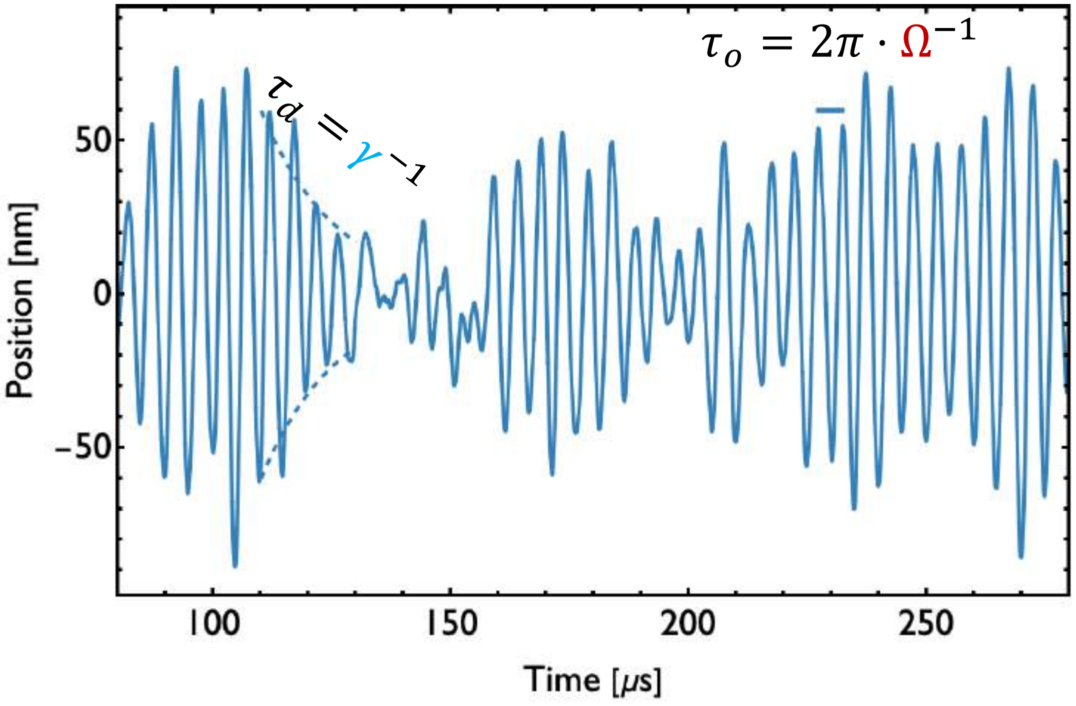
$\sim 10^8, \gamma \sim 1$ mHz

Early work: Ashkin & Dziedzic, Chu..
APL 28:333 (1971), APL 30:202 (1977), Optics Letters 11:288-290 (1986)

Thermal mechanical oscillator

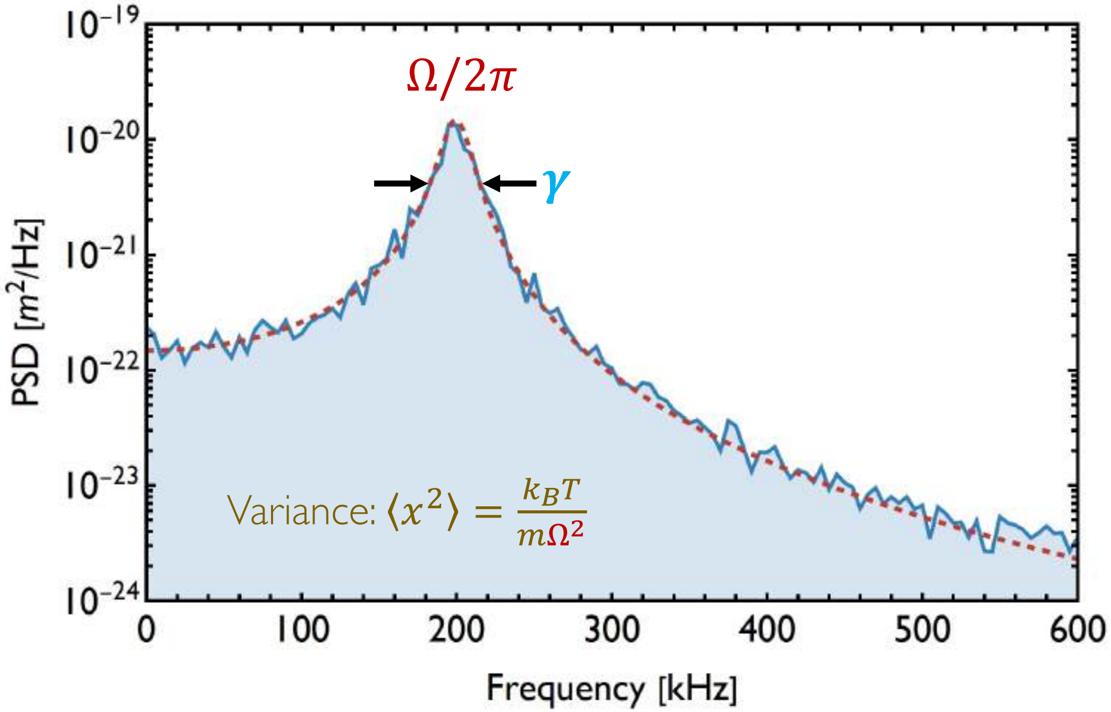
Time domain: position trace

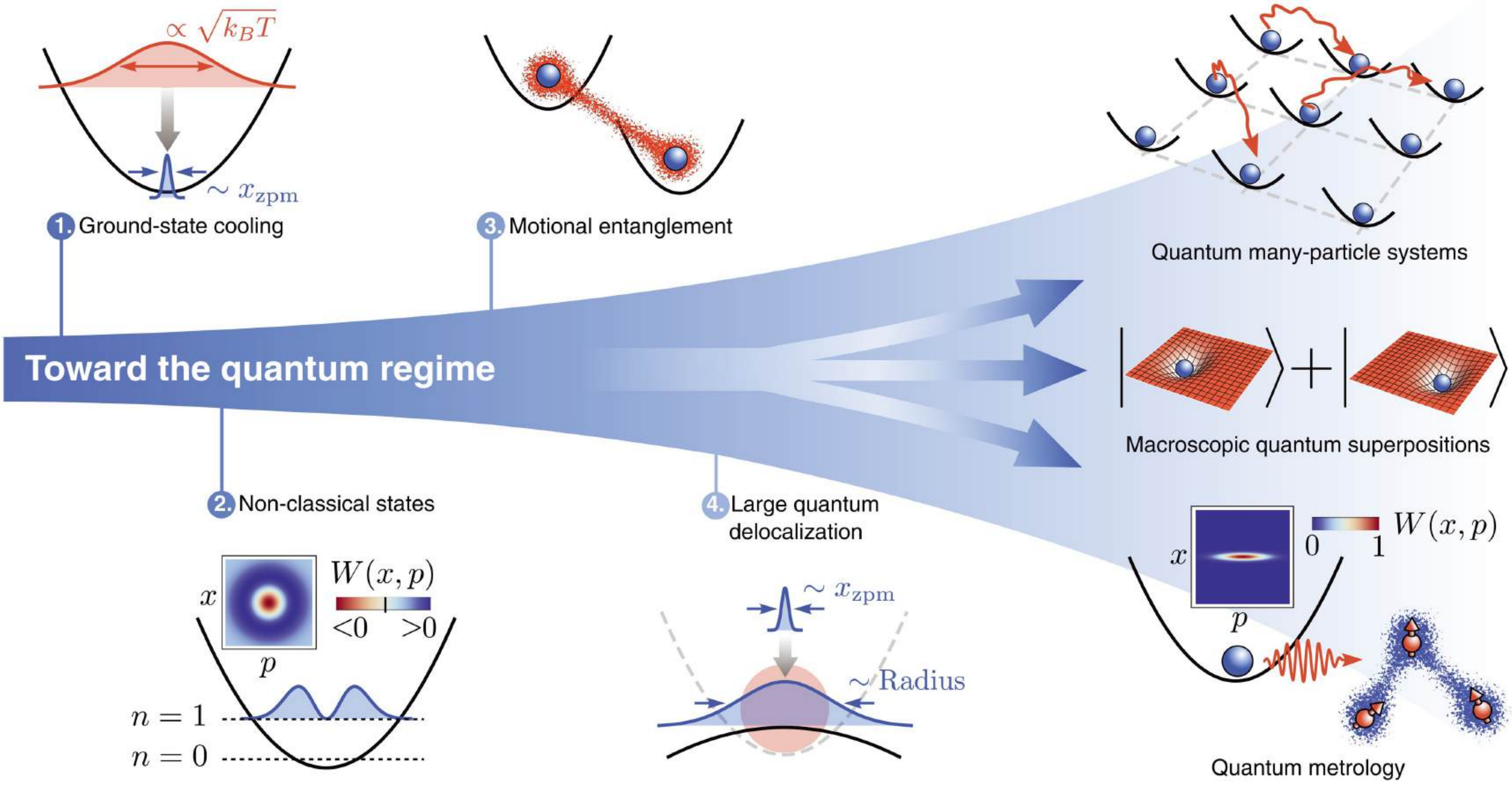
$$\ddot{x}(t) + \gamma\dot{x}(t) + \Omega^2 x(t) = \frac{1}{m}\sqrt{2mk_BT\gamma}\,\xi(t)$$ Thermal noise



Frequency domain: Power spectral density (PSD)

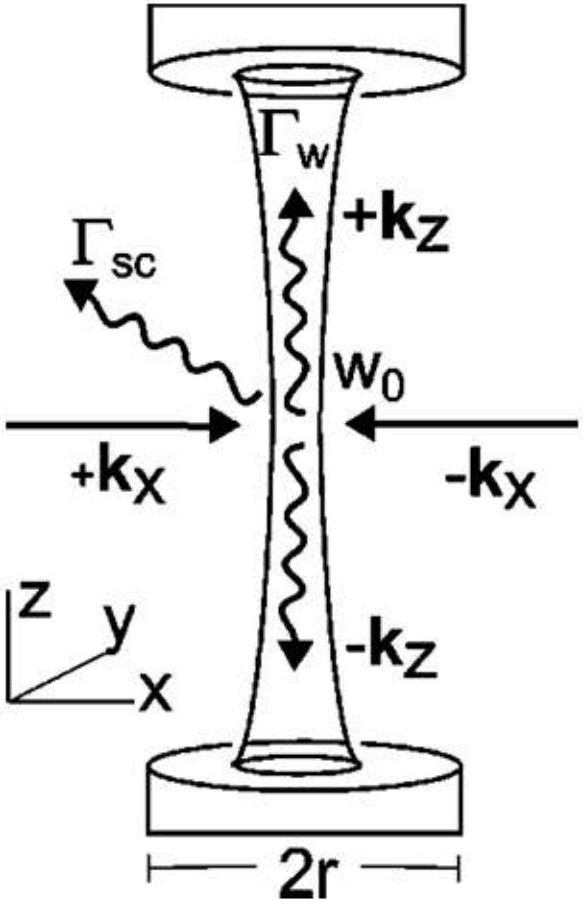
$$S_{xx}(\omega) = \frac{k_BT}{m\Omega^2}\frac{2\gamma\Omega^2}{(\Omega^2-\omega^2)^2+\gamma^2\omega^2}$$





Gonzalez-Ballestero et al, Science 374, eabg3027 (2021)

Cooling: cavity resonance as a transition



Vuletic, Chan, Black, Phys. Rev. A 64, 033405 (2001)

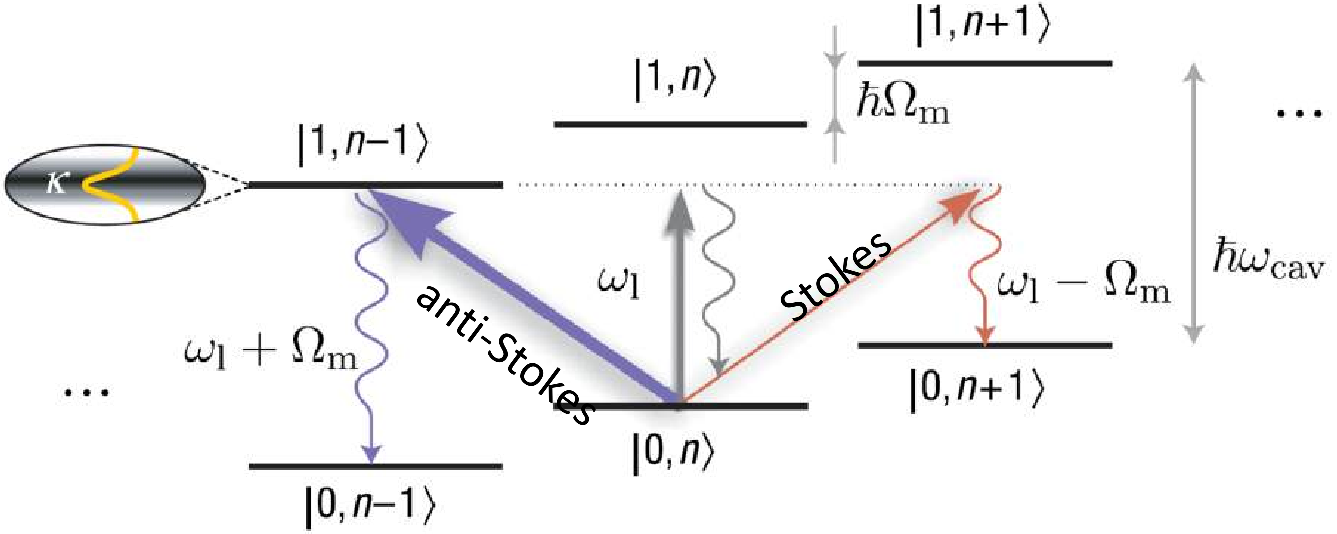
Drive the cavity through the polarizable object:
"far off-resonance atom"

Purcell factor: $\eta = \frac{24}{\pi} \cdot \frac{\mathcal{F}}{k^2 w_{cav}^2}$

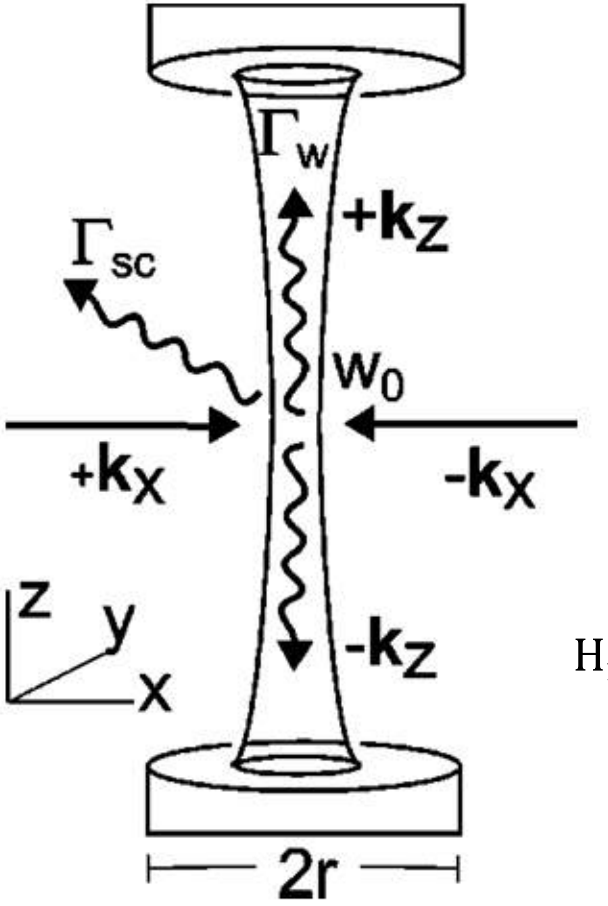
Optomechanical cooperativity: $C = \frac{4g^2}{\kappa\Gamma} = \frac{5}{4}\eta$

Limit of cavity cooling: $n_{min} \approx C^{-1} + \left(\frac{\kappa}{4\Omega}\right)^2$

Vladan Vuletić
MIT



Cooling: optomechanical Hamiltonian



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Limit of cavity cooling: $n_{\min} \approx C^{-1} + \left(\frac{\kappa}{4\Omega}\right)^2$

Vladan Vuletić
MIT

$$H_{int} = -\frac{\mathrm{Re}(\alpha)}{2}|E_{tot}|^2 = -\frac{\mathrm{Re}(\alpha)|E_{tw}|^2}{2} - \frac{\mathrm{Re}(\alpha)|E_{cav}|^2}{2} - \mathrm{Re}(\alpha)\mathrm{Re}(E_{tw} \cdot E_{cav}^*)$$

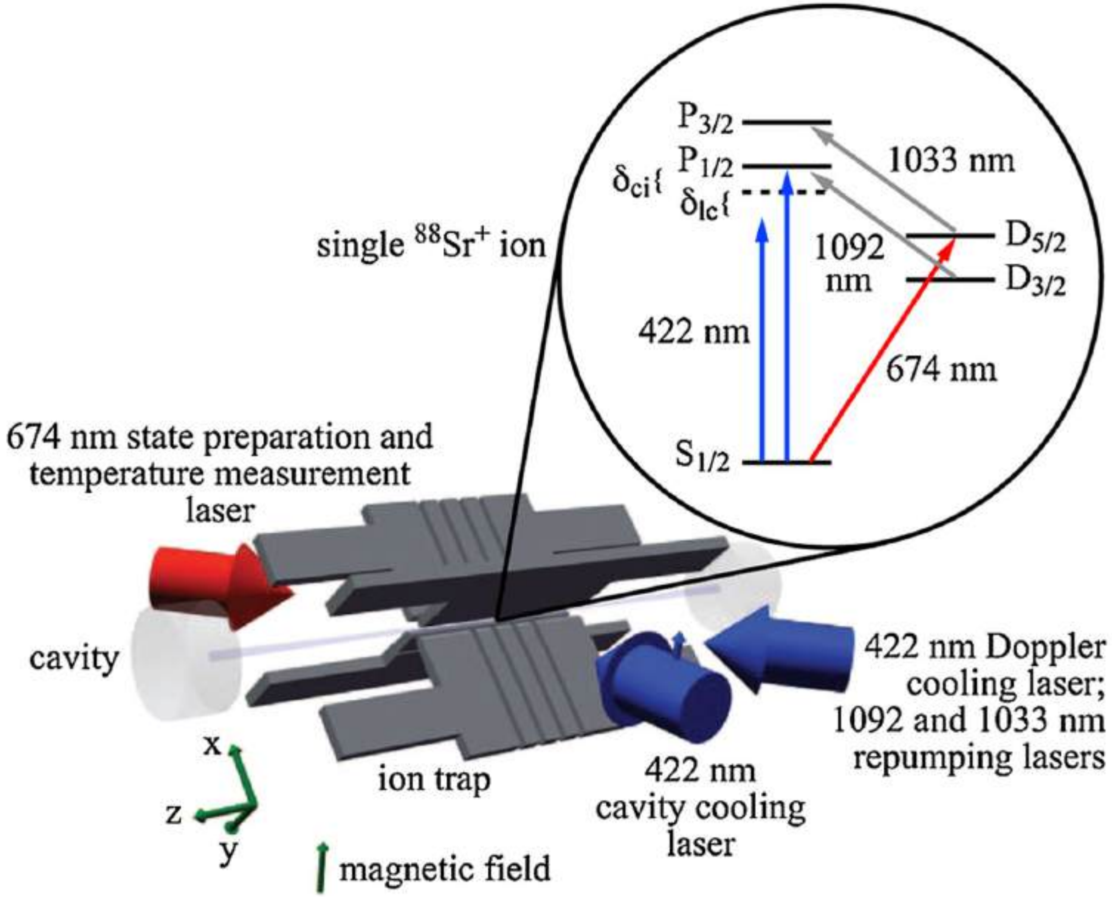
$\propto I_{tw}x^2$
trapping

$\propto I_{cav}x$
Cooling at the intensity slope

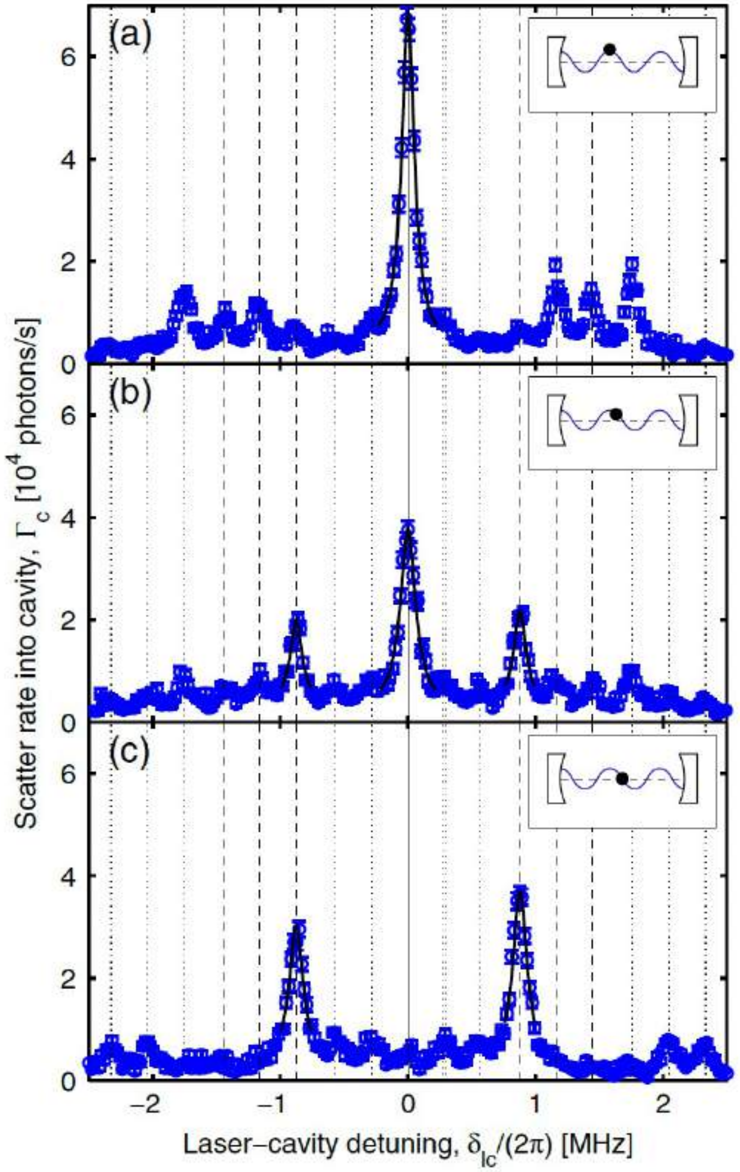
$\propto E_{tw}E_{cav}x$
Cooling at the intensity min.

Delic et al., Phys. Rev. Lett. **122**, 123602 (2019)

Cavity cooling of a single ion



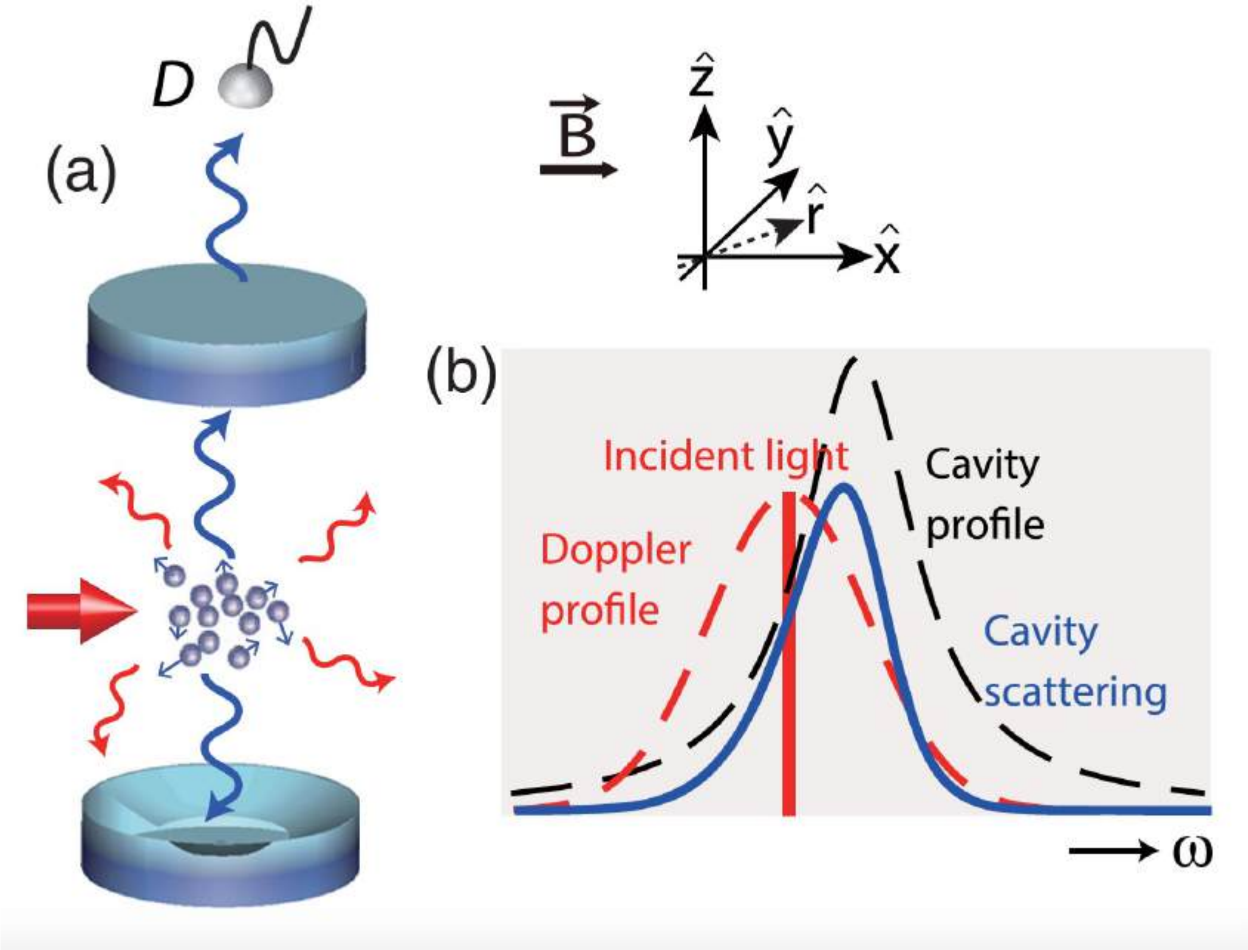
Position-dependent cavity cooling

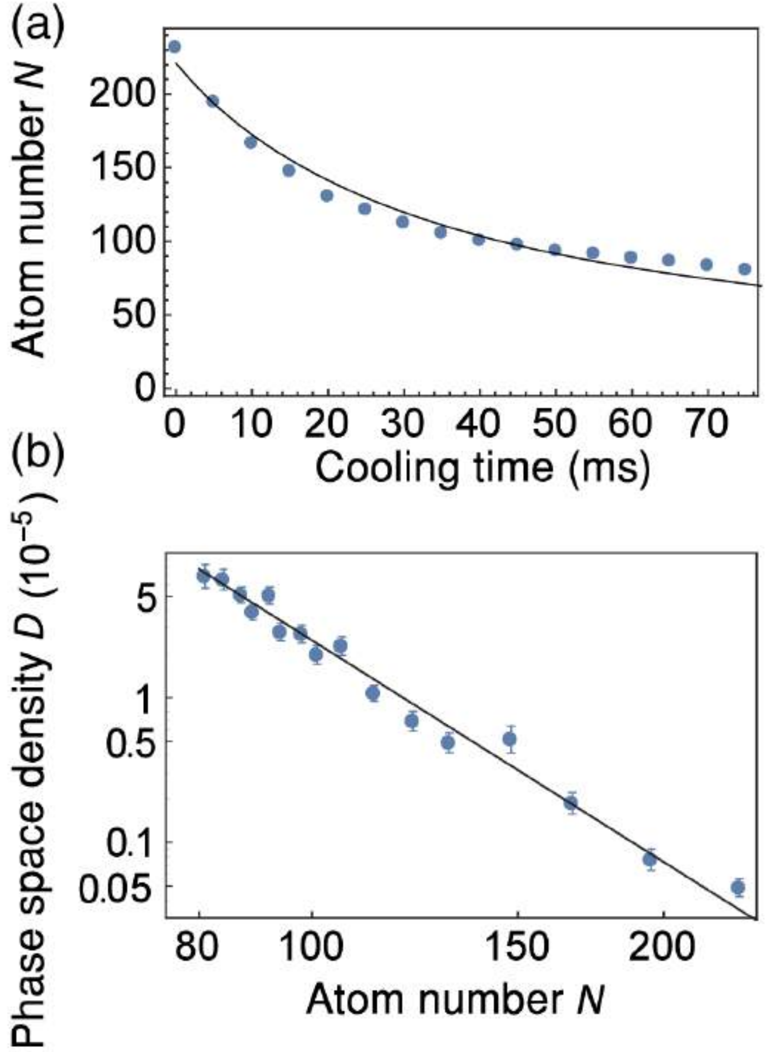


Leibrandt et al., Phys. Rev. Lett. 103, 103001 (2009)

Cavity cooling of many atoms

More efficient cooling than evaporative cooling:
Path toward larger, denser BEC?

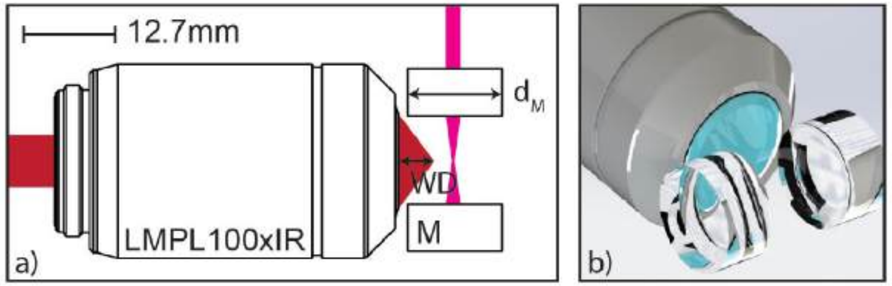




Hosseini et al., Phys. Rev. Lett. 118, 183601 (2017)

Cavity optomechanics with a nanoparticle

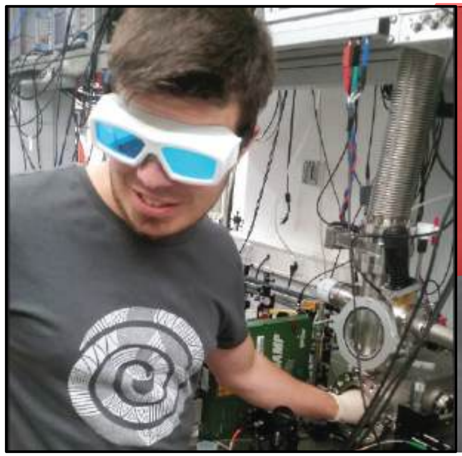
Delić et al., Quantum Sci. Technol. 5 025006 (2020)

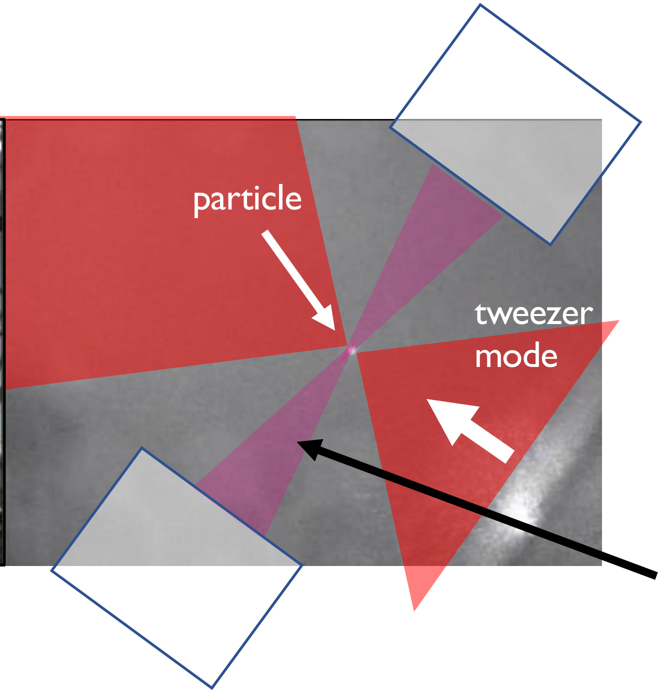


Loading of nanoparticles:
Spraying isopropanol droplets
with <1 nanoparticle per droplet

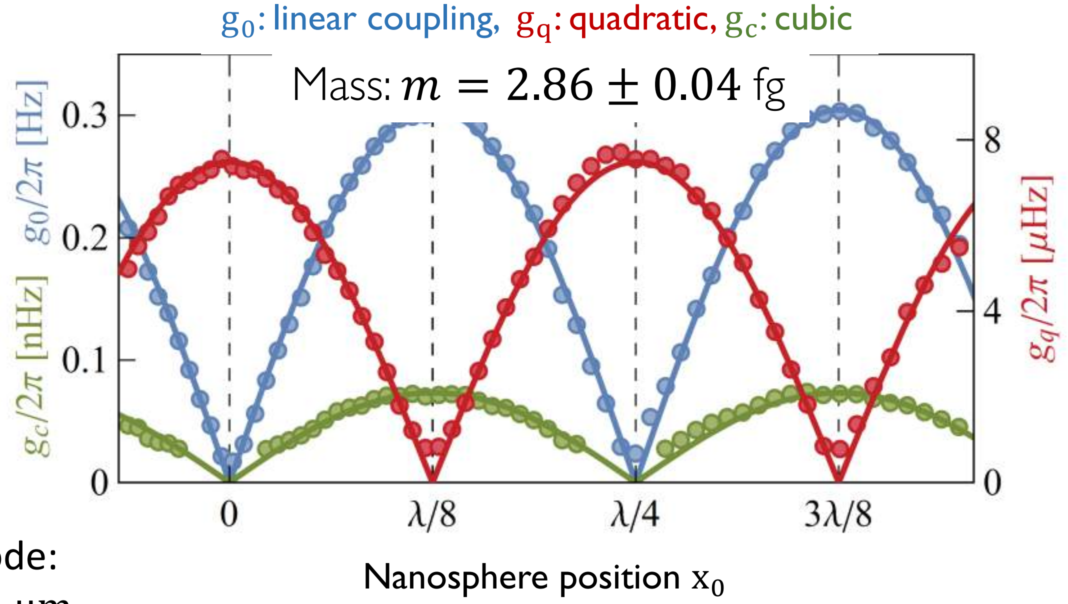
Bad for the cavity and vacuum

Working distance of tweezer < Radius of cavity mirrors
→ mirror cutting





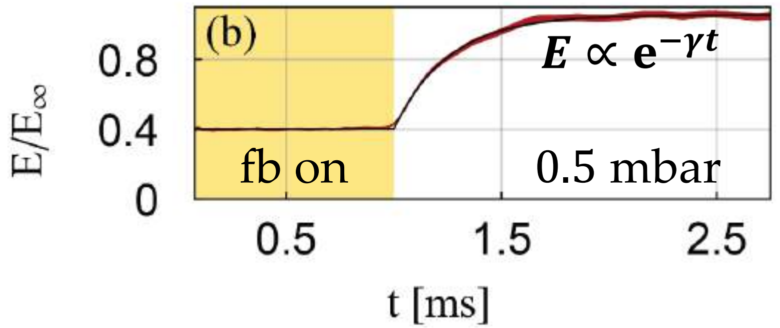
Cavity mode:
$w_0 = 41\ \mu m$

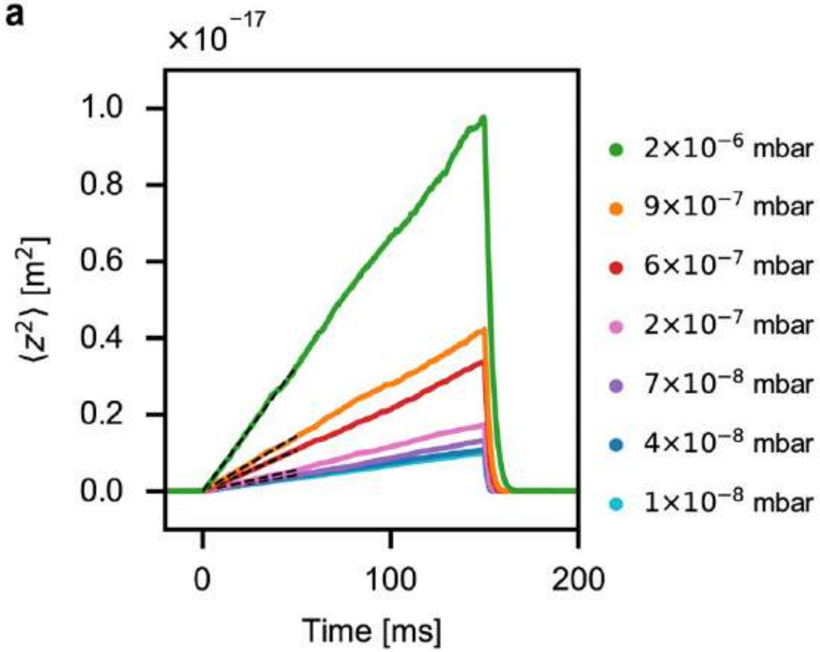


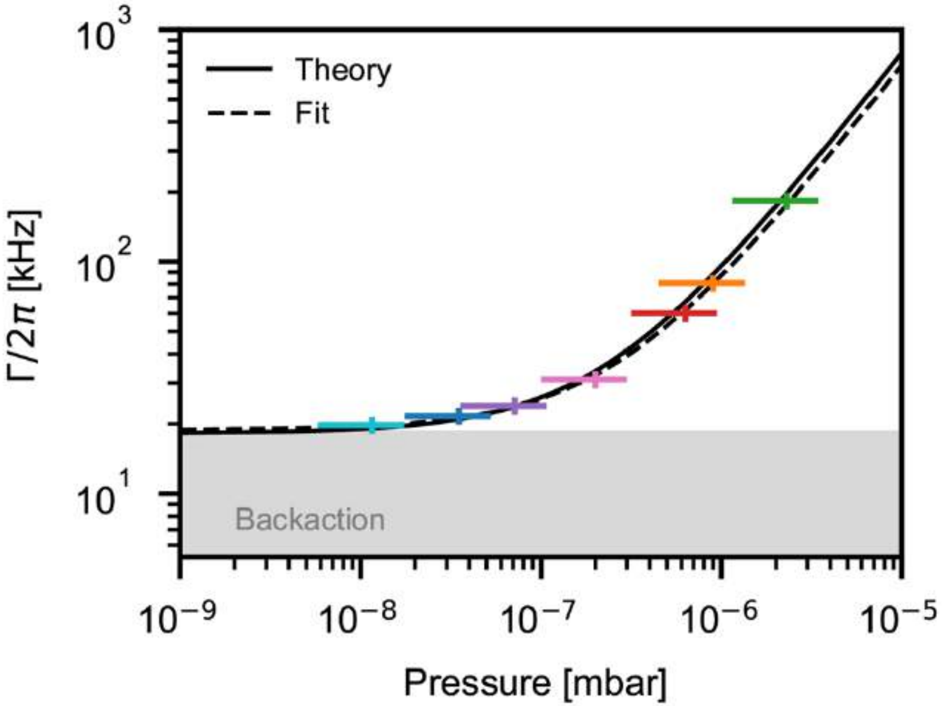
Recoil heating

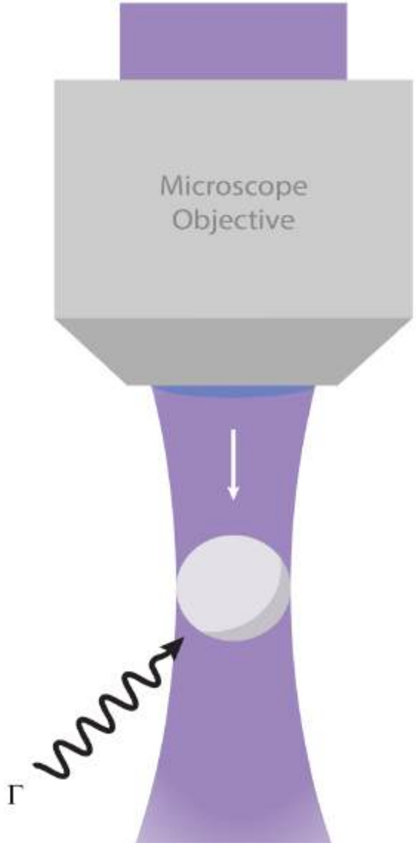
Measured by switching off the feedback:

$$E(t) = E_\infty - (E_\infty - E_{cool})\, e^{-\gamma t}$$







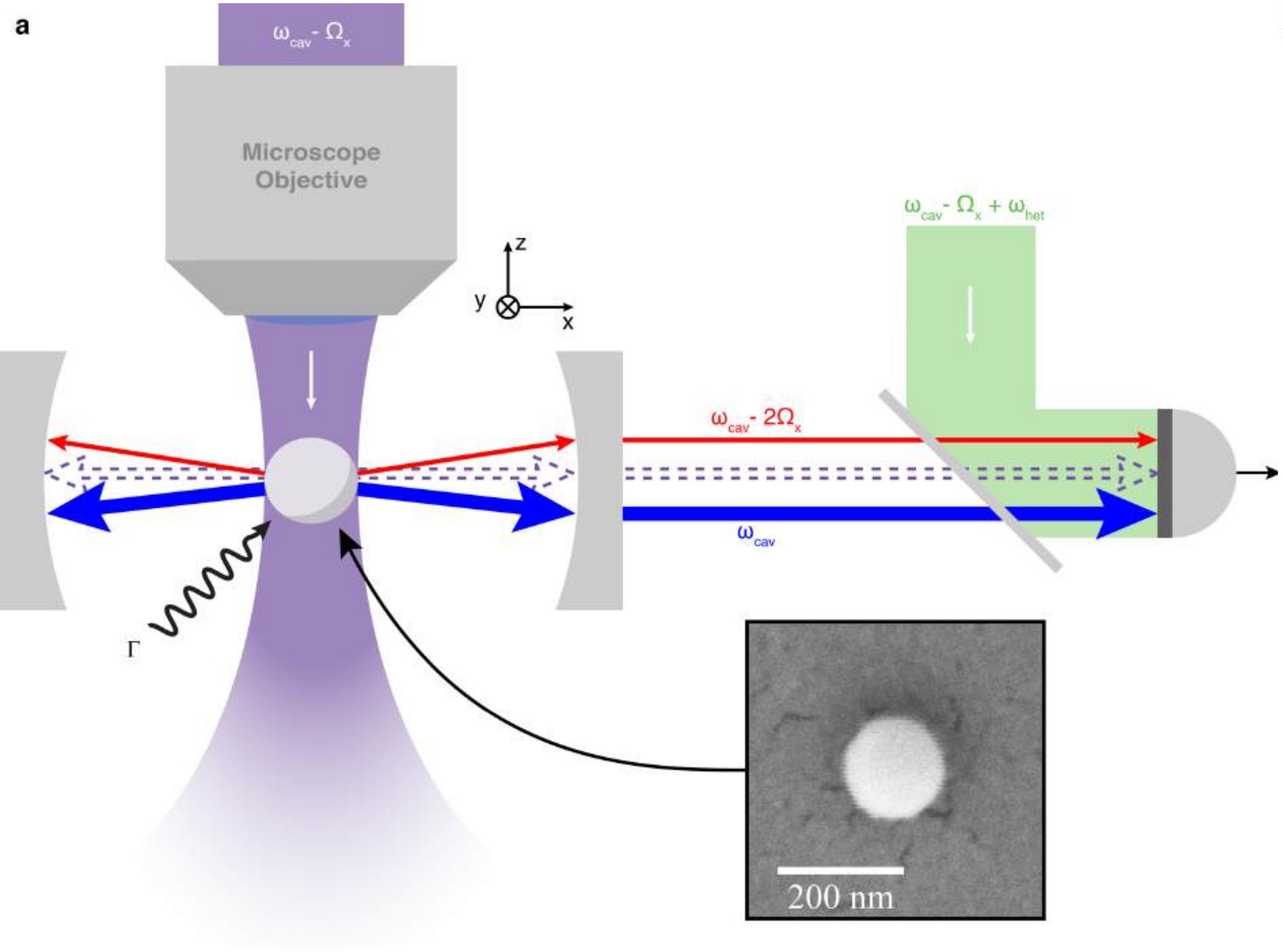


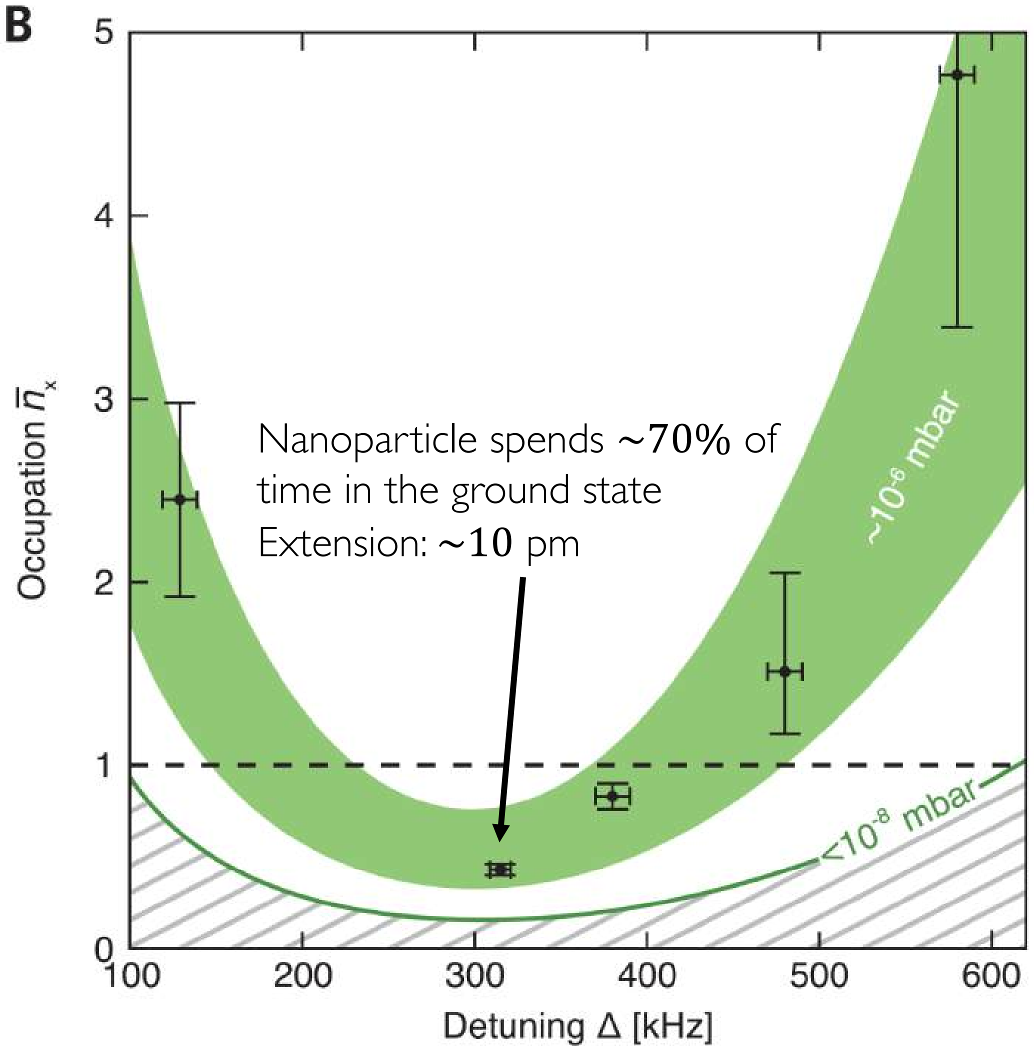
Gas: $\Gamma_{gas} = \gamma_g n_{th}$

Photons: Γ_{rec}

Jain et al., PRL 116, 243601 (2016)
Delic, QST (2020), Magrini, Nature (2021)

Ground-state cavity cooling of a nanoparticle



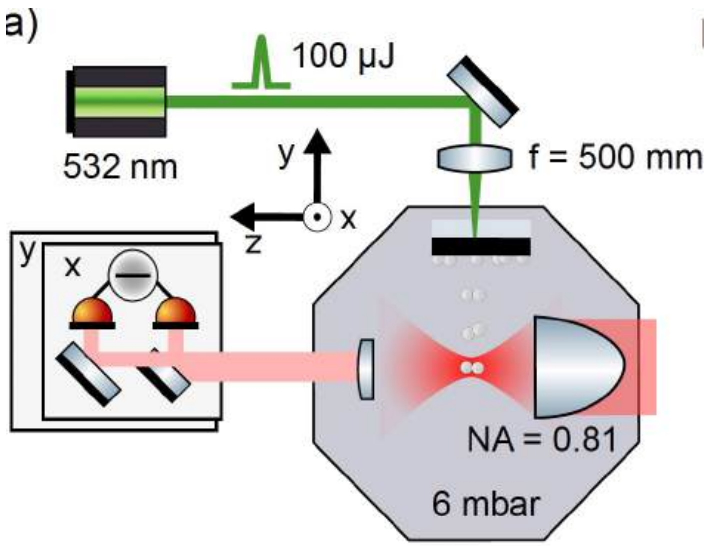


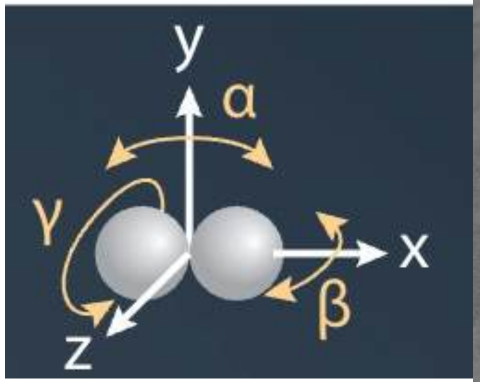
Delić et al., Science 367, 892 (2020)

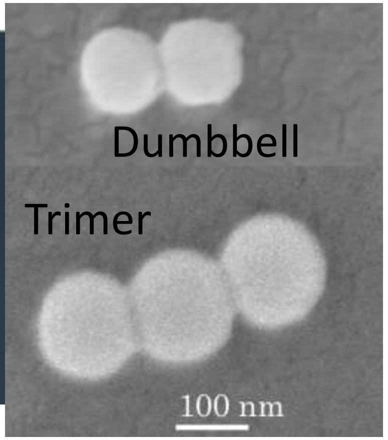
Ground state cooling of librations

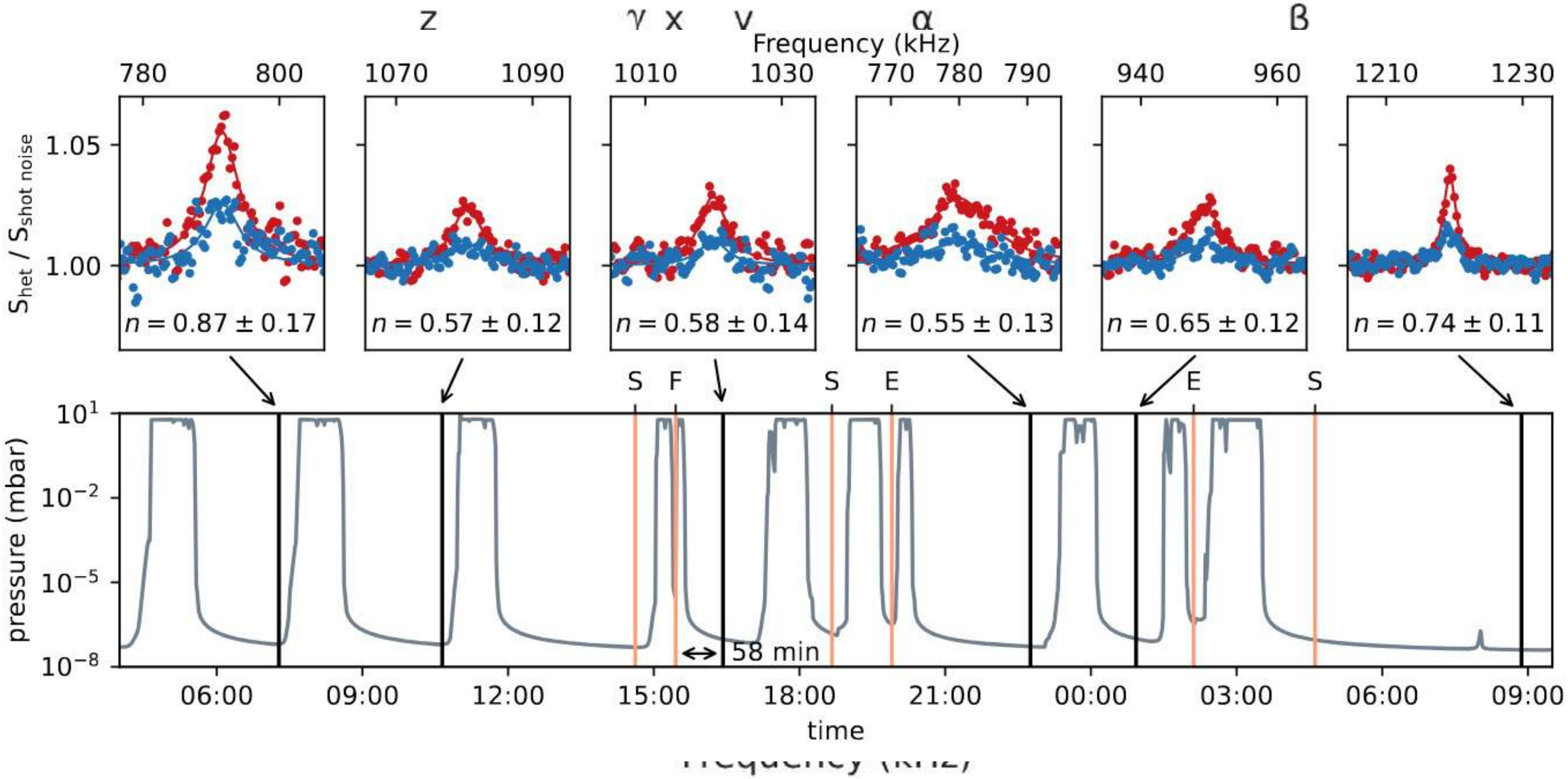
Solid-state objects can also rotate (free rotations) or librate (trapped rotations)

Robust loading:









Together with the Arndt group, soon on arXiv

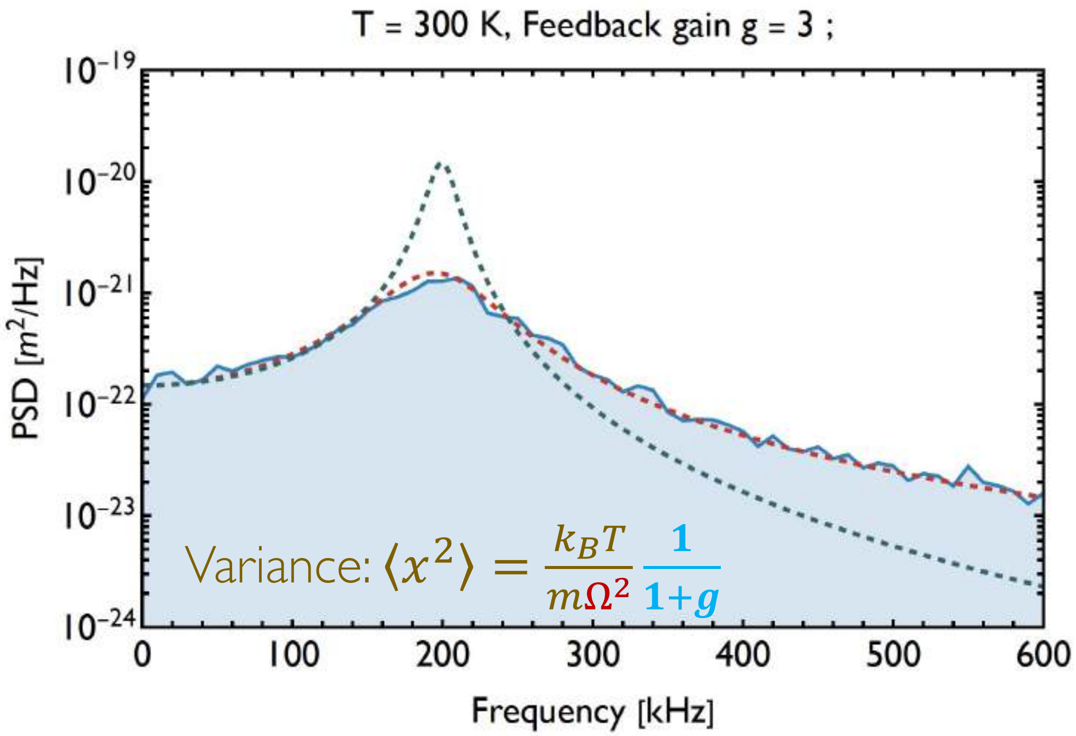
Feedback cooling

Time domain: position trace

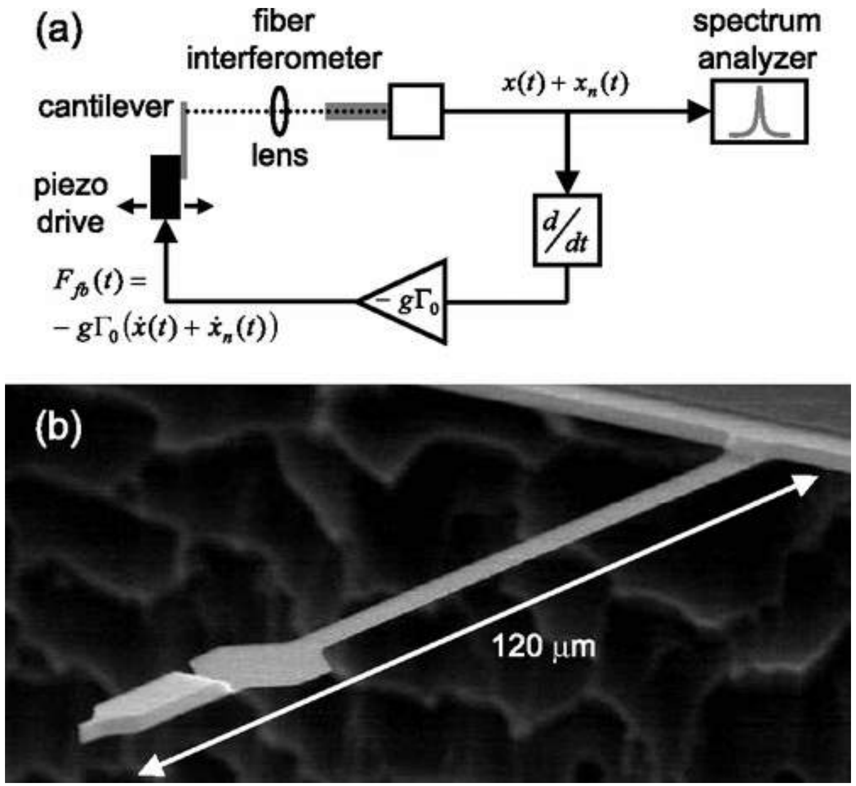
$$\ddot{x}(t) + \gamma\dot{x}(t) + \Omega^2 x(t) = \frac{1}{m}\sqrt{2mk_B T\gamma}\,\xi(t) - g\gamma\dot{x}(t)$$

Frequency domain: Power spectral density (PSD)

$$S_{xx}(\omega) = \frac{k_B T}{m\Omega^2}\frac{2\gamma\Omega^2}{(\Omega^2 - \omega^2)^2 + (1+g)^2\gamma^2\omega^2}$$

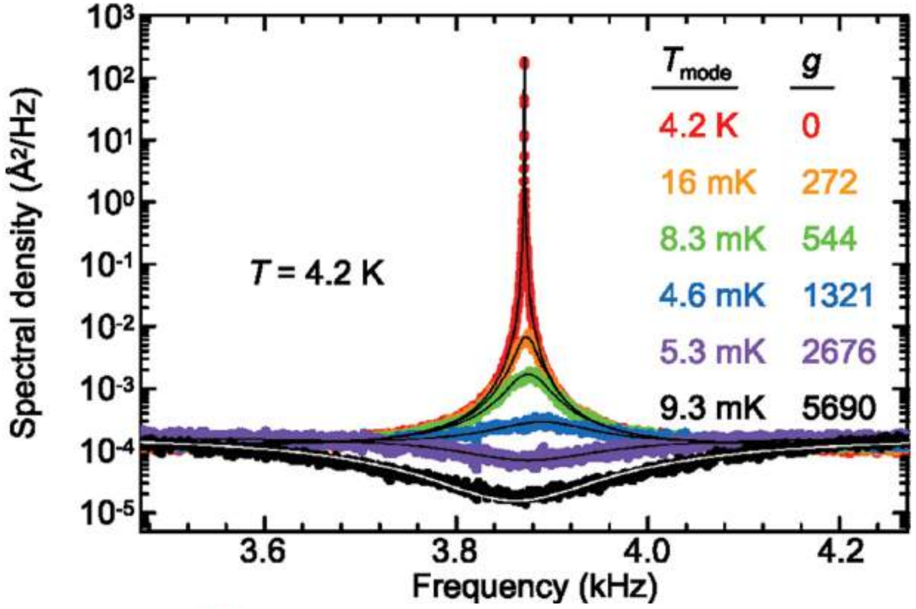


Limitations of feedback cooling



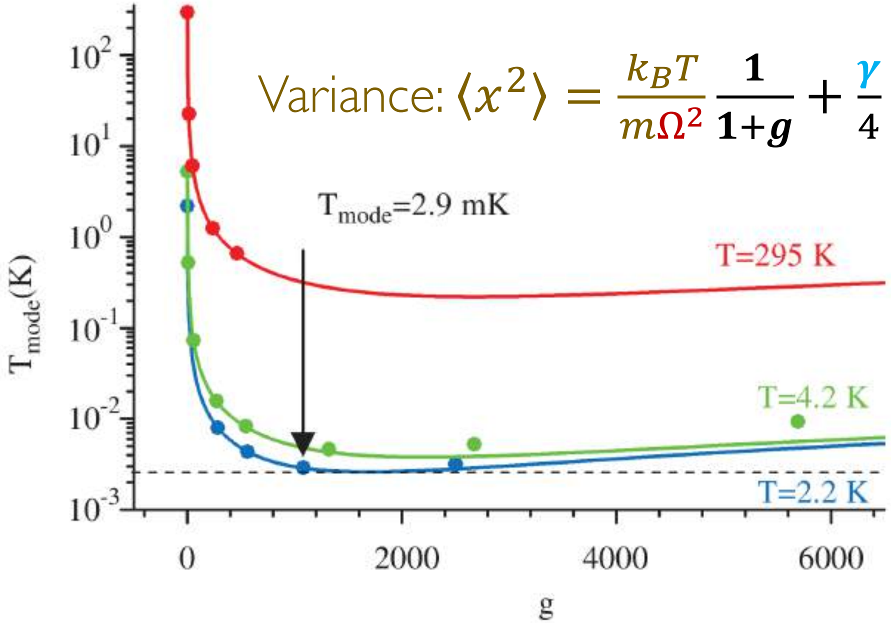
$$\ddot{x}(t) + \gamma\dot{x}(t) + \Omega^2 x(t) = \frac{1}{m}\sqrt{2mk_BT\gamma}\,\xi(t) - g\gamma(\dot{x}(t) + \dot{x}_n)$$

But the detection is: $\propto x(t) + x_n(t) \rightarrow$ correlated

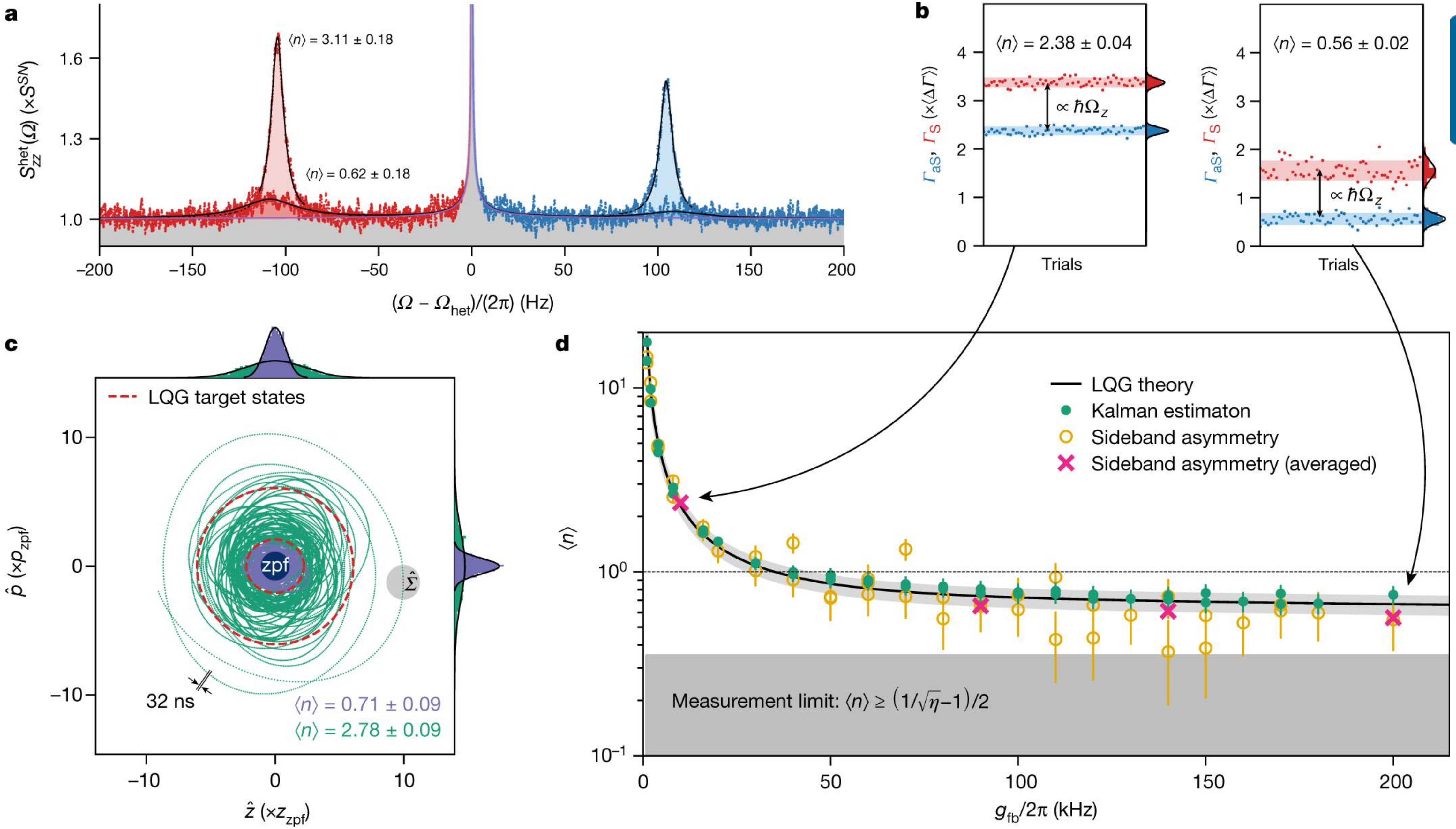


Below the detection background?

Variance: $\langle x^2\rangle = \frac{k_BT}{m\Omega^2}\frac{\mathbf{1}}{\mathbf{1+g}} + \frac{\gamma}{4}\left(\frac{g^2}{1+g}\right)S_{x_n}$



Poggio et al., Phys. Rev. Lett. **99**, 017201

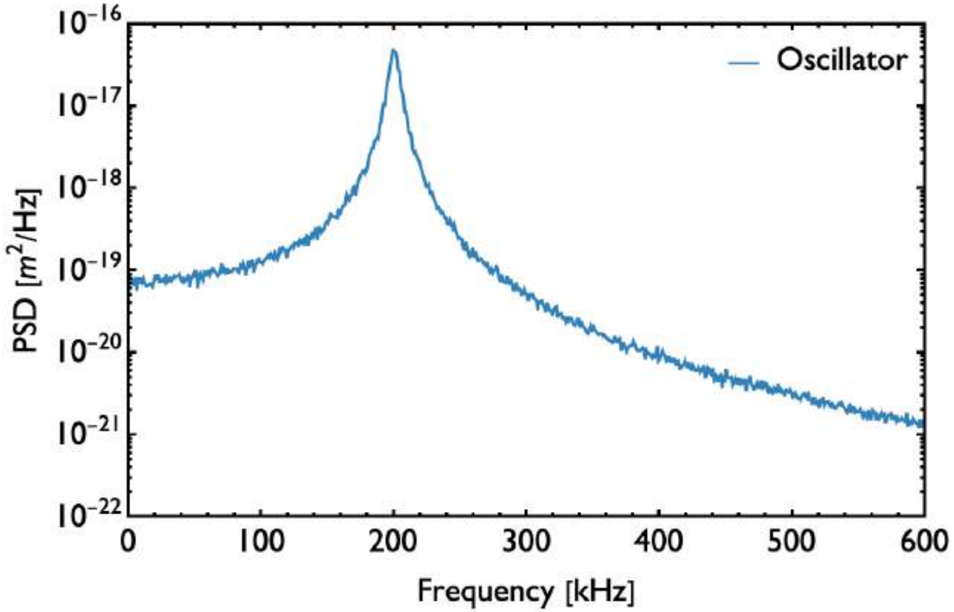


Magrini et al., Nature 595, 373 (2021)

Optimal state estimation

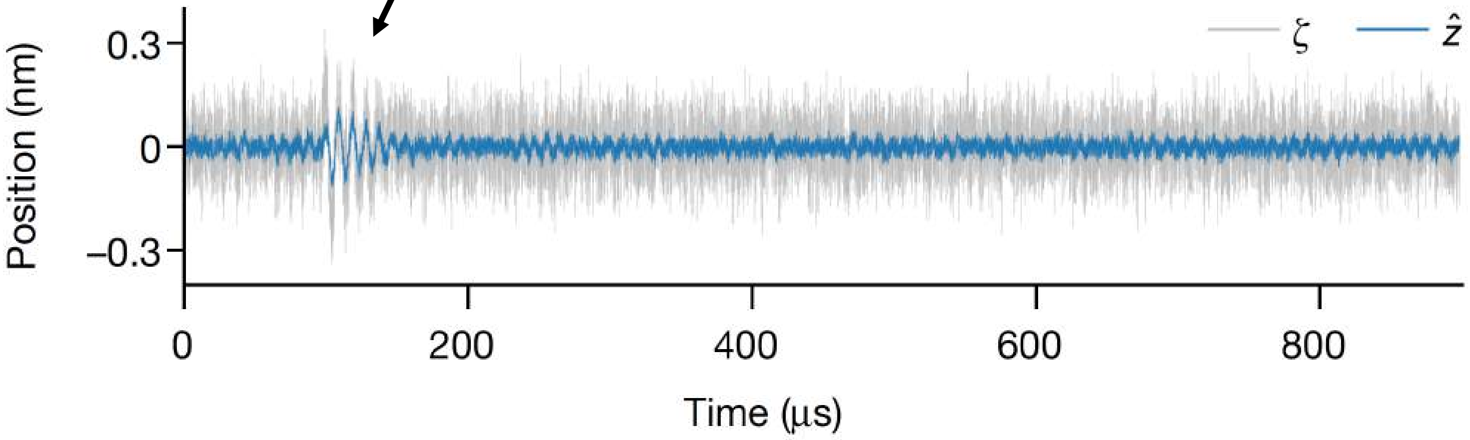
We detect $x(t) + x_n(t)$

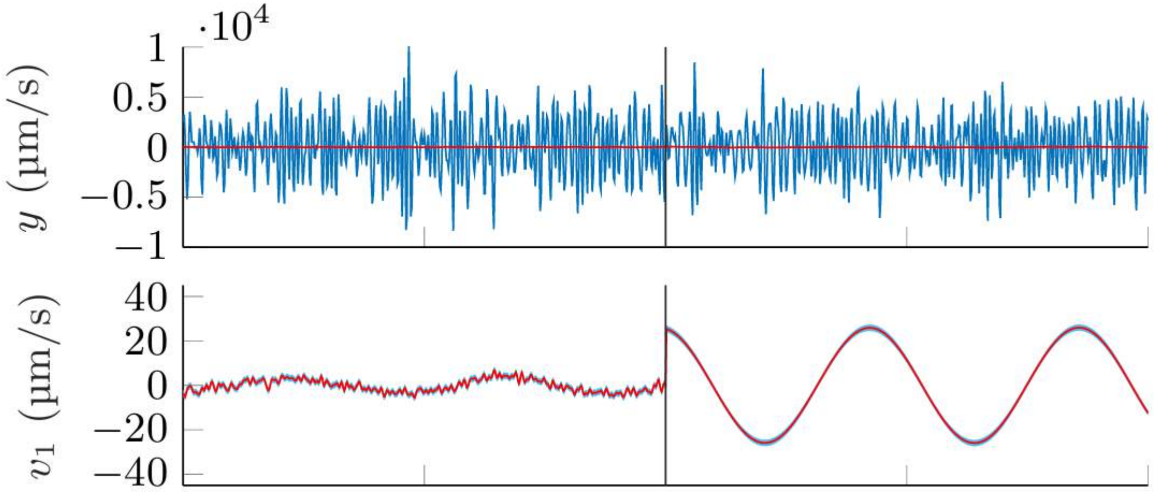
x_n: detection (white) noise
Optimal case: laser shot noise



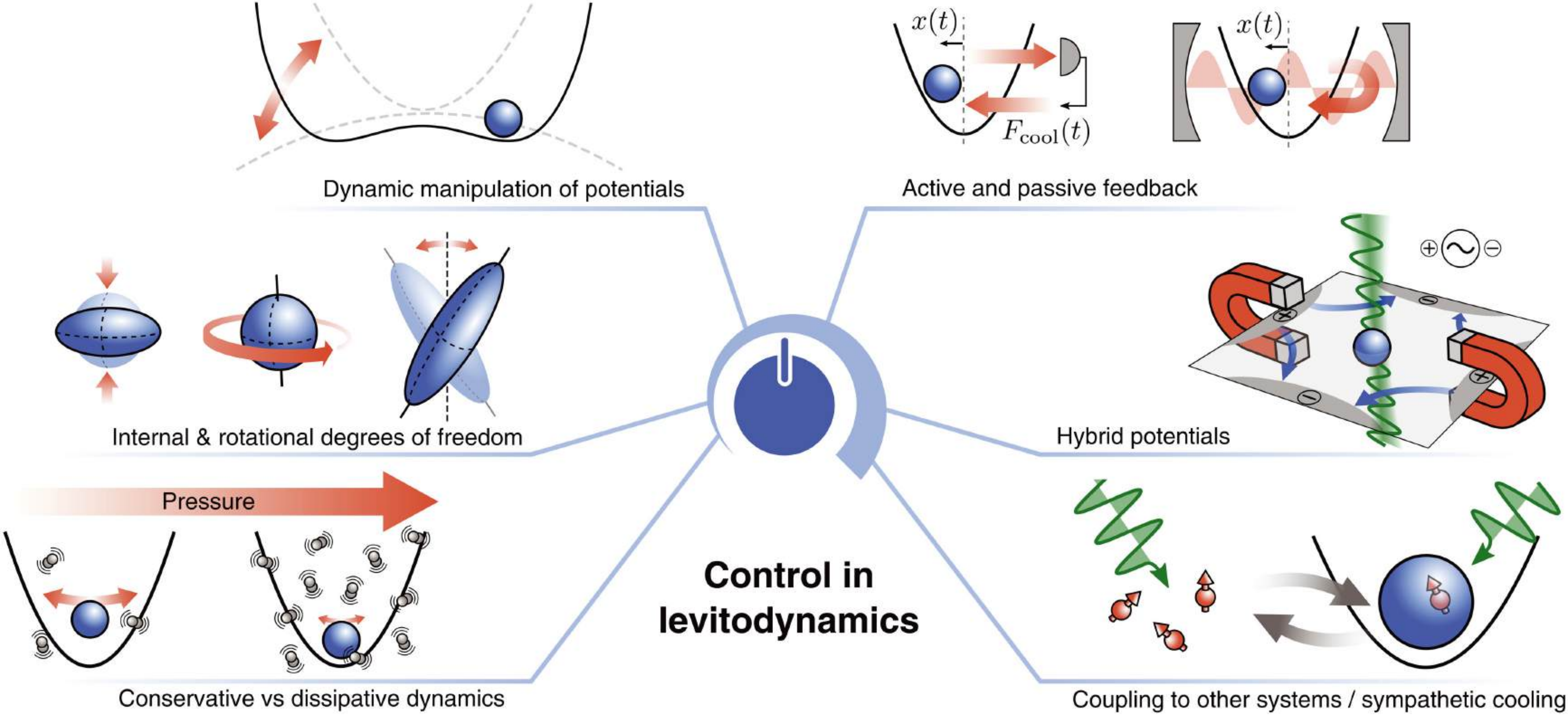
Detection at the Heisenberg limit with a Kalman filter:
Magrini et al., Nature 595, 373 (2021)
Jointly with Kugi & Deutschmann-Olek groups

Gas kick: momentum transfer



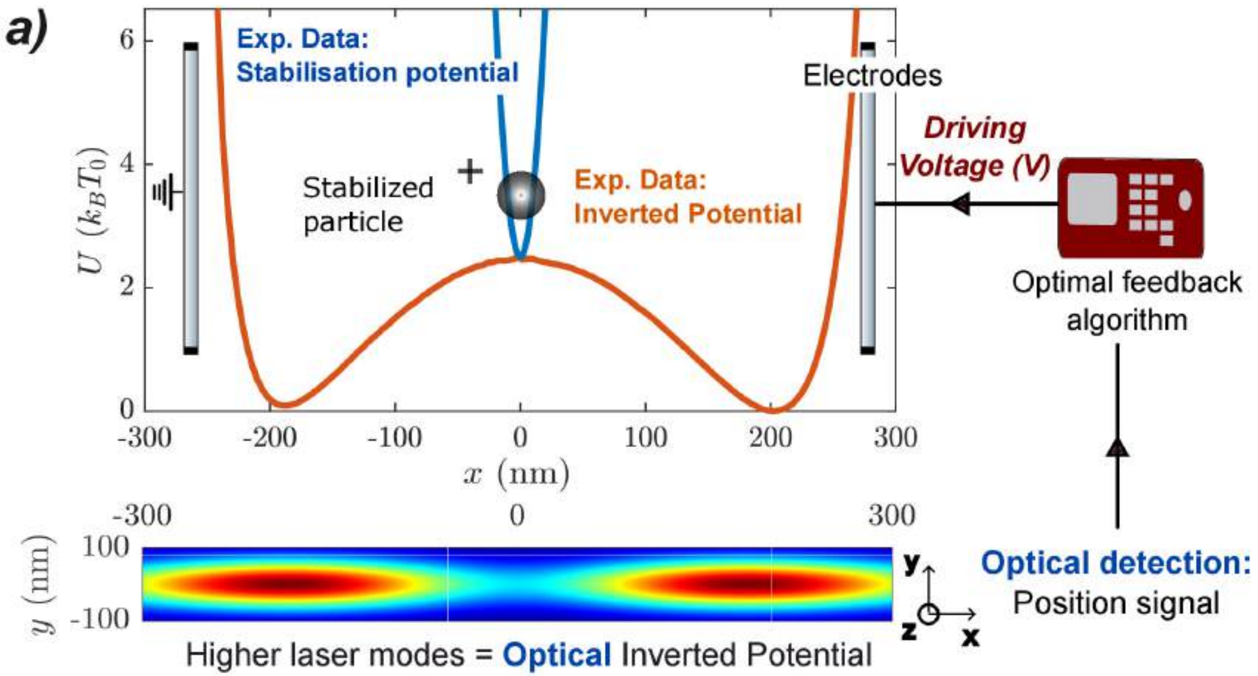


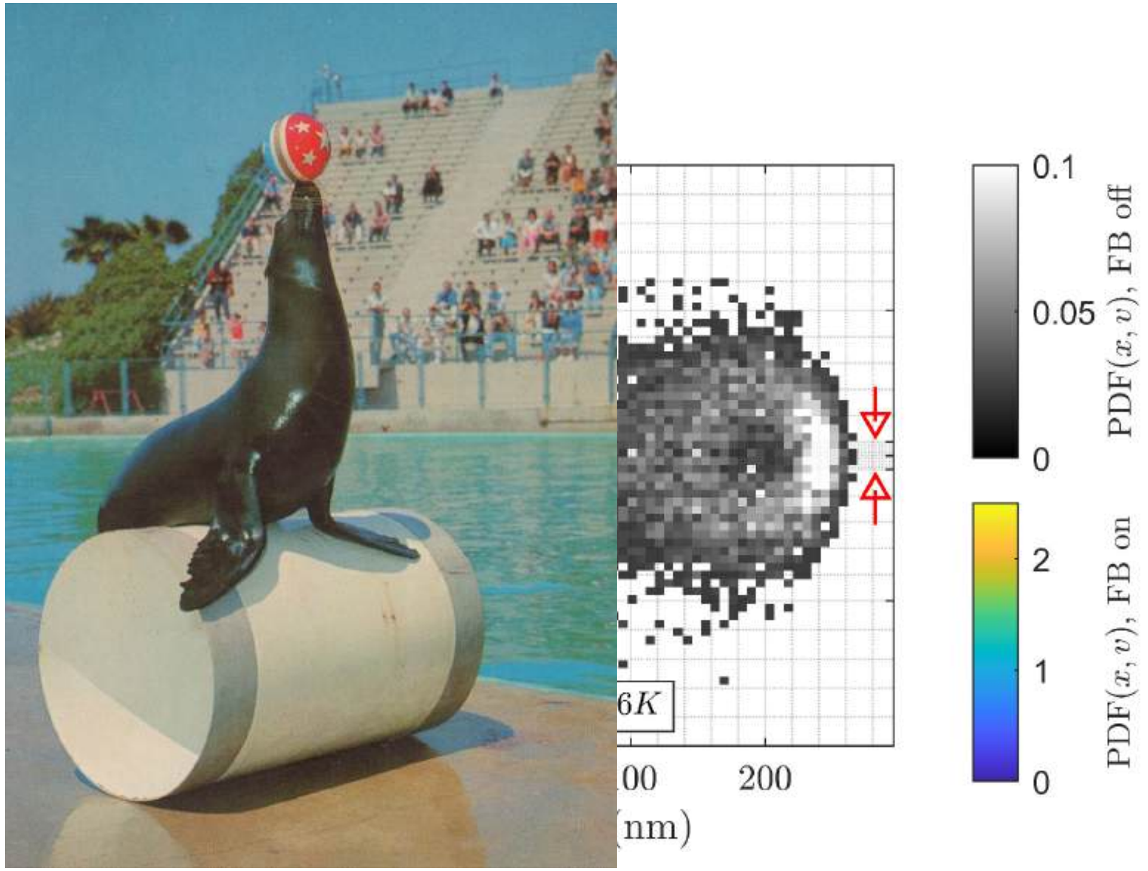
Schmerling et al., arXiv:2411.02215



$x(t)$
$F_{\mathrm{cool}}(t)$
$x(t)$
Dynamic manipulation of potentials
Active and passive feedback
Internal & rotational degrees of freedom
Hybrid potentials
Pressure
Control in levitodynamics
Conservative vs dissipative dynamics
Coupling to other systems / sympathetic cooling

Manipulation of optical potentials

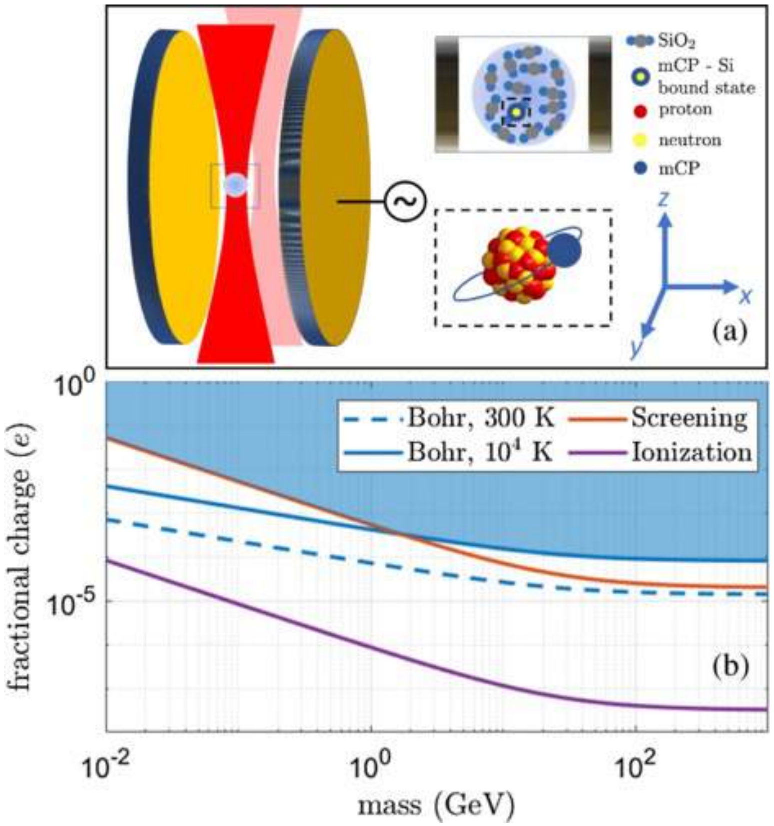




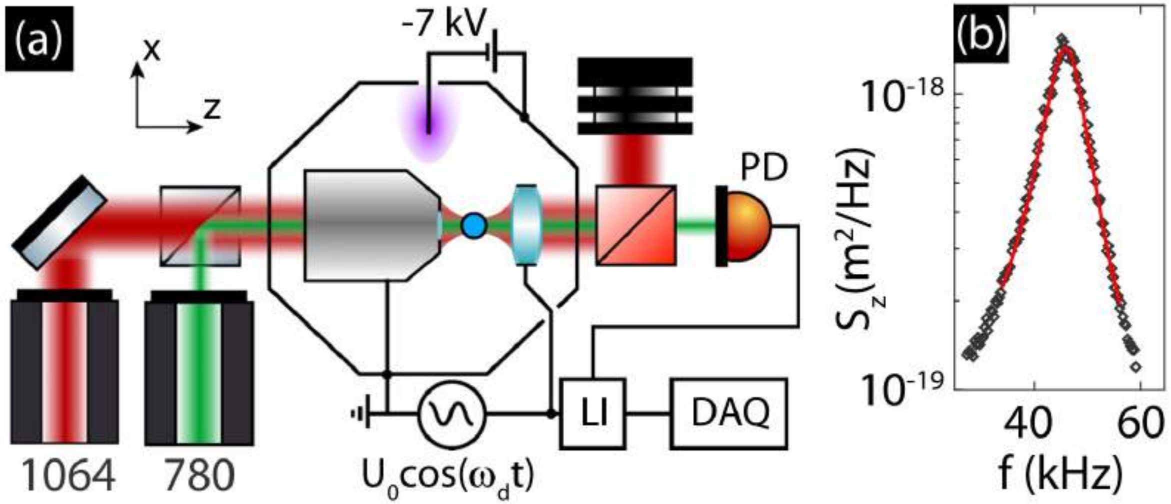
Dago, …, Kiesel, **Optics Express** 32, 45133 (2024)

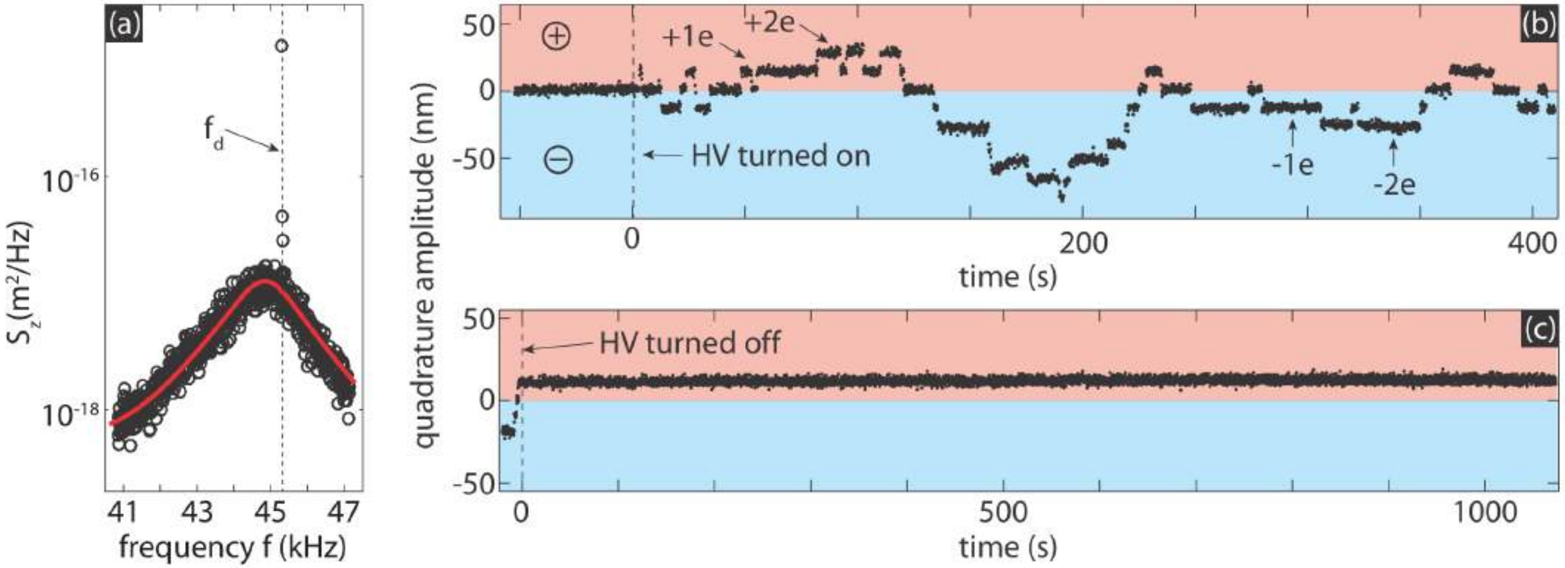
Sensing electrostatic forces

Do fractional charges exist?



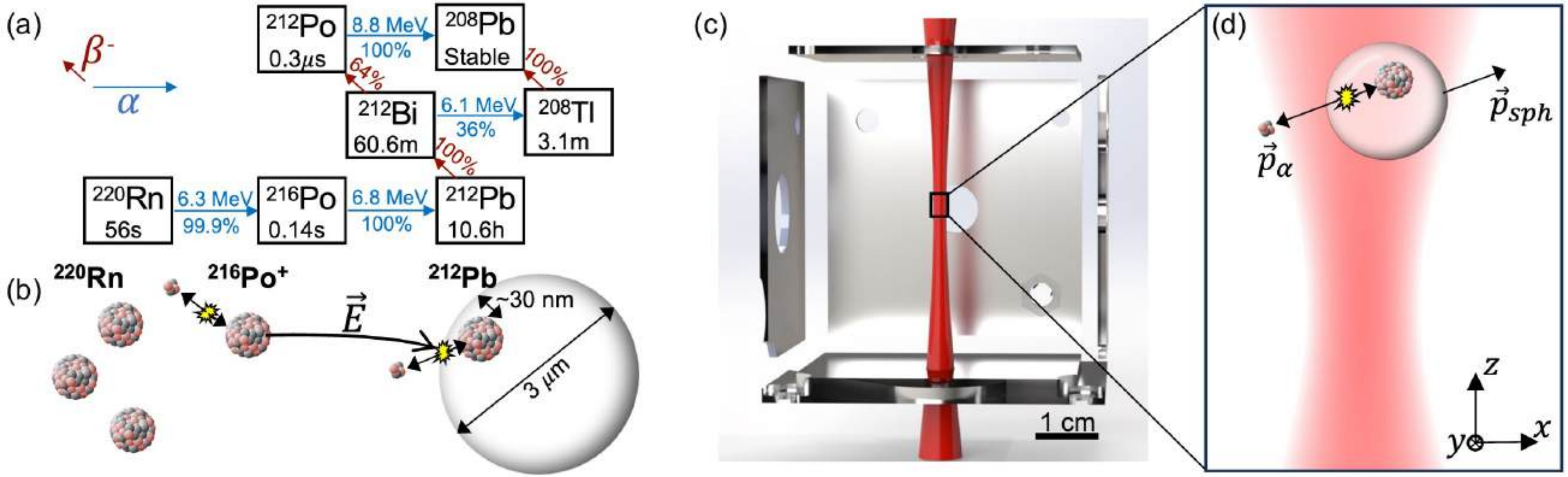
Afek et al., Phys. Rev. D **104**, 012004

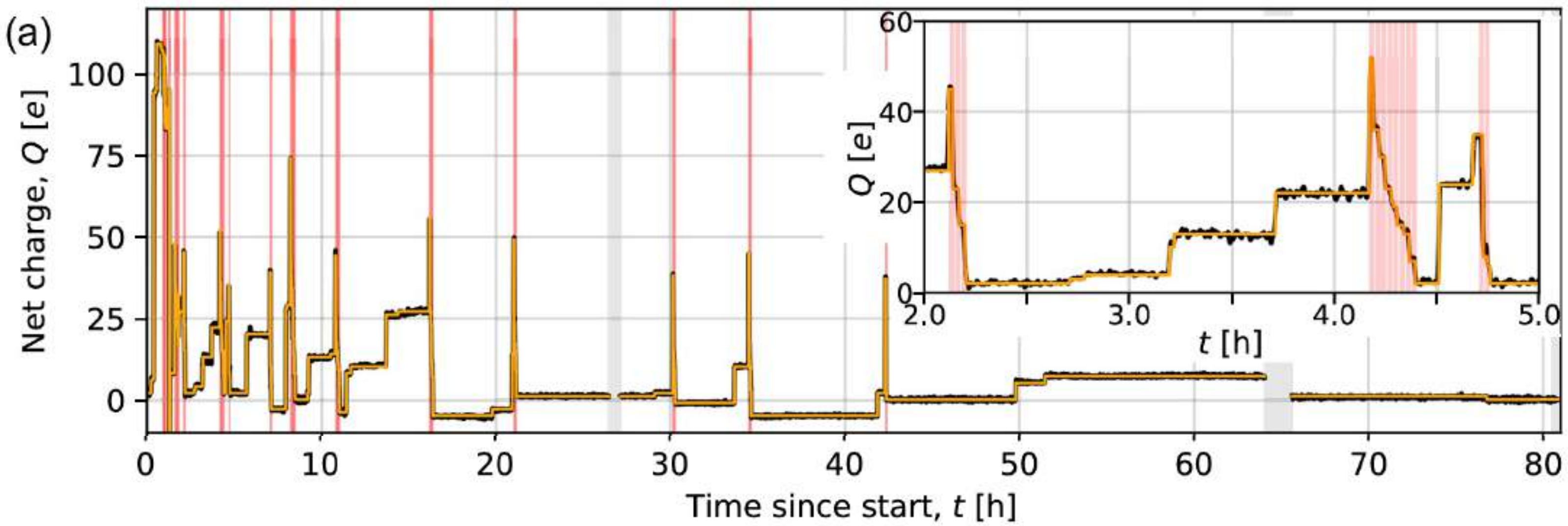




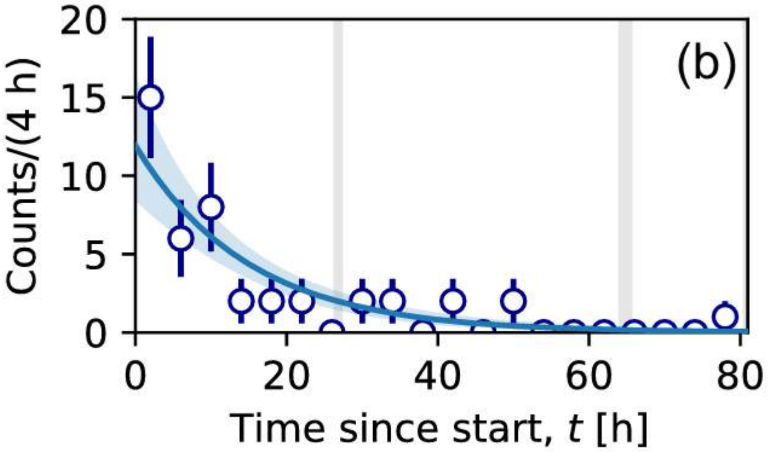
Frimmer et al., Phys. Rev. A 95, 061801(R)

Using levitated nanoparticles to observe nuclear reactions





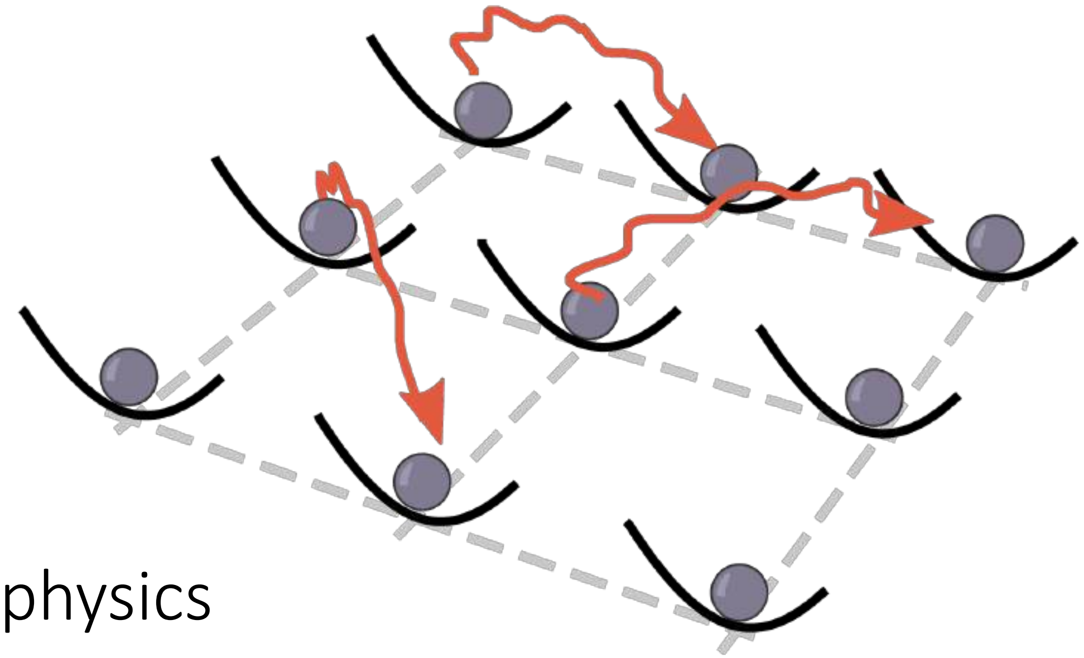
Decay time of ~10.3 hours
Consistent with Pb decay



David Moore group, Yale

Future experiments with trapped nanoparticles

Multi-particle experiments?



- Large delocalization

  First steps: arXiv:2408.01264

- Free-fall experiments

  First steps: arXiv:2507.12995

- Table-top experiments for fundamental physics

- Non-Gaussian states? Matter-wave interference?

  Long road ahead, but unique optical control techniques are available