Introduction to Plasma Physics

Hartmut Zohm
Max-Planck-Institut für Plasmaphysik
85748 Garching

DPG Advanced Physics School
'The Physics of ITER'
Bad Honnef, 22.09.2014
A simplistic view on a Fusion Power Plant

The 'amplifier' is a thermonuclear plasma burning hydrogen to helium
Centre of the sun: $T \sim 15\text{ Mio K}$, $n \leq 10^{32}\text{ m}^{-3}$, $p \sim 2.5 \times 10^{11}\text{ bar}$

$P_{\text{in}} = 50\text{ MW}$
(Initiate and control burn)

$P_{\text{out}} = 2-3\text{ GW}_{\text{th}}$
(Aiming at $1\text{ GW}_{\text{th}}$)
A bit closer look…

Fusion reactor: magnetically confined plasma, $D + T \rightarrow He + n + 17.6$ MeV

Centre of reactor: $T = 250$ Mio K, $n = 10^{20}$ m$^{-3}$, $p = 8$ bar
Plasma = ionised gas

- degree of ionisation $n_e/(n_e+n_0)$, depends on temperature (Saha equation)
- because of Maxwell distribution: $n_e/(n_e+n_0) \sim 1$ at $k_B T \sim 1/10 W_{ion}$
Plasmas occur in large range of \( n \) and \( T \)

- **ideal plasma condition** \( E_{\text{therm}} \gg E_{\text{interaction}} \) in large range
- **fusion plasma** can be treated as ideal gas of ions and electrons \((p = n k_B T)\)

Note: 
1 eV = 11600 K

\[
\frac{3}{2}kT > \frac{e^2}{4\pi\varepsilon_0} n^{1/3} \quad \Rightarrow \quad T[\text{eV}] > 0.97 \times 10^{-9} (n[\text{m}^{-3}])^{1/3}
\]
Existence diagram: density $n$ and temperature $T$

Astrophysical plasmas

$pulsar~magnetosphere$

$solar~wind$

$interstellar~medium$

$chromosphere$

$corona$

$lightning$

$photosphere$

$ionosphere$

$white~dwarf$

Note: $1~eV = 11600~K$
Existence diagram: density $n$ and temperature $T$

lab plasmas

- $n k_B T \sim 1$ bar
- Magnetically Confined Fusion plasmas
- Inertially Confined Fusion plasmas
- Semiconductor plasmas
- Glow discharge
- Flame
- Arcs
- Electron gas in metals

Note: $1$ eV = $11600$ K
Large number of freely movable charges: charge separation leads to strong electric fields – strong restoring force

- quasineutrality \( n_e = Z n_i \) can only be violated on Debye length \( \lambda_D \)
- on a scale \( L \gg \lambda_D \), plasmas are always quasineutral

\[
\lambda_D = \sqrt{\frac{\varepsilon_0 k T}{n_e e^2}} = 7437 \sqrt{\frac{T [eV]}{n [m^{-3}]} [m]}
\]
Displacement of electrons leads to large restoring force - oscillation

- below $\omega_p$, electrons can follow oscillating e-field – reflection of wave (cut-off)
- above $\omega_p$, electrons can no longer follow - plasma transparent to $\omega > \omega_p$

Used for density measurement (‘reflectometry’) – cut-off important for heating
Coulomb collisions are the main interaction between plasma particles

- thermodynamic equilibrium through Coulomb collisions
- dissipation by Coulomb collisions – electrical and thermal resistance

Collision frequency decreases with increasing temperature

- mean free path increases with $T$ – ‘collisionless plasma’
- electrical (‘Spitzer’) and thermal conductivity of fusion plasma very high

\[
\tau_{ee} = \frac{3 \sqrt{m_e (4\pi\epsilon_0)^2 (kT)^{3/2}}}{4 \sqrt{\pi e^4 \ln \Lambda}} \frac{1}{n_e}
\]

\[
\sigma = \frac{3 (4\pi\epsilon_0)^2}{4 \sqrt{2\pi m_e e^2 \ln \Lambda}} (kT)^{3/2}
\]
Thermalisation of a fast particle ensemble

‘Isotropisation’ – collisions randomise velocity components
‘Slowing down’ – collisions transfer energy to Maxwellian bulk
Application: Neutral Beam Heating (NBI)
Magnetised plasmas – single particle picture

Charged particles gyrate perpendicular B, but move freely along B

- cyclotron frequency $\omega_c$ used for diagnostics and heating (ECRH)
- for $k_B T \sim 1 \text{ keV}$ and $B = 2 \text{T}$: $r_{Le} \sim 50 \text{ \(\mu\)m}, r_{Li} \sim 2 \text{ mm} \Rightarrow$ magnetised plasma
On timescales much longer than $1/\omega_c$, motion of gyrocentre is relevant.

For an external force $F$, a drift perpendicular to $v_D = \frac{F \times B}{qB^2}$ is obtained.

Example: Plasma confinement in purely toroidal field

- curved magnetic field leads to vertical drift (centrifugal force)
- resulting $E$ field leads to a net outward drift

Plasma confinement in a purely toroidal field is not possible (see later).
For ‘adiabatic changes of gyromotion (gyro-circles almost closed):

- magnetic moment $\mu \sim mv_\perp^2 / B = \text{const. along trajectory}$
- since total energy is conserved, $||$ energy converted to $\perp$ if $B$ increases
Particle orbits: mirror in the Earth's magnetic field

- Proton drift
- Electron drift
- Trapped particle orbit
- Reflection point
- B-field line

$R_E$

$R_0$
Magnetised plasmas – many body picture

A comprehensive approach deals with a description in 6-d phase space

• ‘kinetic theory’ of distribution function \( f(\mathbf{v}, \mathbf{x}, t) \) – too complicated for today 😊

If thermodynamic equilibrium is assumed (\( f = \text{Maxwellian} \)), the set of MagnetoHydroDynamic (MHD) equations can be used:

\[
\frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{v}) = 0 \quad \text{continuity equation}
\]

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = -\nabla \cdot \mathbf{P} + \mathbf{j} \times \mathbf{B} \quad \text{force (Euler) equation}
\]

\[
\mathbf{E} + \mathbf{v} \times \mathbf{B} = \frac{1}{\sigma} \mathbf{j} \quad \text{Ohm’s law}
\]

+ equation of state (e.g. adiabatic)

+ Maxwell’s equations

• mostly adequate for motion perpendicular to \( B \), usually not along \( B \)
In equilibrium, there is no time dependence. If in addition, no flow:

\[ \nabla p = \mathbf{j} \times \mathbf{B} \Rightarrow \nabla p = \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} \Rightarrow \nabla_\perp (p + \frac{B^2}{2\mu_0}) + \frac{B^2}{\mu_0 R_c} e_{R_c} = 0 \]

One can identify two contributions to the force balance:

- **magnetic pressure**
- **field line tension**

N.B.: these two forces lead to two branches of MHD (Alfvén) waves
Ohm’s law and the ‘frozen fieldlines’

The change of magnetic field is governed by Ohm’s law:

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} = \nabla \times (\vec{v} \times \vec{B}) - \frac{1}{\mu_0 \sigma} \nabla \times (\nabla \times \vec{B})$$

$$\Rightarrow \frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \frac{1}{\mu_0 \sigma} \Delta \vec{B}$$

- in ideal MHD, plasma and field lines move together (Alfvén time scale, fast)
- resistivity leads to a diffusion of the magnetic field through the plasma (resistive MHD time scale, arbitrarily slow for arbitrary high conductivity)

Important example: collapse of a star leads to enormous field amplification!
Ohm’s law and the ‘frozen fieldlines’

Concept of ‘flux tubes’:

- in ideal MHD, plasma and field lines move together (Alfvén time scale, fast)
- flux tubes move with fluid and cannot intersect – topology conserved
- example: collapse of a neutron star
Emission from pulsars validates high B-fields

Beamed (relativistic) 'curvature radiation' from parallel e⁻ motion
A change of magnetic topology is only possible through reconnection

- opposing field lines reconnect and form new topological objects
- requires finite resistivity in the reconnection region

Example: Coronal Mass Ejection (CME) from the sun

Due to high electrical conductivity, magnetic flux is frozen into plasma

⇒ magnetic field lines and plasma move together
A change of magnetic topology is only possible through reconnection

- opposing field lines reconnect and form new topological objects
- requires finite resistivity in the reconnection region

Example: Coronal Mass Ejection (CME) from the sun

Due to high electrical conductivity, magnetic flux is frozen into plasma

⇒ magnetic field lines and plasma move together
Plasma waves: two fluid equations

\[
(1 - \frac{k^2 c^2}{\omega^2})E + \frac{c^2}{\omega^2}k(k \cdot E) = -i \frac{\omega_p^2 m_e}{\omega} e (u_i - u_e) \quad \text{wave eqn.}
\]

\[
u_i - i \frac{\omega_ci}{\omega} u_i \times b - \frac{c_i^2}{\omega^2} k(k \cdot u_i) = i \frac{e}{\omega m_i} E \quad \text{ion force balance}
\]

\[
u_e + i \frac{\omega_ce}{\omega} u_e \times b - \frac{c_e^2}{\omega^2} k(k \cdot u_e) = -i \frac{e}{\omega m_e} E \quad \text{electron force balance}
\]

After linearisation and Fouriertransform, a 9 x 9 Matrix system is obtained. Matrix is the equivalent of the 'dielectric tensor'.

Solutions give dispersion relation \( \omega = \omega(k) \)

- neutral gas: e-m waves and sound waves uncoupled
- plasma: sound waves couples to electrostatic wave (charge density)
Waves in an Unmagnetised Plasma

Electromagnetic wave

\[ \omega^2 = \omega_{pe}^2 + k^2 c^2 \]

Electron sound wave

\[ \omega = k \cdot c \]

\[ v_p = c \]

Ion sound wave

\[ \omega = k \cdot (3/2)^{1/2} \cdot v_{th} \]

\[ \omega = k \cdot \left[ k_B \cdot T_e/(1 + k^2 \lambda_D^2) + 3k_B T_i \right]^{1/2} / m_i^{1/2} \]
Waves in a Magnetised Plasma: propagation \parallel \text{ to } B

- right-handed circular polarised wave
- left-handed circular polarised wave
- electron cyclotron wave
- Whistler wave
- Magneto-acoustic wave
- ion cyclotron wave
- Shear Alfvén wave

\omega, k
Waves in a Magnetised Plasma: propagation $\perp$ to $B$

- extraordinary wave (X-mode)
- ordinary wave (O-mode)
- upper hybrid wave
- lower hybrid wave
- compressional Alfvén wave
Electron Cyclotron Resonance Heating (ECRH)

Microwave beam absorbed at $\omega = \omega_{ce}$ – good localisation and control
Ion Cyclotron Resonance Heating (ICRH)

Example: ion Bernstein wave, electrostatic ion cyclotron wave

- In the vicinity of the ion-ion hybrid layer, mode conversion to shorter wavelength waves occurs.

IBW : Ion Bernstein Wave
  Propagates towards the high field side

ICW : Ion Cyclotron Wave
  Propagates towards the low field side
The Plasma Universe

Note: 1 eV = 11600 K