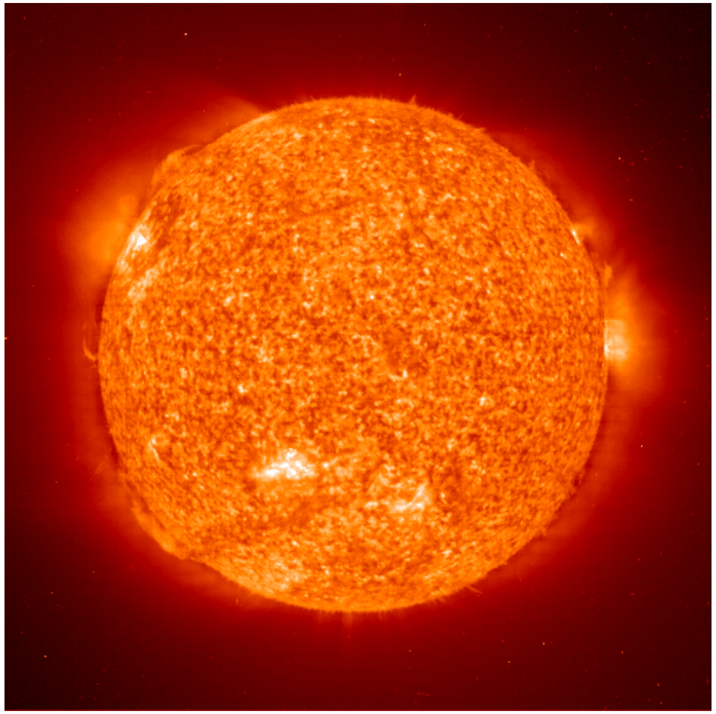
Introduction to Plasma Physics



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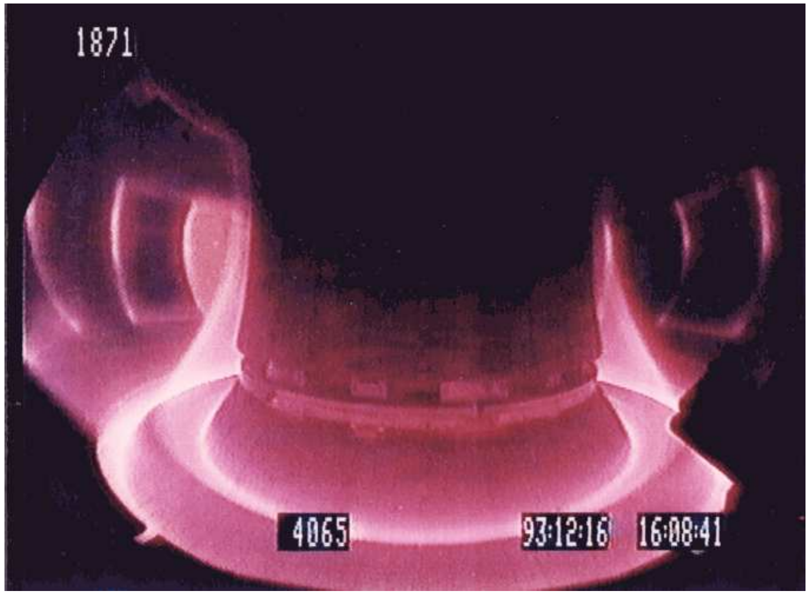
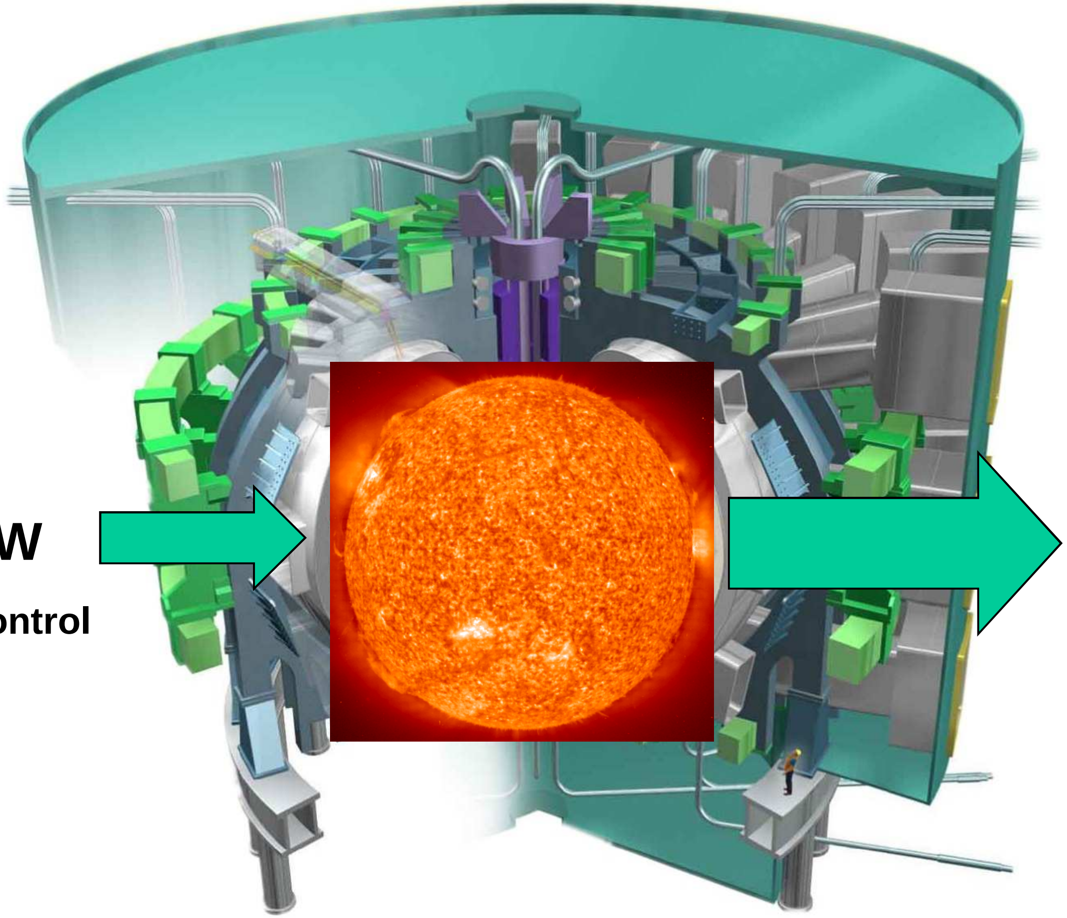


DPG Advanced Physics School
‚The Physics of ITER‘
Bad Honnef, 22.09.2014

A simplistic view on a Fusion Power Plant

P_{in} = 50 MW

(initiate and control burn)



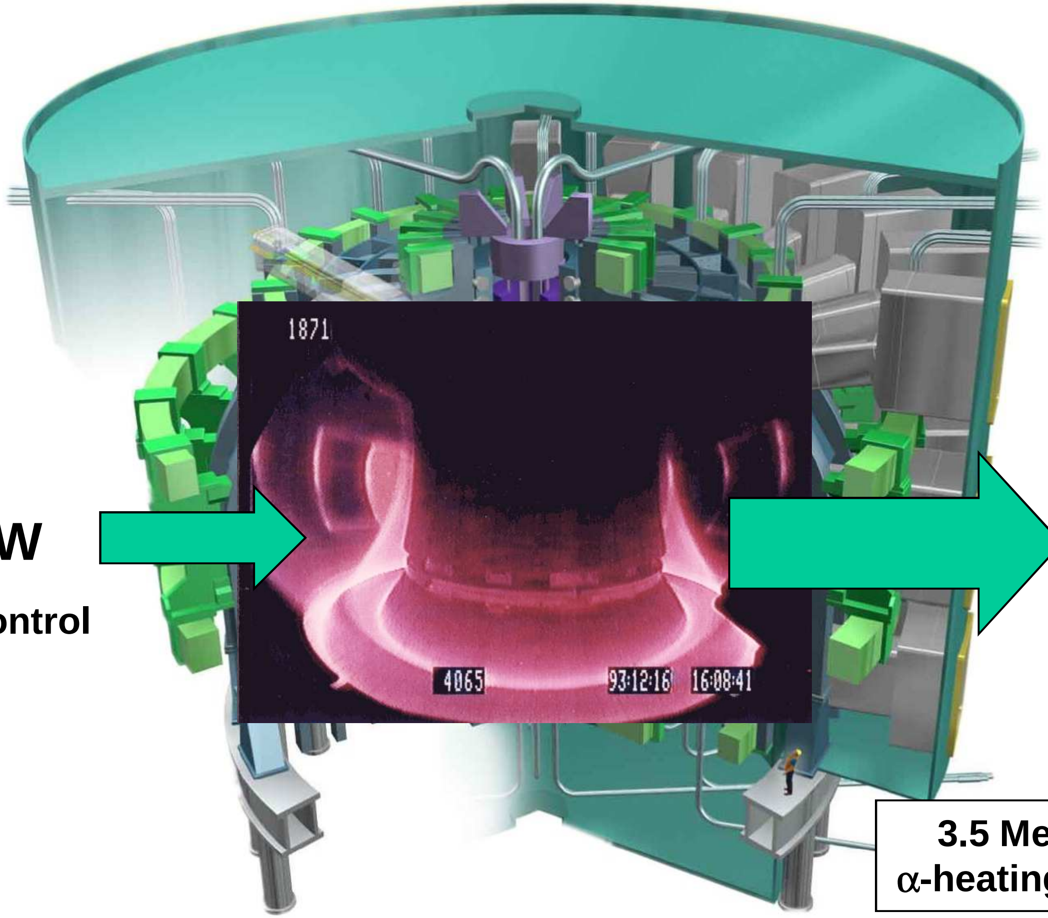
P_{out} = 2-3 GW_{th}

(aiming at 1 GW_e)

The ‚amplifier' is a thermonuclear plasma burning hydrogen to helium

Centre of the sun: T ~ 15 Mio K, $n \leq 10^{32}$ m^{-3}, p ~ 2.5 x 10^{11} bar

A bit closer look…



P_{in} = 50 MW

(initiate and control burn)

P_{out} = 2-3 GW_{th}

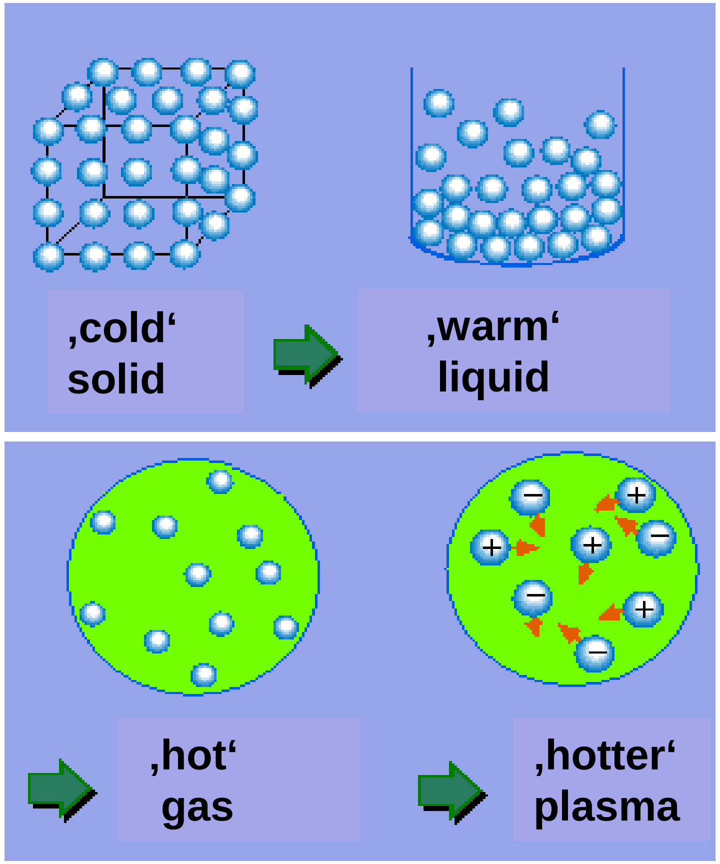
(aiming at 1 GW_e)

3.5 MeV α-heating	14.1 MeV wall loading

Fusion reactor: magnetically confined plasma, D + T → He + n + 17.6 MeV

Centre of reactor: T = 250 Mio K, $n = 10^{20}$ m^{-3}, p = 8 bar

What is a plasma?

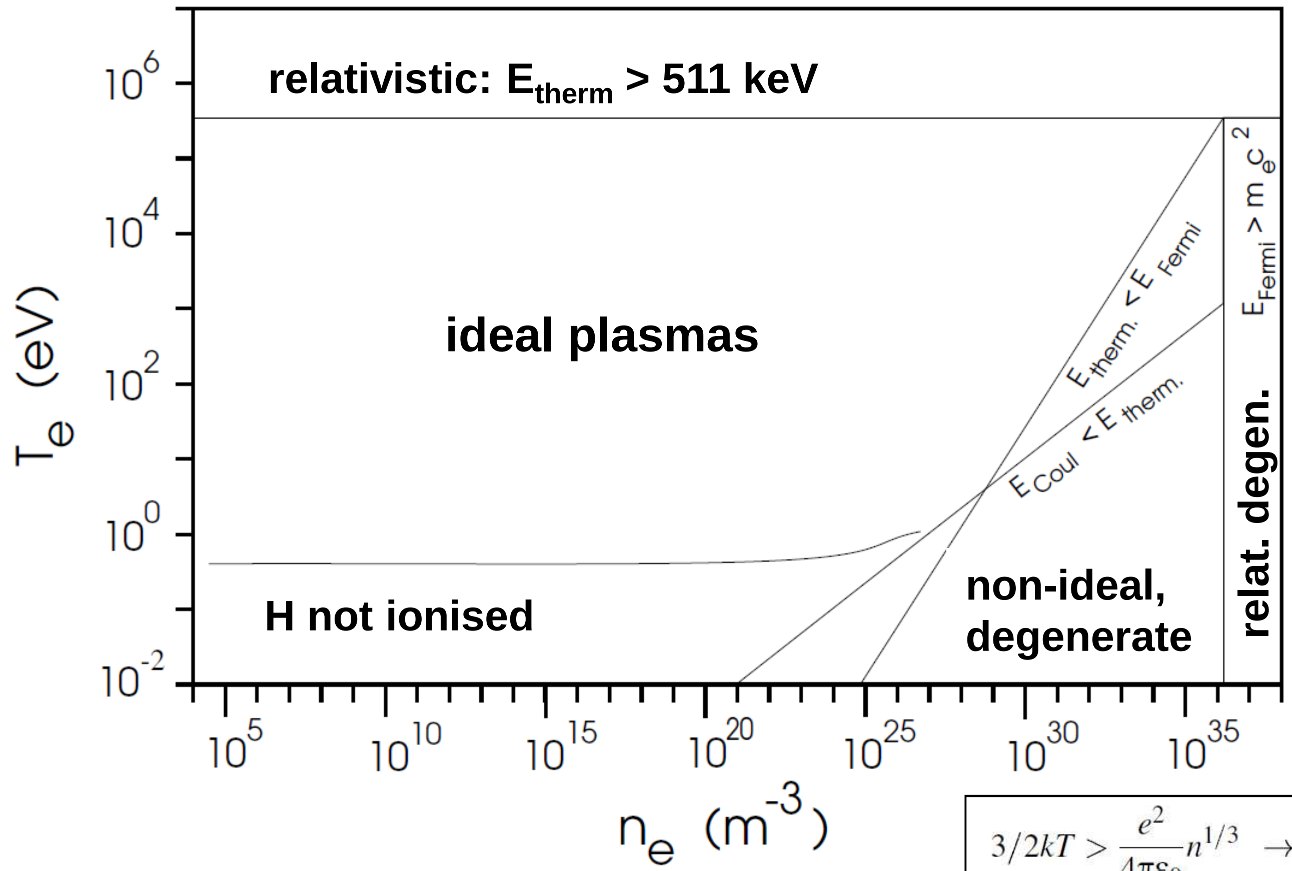


$$\frac{n_{Z+1} n_e}{n_Z} = \frac{g_{Z+1}}{g_Z} \frac{(2\pi m_e kT)^{3/2}}{h^3} e^{-\frac{W_{i,Z}}{kT}}$$

Plasma = ionised gas

- degree of ionisation $n_e/(n_e+n_0)$, depends on temperature (Saha equation)
- because of Maxwell distribution: $n_e/(n_e+n_0) \sim 1$ at $k_B T \sim 1/10\ W_{ion}$

Existence diagram: density n and temperature T



Note:
1 eV = 11600 K

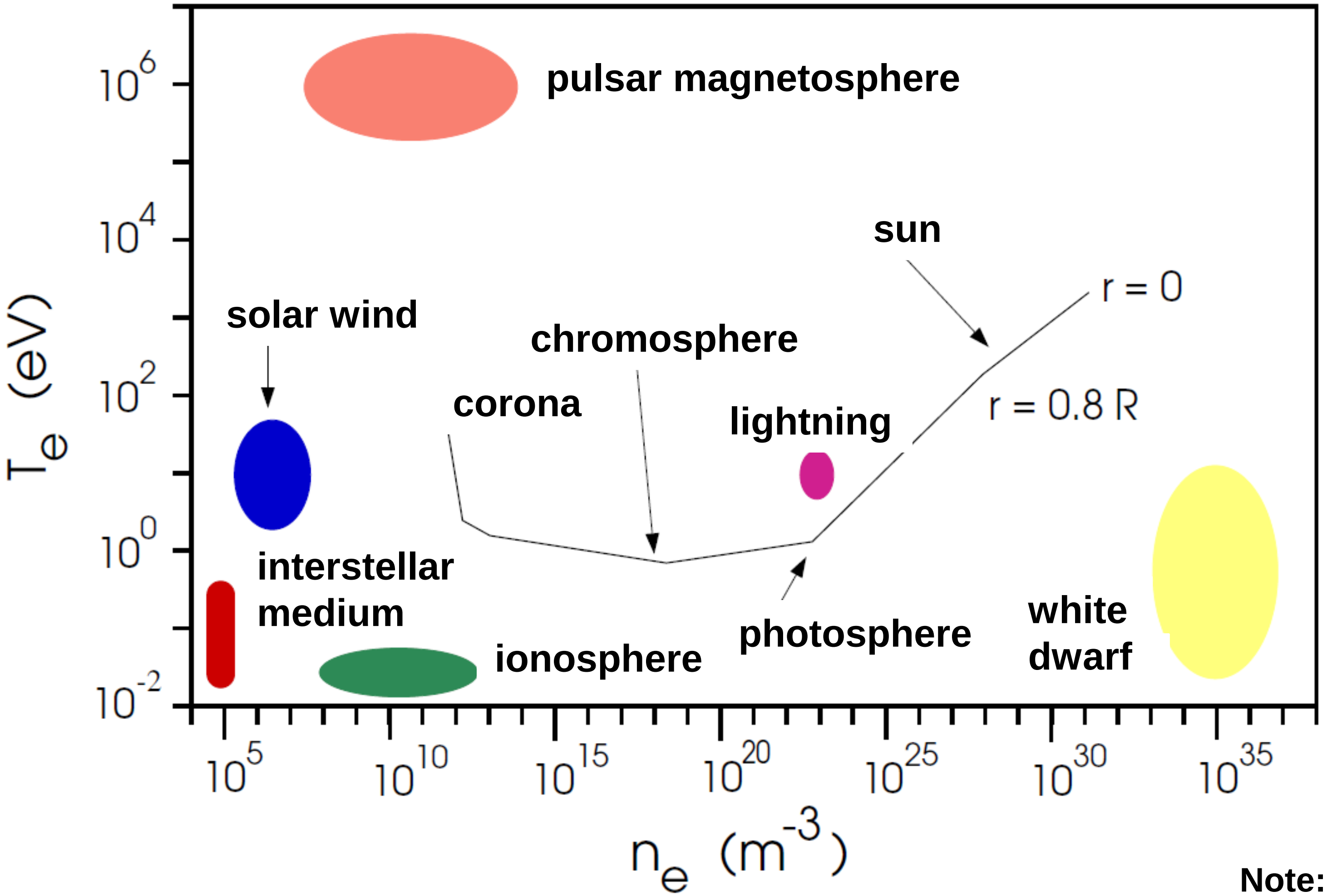
$$3/2kT > \frac{e^2}{4\pi\varepsilon_0} n^{1/3} \rightarrow T[\text{eV}] > 0.97 \times 10^{-9} (n[\text{m}^{-3}])^{1/3}$$

Plasmas occur in large large range of n and T

- ideal plasma condition $E_{therm} >> E_{interaction}$ in large range
- fusion plasma can be treated as ideal gas of ions and electrons ($p = n k_B T$)

Existence diagram: density n and temperature T

Astrophysical plasmas



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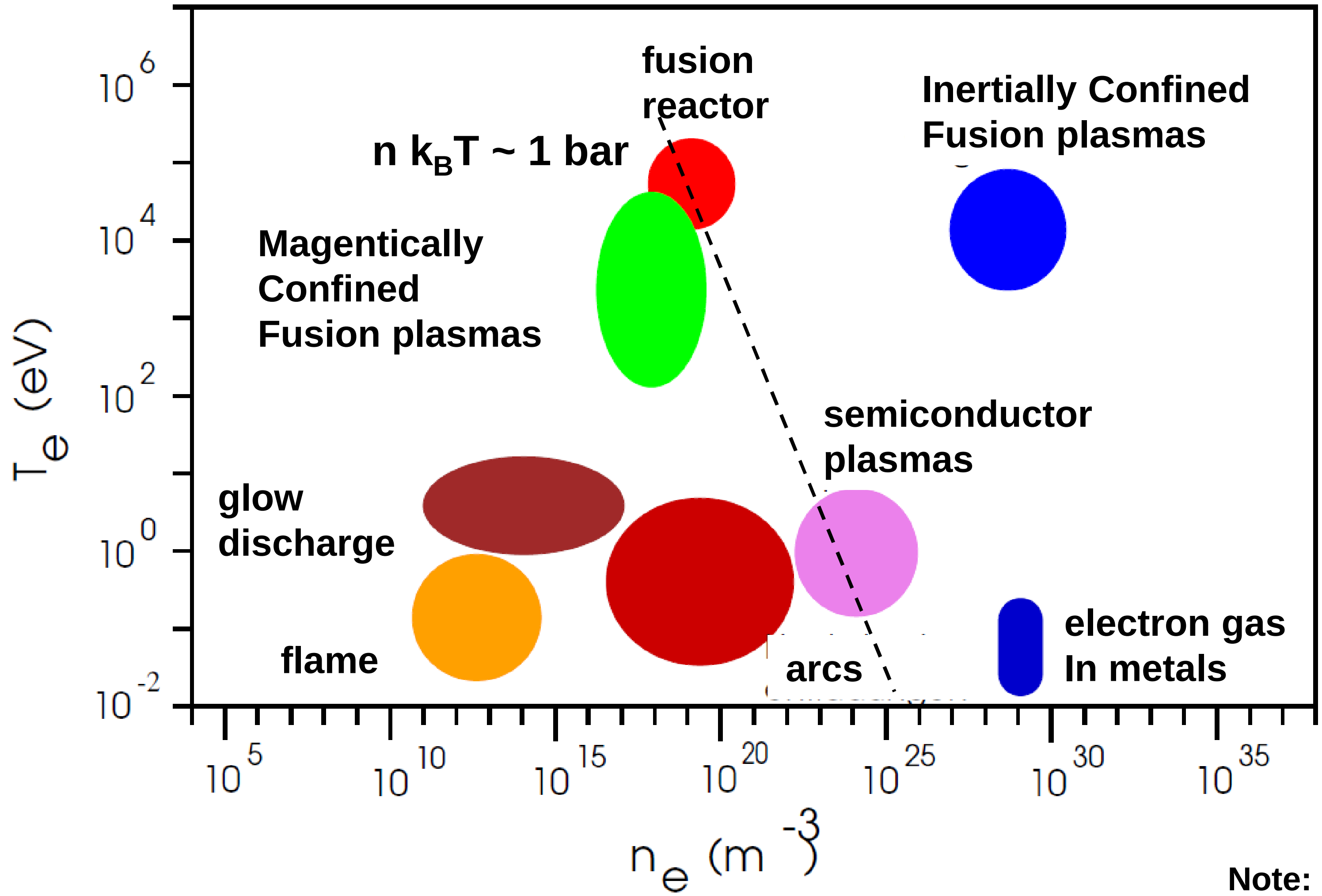
Existence diagram: density n and temperature T

lab plasmas



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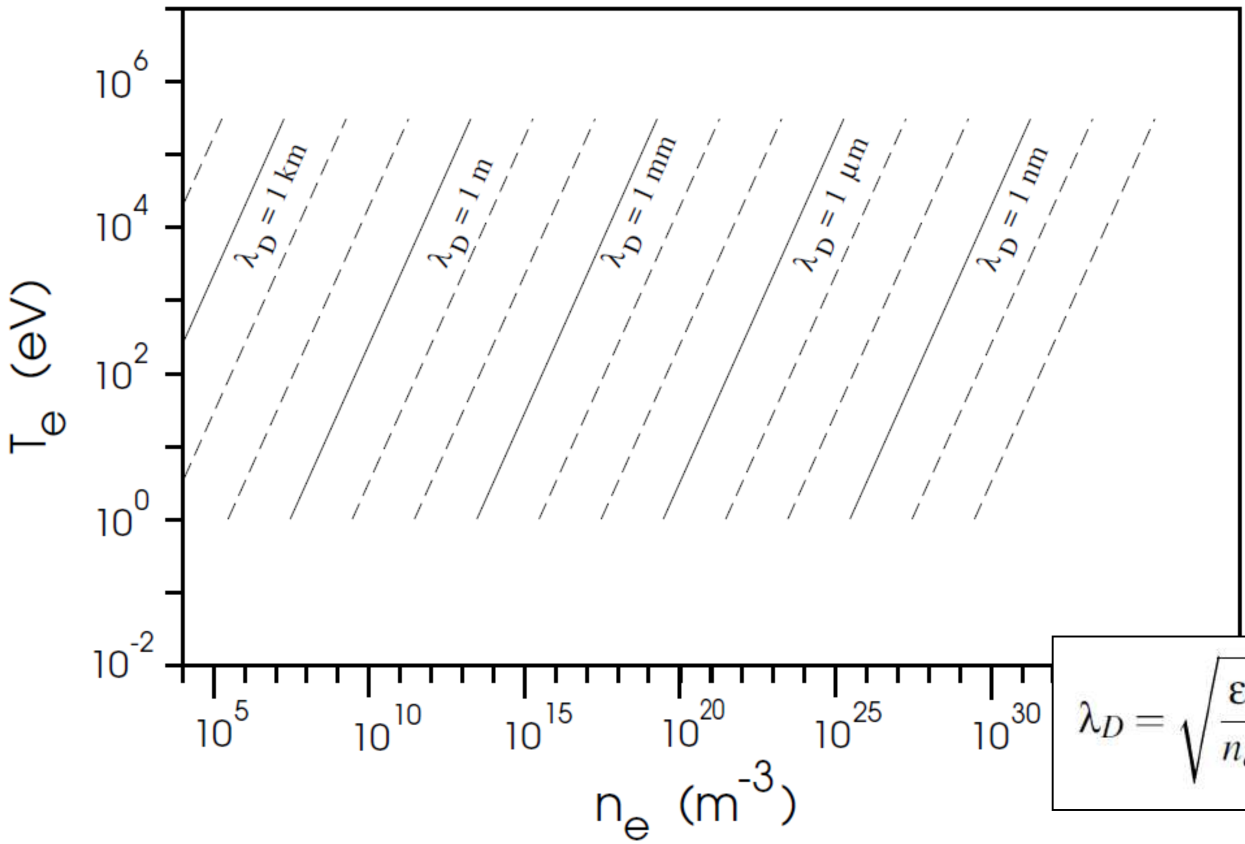
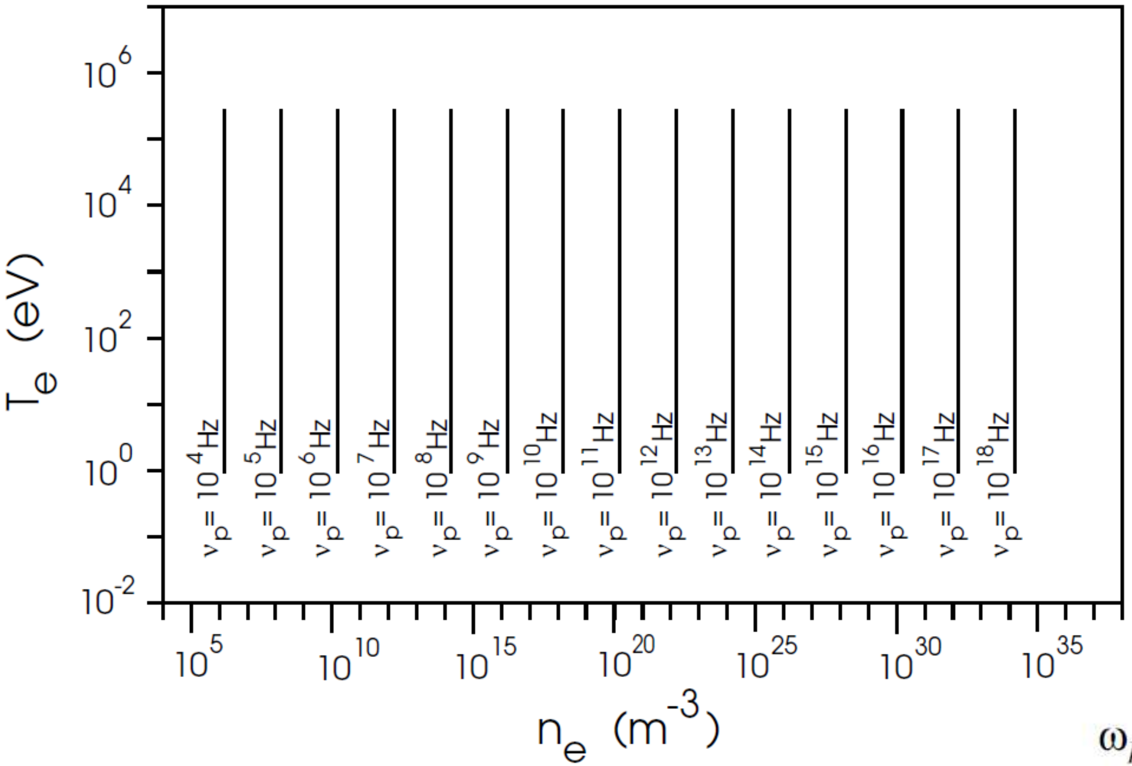


$$\lambda_D = \sqrt{\frac{\varepsilon_0 kT}{n_{e,0}e^2}} = 7437\sqrt{\frac{T[\mathrm{eV}]}{n[\mathrm{m}^{-3}]}}[\mathrm{m}]$$

Large number of freely movable charges: charge separation leads to strong electric fields – strong restoring force

- quasineutrality $n_e = Zn_i$ can only be violated on Debye length λ_D
- on a scale L >> λ_D, plasmas are always quasineutral

Dynamic shielding – plasma frequency



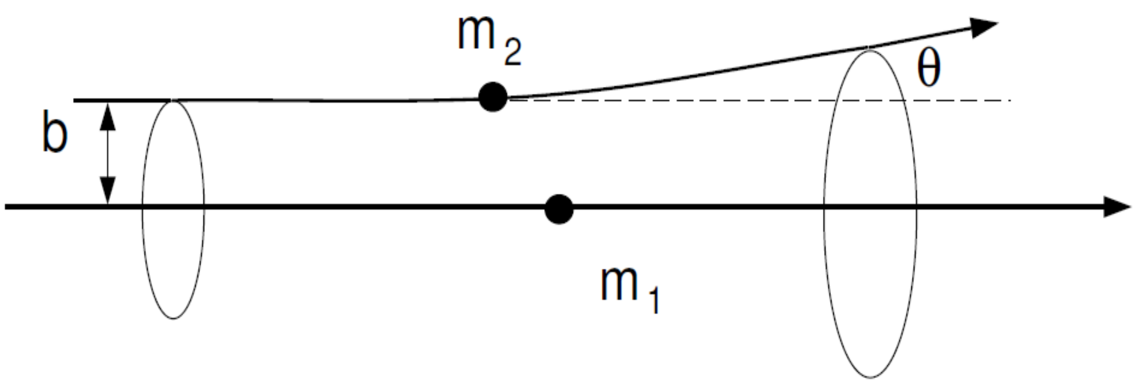
$$\omega_p = \sqrt{\frac{n_e e^2}{m_e \varepsilon_0}} = 56.4(n_e[\mathrm{m}^{-3}])^{1/2} s^{-1}$$

Displacement of electrons leads to large restoring force - oscillation

- below ω_p, electrons can follow oscillating e-field – reflection of wave (cut-off)
- above ω_p, electrons can no longer follow - plasma transparent to $\omega > \omega_p$

Used for density measurement (‘reflectometry’) – cut-off important for heating

Coulomb collisions



$$\tau_{ee} = \frac{3\sqrt{m_e}(4\pi\varepsilon_0)^2}{4\sqrt{\pi}e^4 \ln\Lambda} \frac{(kT)^{3/2}}{n_e}$$

$$\sigma = \frac{3(4\pi\varepsilon_0)^2}{4\sqrt{2\pi m_e} e^2 \ln\Lambda}(kT)^{3/2}$$

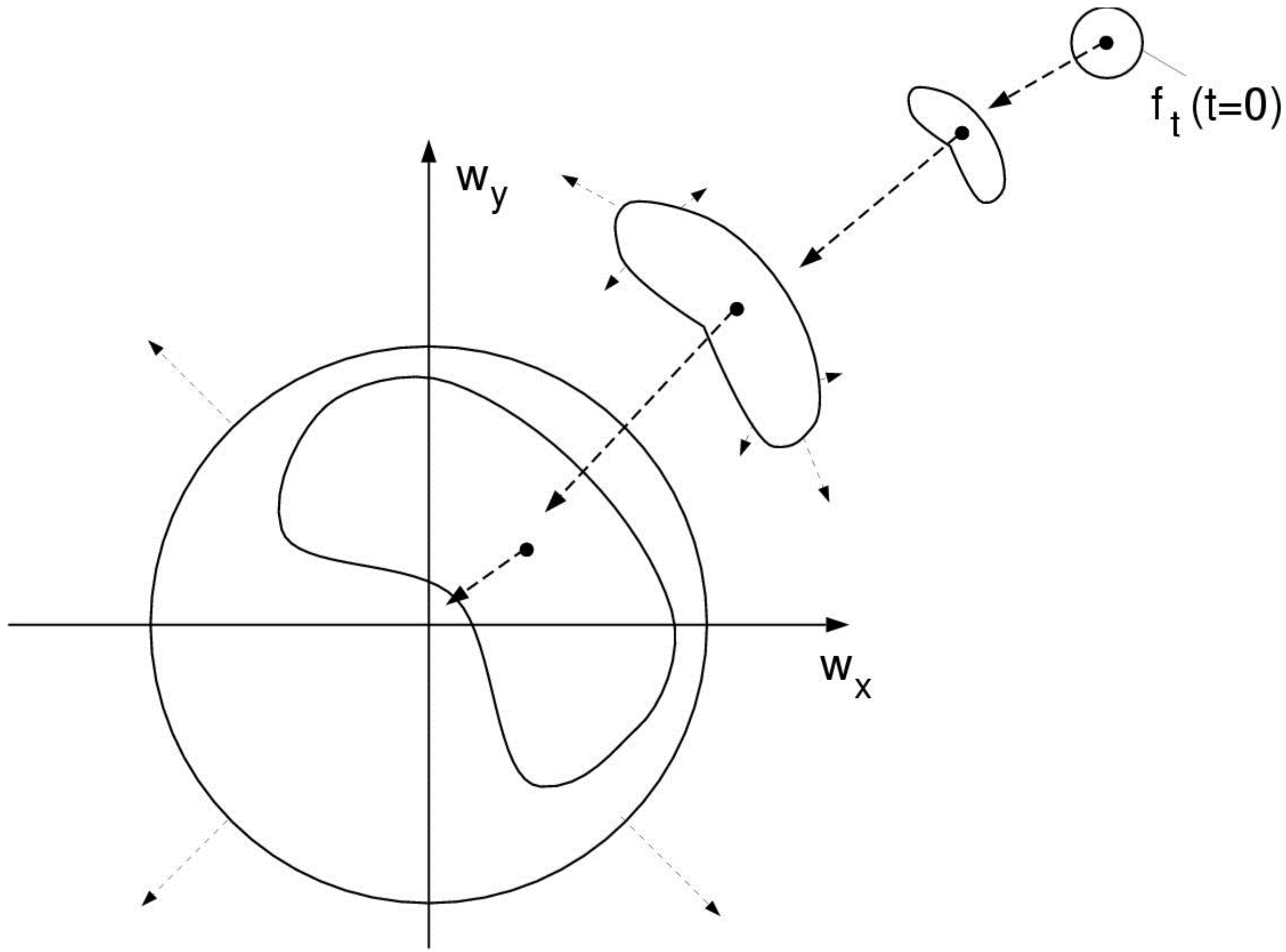
Coulomb collisions are the main interaction between plasma particles

- thermodynamic equilibrium through Coulomb collisions
- dissipation by Coulomb collisions – electrical and thermal resistance

Collision frequency decreases with increasing temperature

- mean free path increases with T – ‘collisionless plasma’
- electrical (‘Spitzer’) and thermal conductivity of fusion plasma very high

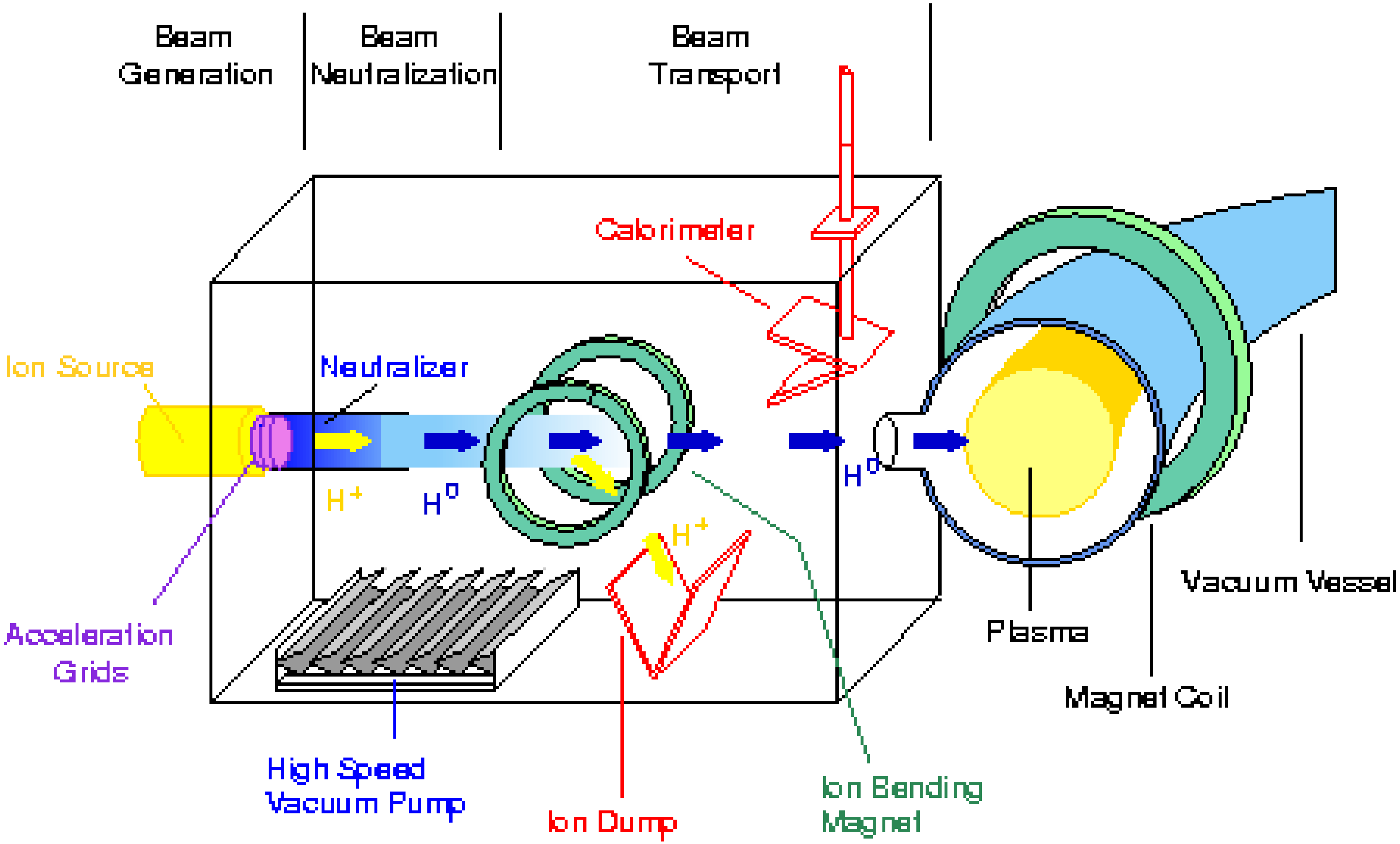
Thermalisation of a fast particle ensemble



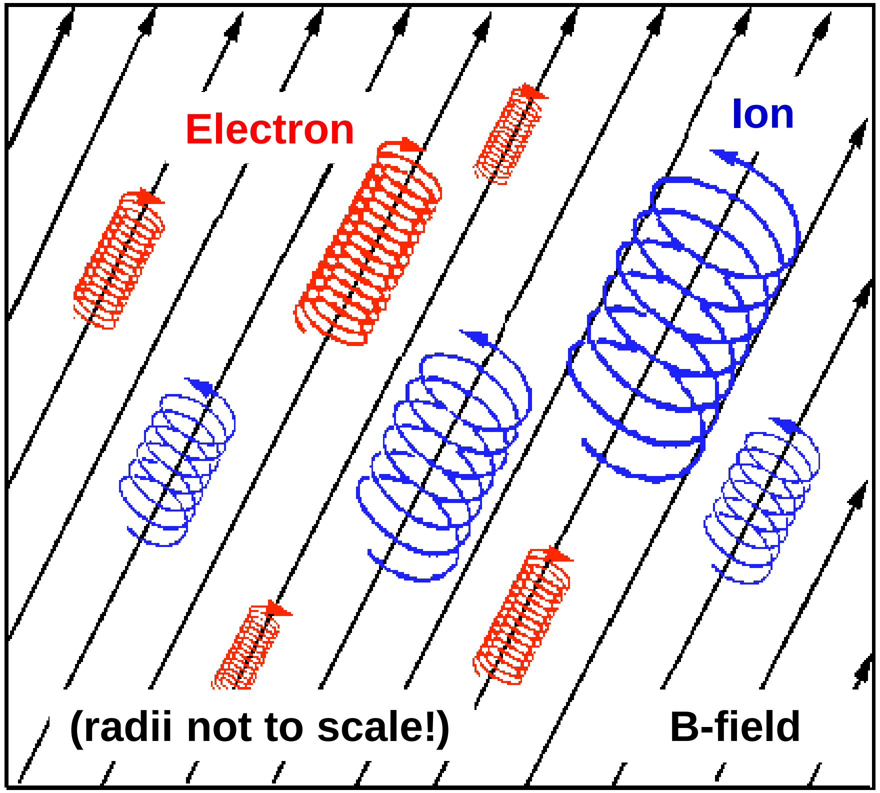
'Isotropisation' – collisions randomise velocity components

'Slowing down' – collisions transfer energy to Maxwellian bulk

Application: Neutral Beam Heating (NBI)



Magnetised plasmas – single particle picture



$$\omega_c = \frac{|q|B}{m}$$

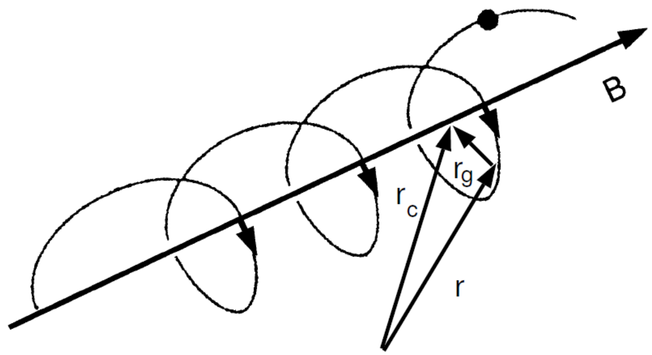
$$r_L = \frac{mv_\perp}{|q|B} = \frac{\sqrt{2mkT}}{|q|B}$$

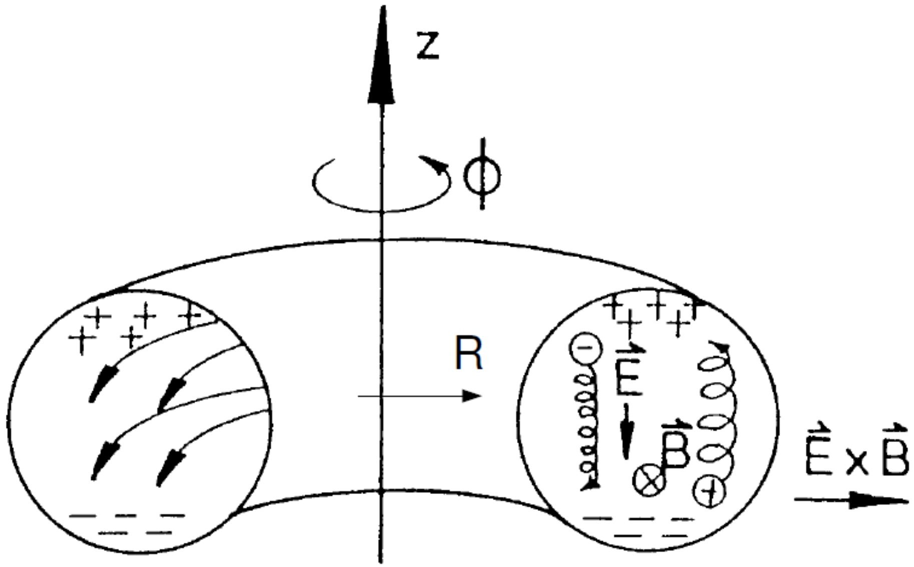
$$r_{Le}[\mathrm{m}] = 3.38 \times 10^{-6} \frac{\sqrt{T_e[\mathrm{eV}]}}{B[\mathrm{T}]}$$

Charged particles gyrate perpendicular B, but move freely along B

- cyclotron frequency ω_c used for diagnostics and heating (ECRH)
- for $k_b T \sim 1$ keV and B = 2T: $r_{Le} \sim 50\ \mu m$, $r_{Li} \sim 2$ mm $\Rightarrow$ magnetised plasma

Magnetised plasmas – particle drifts





On timescales much longer than $1/\omega_c$, motion of gyrocentre is relevant

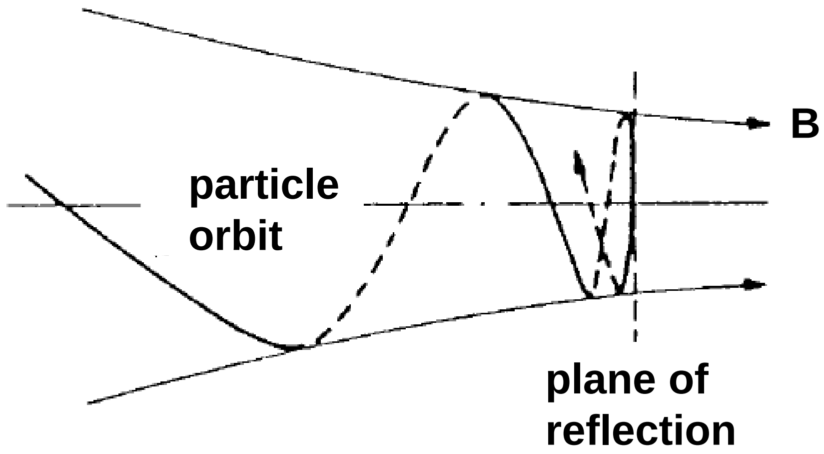
For an external force F, a drift perpendicular drift $\mathbf{v}_D = \dfrac{\mathbf{F} \times \mathbf{B}}{qB^2}$ is obtained

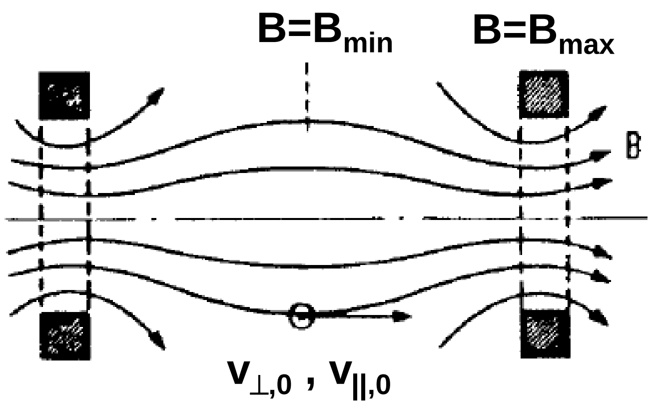
Example: Plasma confinement in purely toroidal field

- curved magnetic field leads to vertical drift (centrifugal force)
- resulting E field leads to a net outward drift

Plasma confinement in a purely toroidal field is not possible (see later)

Single particle picture – magnetic mirror



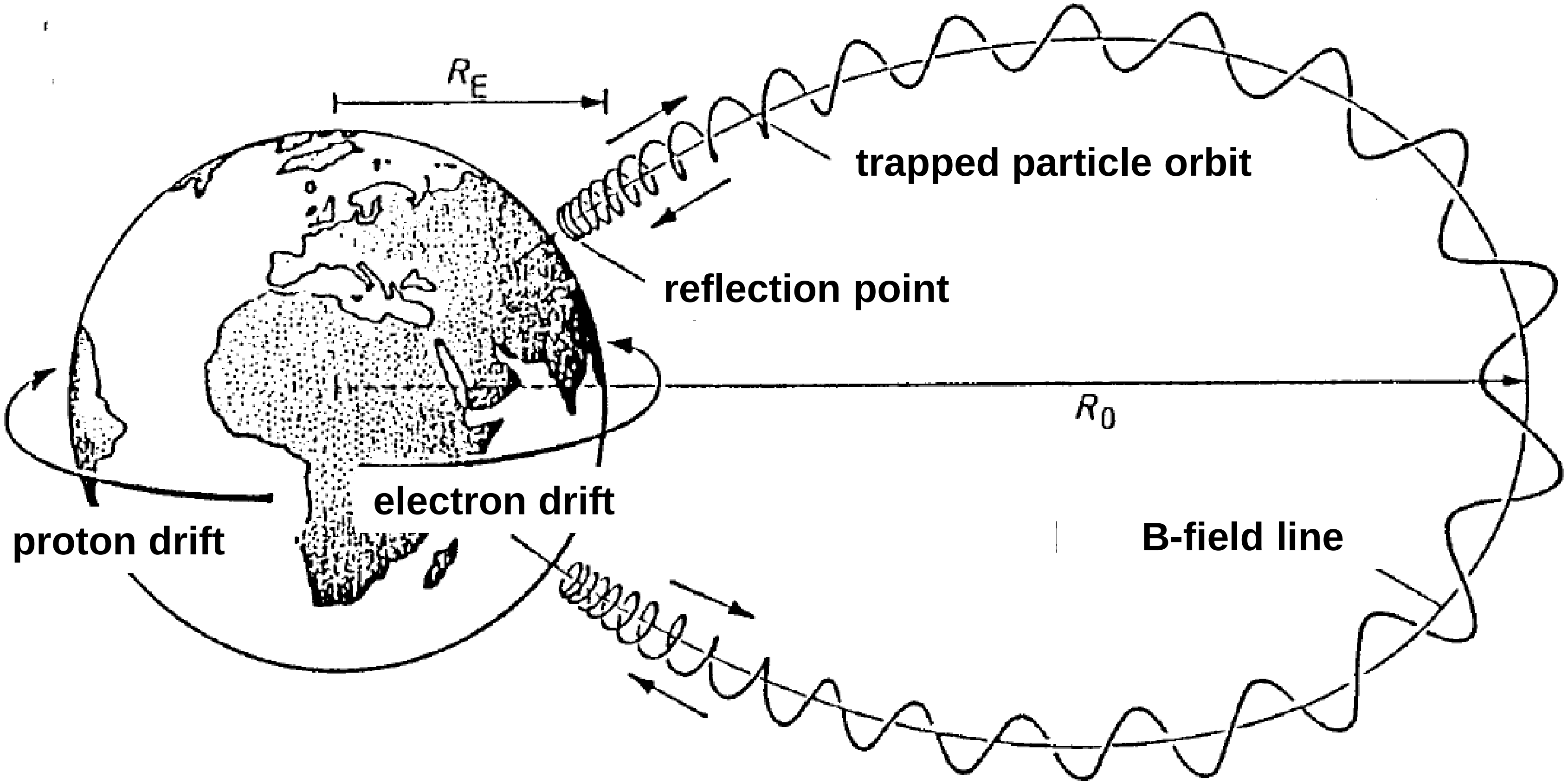


$$\oint p\,dq = \int_0^{2\pi} m\omega_c r_L^2 d\theta = 2\pi m\omega_c r_L^2 = 4\pi\frac{m}{q}\frac{mv_\perp^2}{2B} = 4\pi\frac{m}{q}\mu$$

For ‘adiabatic changes of gyromotion (gyro-circles almost closed):

- magnetic moment $\mu \sim mv_\perp^2 / B$ = const. along trajectory
- since total energy is conserved, || energy converted to $\perp$ if B increases

Particle orbits: mirror in the Earth's magnetic field



Magnetised plasmas – many body picture

A comprehensive approach deals with a description in 6-d phase space

- 'kinetic theory' of distribution function f(**v**,**x**,t) – too complicated for today ☺

If thermodynamic equilibrium is assumed (f = Maxwellian), the set of MagnetoHydroDynamic (MHD) equations can be used:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho\vec{v}) = 0$$ continuity equation

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}\right) = -\nabla\cdot\mathbf{P} + \vec{j}\times\vec{B}$$ force (Euler) equation

$$\vec{E} + \vec{v}\times\vec{B} = \frac{1}{\sigma}\vec{j}$$ Ohm's law

+ equation of state (e.g. adiabatic)

+ Maxwell's equations

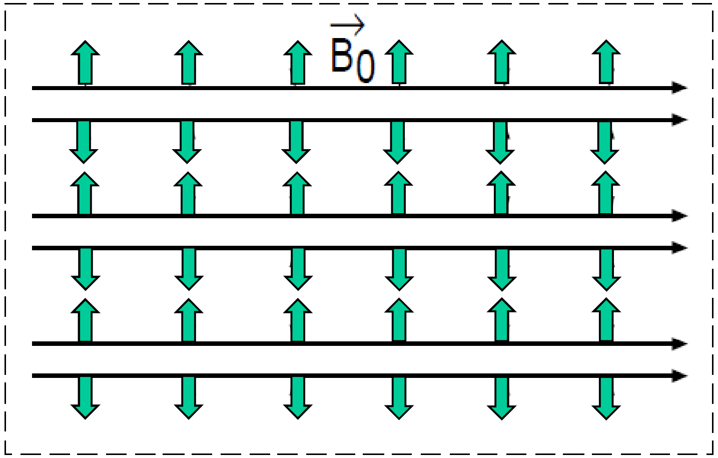
- mostly adequate for motion perpendicular to B, usually not along B

Magnetohydrodynamic (MHD) - equilibrium

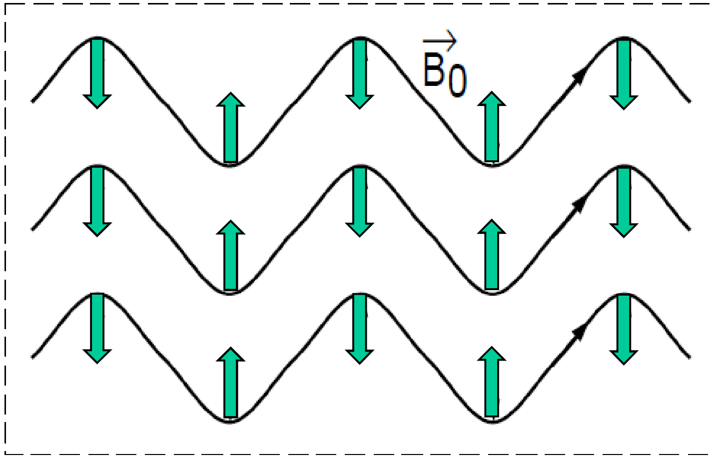
In equilibrium, there is no time dependence. If in addition, no flow:

$$\nabla p = \mathbf{j} \times \mathbf{B} \quad \Rightarrow \quad \nabla p = \frac{1}{\mu_0}(\nabla \times \mathbf{B}) \times \mathbf{B} \quad \Rightarrow \quad \nabla_\perp \left(p + \frac{B^2}{2\mu_0}\right) + \frac{B^2}{\mu_0 R_c}\mathbf{e}_{R_c} = 0$$

One can identify two contributions to the force balance:



magnetic pressure



field line tension

N.B.: these two forces lead to two branches of MHD (Alfvén) waves

Ohm's law and the 'frozen fieldlines'

The change of magnetic field is governed by Ohm's law:

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} = \nabla \times \left(\vec{v} \times \vec{B}\right) - \frac{1}{\mu_0 \sigma} \nabla \times \left(\nabla \times \vec{B}\right)$$

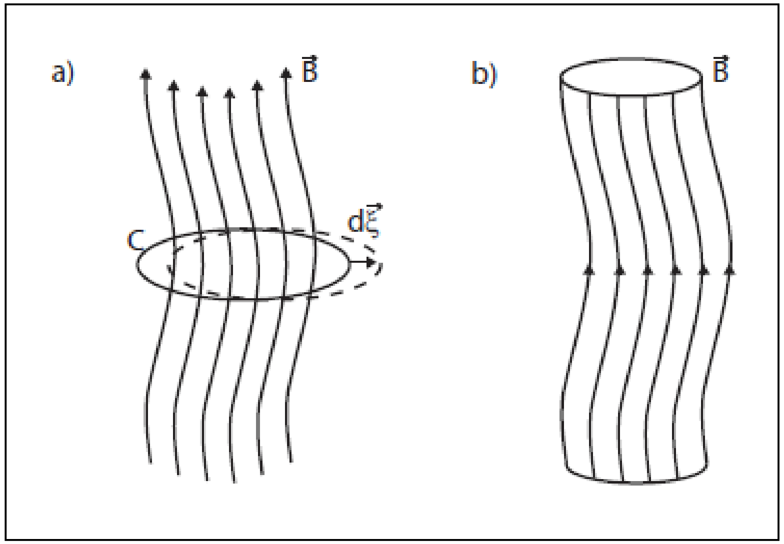
$$\Rightarrow \frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{v} \times \vec{B}\right) + \frac{1}{\mu_0 \sigma} \Delta \vec{B}$$

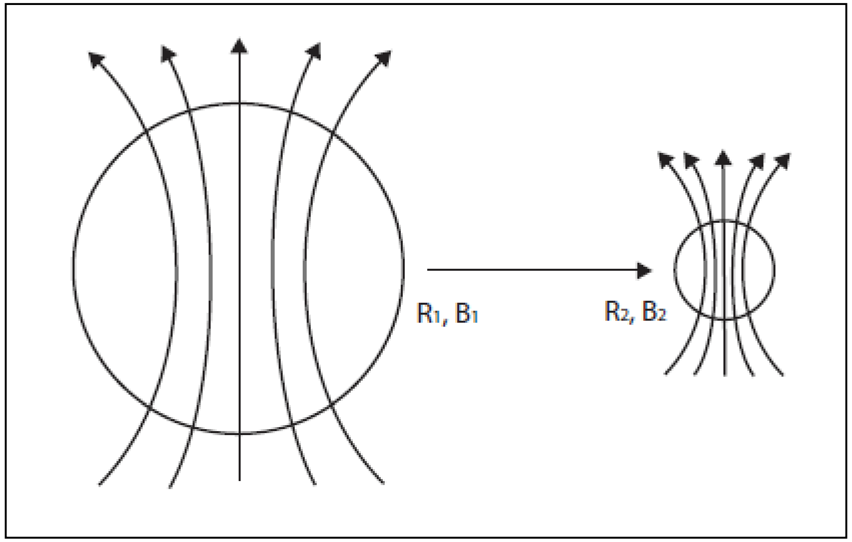
- in ideal MHD, plasma and field lines move together (Alfvén time scale, fast)
- resistivity leads to a diffusion of the magnetic field through the plasma (resistive MHD time scale, arbitrarily slow for arbitrary high conductivity)

Important example: collapse of a star leads to enormous field amplification!

Ohm's law and the 'frozen fieldlines'

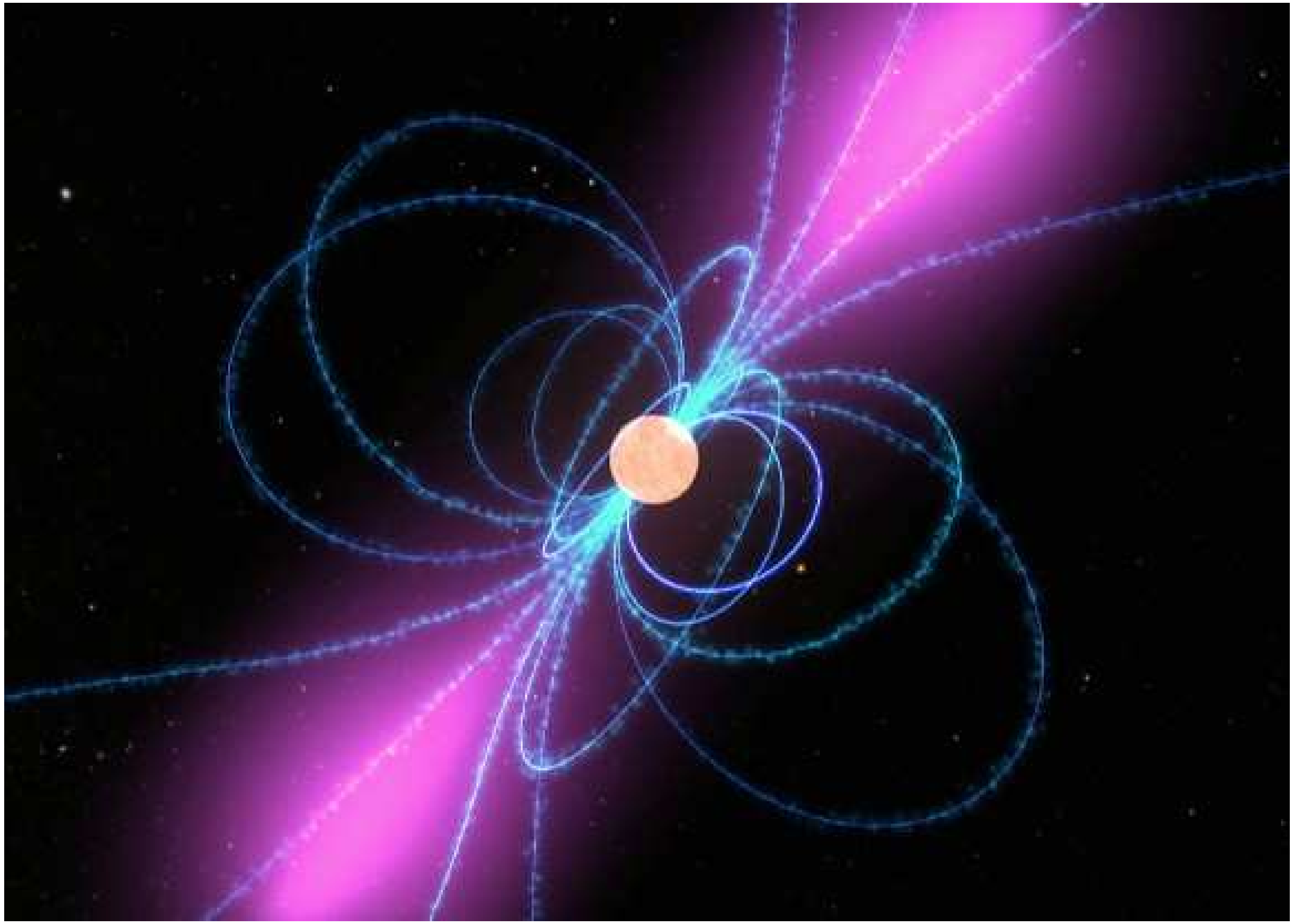
Concept of 'flux tubes':





- in ideal MHD, plasma and field lines move together (Alfvén time scale, fast)
- flux tubes move with fluid and cannot intersect – topology conserved
- example: collapse of a neutron star

Emission from pulsars validates high B-fields

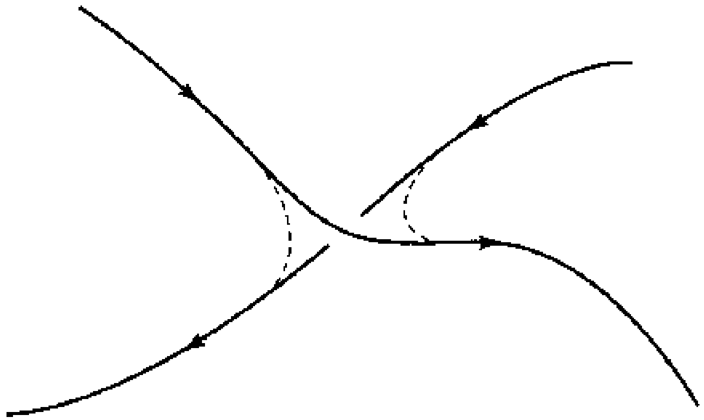


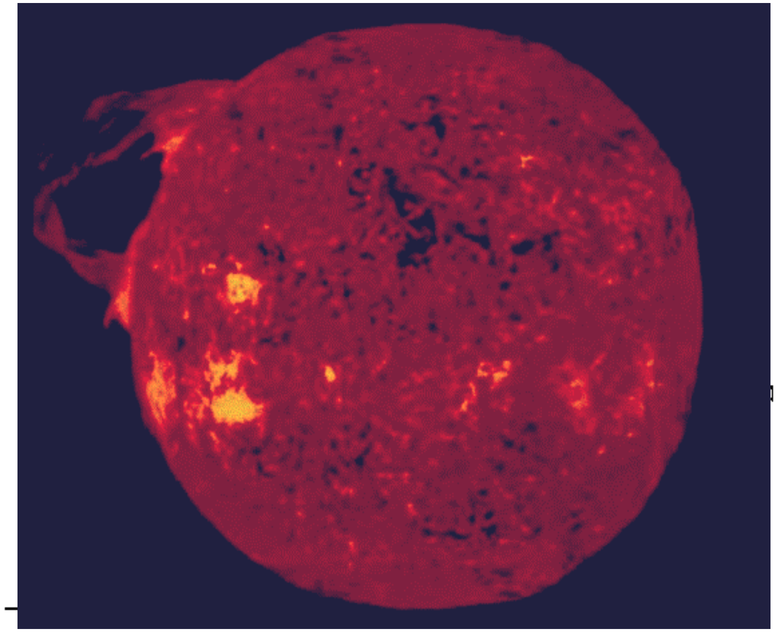
Beamed (relativistic) ‚curvature radiation‘ from parallel e^- motion

Reconnection in a hot fusion plasma

Due to high electrical conductivity, magnetic flux is frozen into plasma

$\Rightarrow$ magnetic field lines and plasma move together





A change of magnetic topology is only possible through reconnection

- opposing field lines reconnect and form new topological objects
- requires finite resistivity in the reconnection region

Example: Coronal Mass Ejection (CME) from the sun

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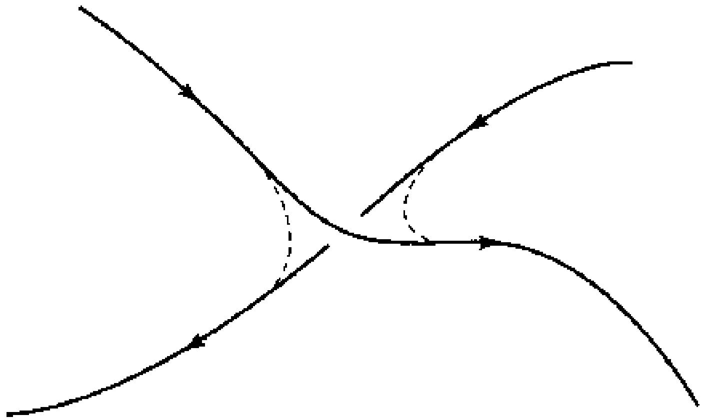

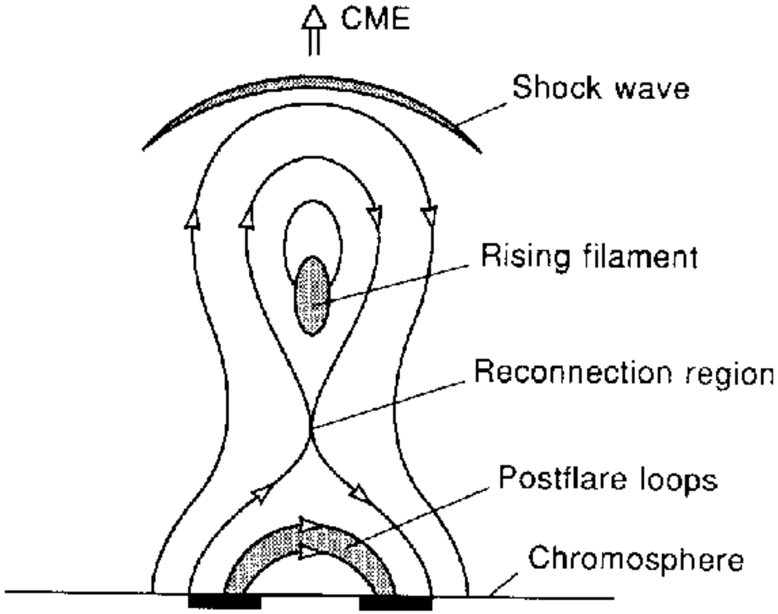

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Plasma waves: two fluid equations

$$\left(1-\frac{k^2c^2}{\omega^2}\right)\mathbf{E}+\frac{c^2}{\omega^2}\mathbf{k}(\mathbf{k}\cdot\mathbf{E}) = -i\frac{\omega_{pe}^2}{\omega}\frac{m_e}{e}(\mathbf{u}_i-\mathbf{u}_e) \quad \text{wave eqn.}$$

$$\mathbf{u}_i - i\frac{\omega_{ci}}{\omega}\mathbf{u}_i\times\mathbf{b}-\frac{c_i^2}{\omega^2}\mathbf{k}(\mathbf{k}\cdot\mathbf{u}_i) = i\frac{e}{\omega m_i}\mathbf{E} \quad \text{ion force balance}$$

$$\mathbf{u}_e + i\frac{\omega_{ce}}{\omega}\mathbf{u}_e\times\mathbf{b}-\frac{c_e^2}{\omega^2}\mathbf{k}(\mathbf{k}\cdot\mathbf{u}_e) = -i\frac{e}{\omega m_e}\mathbf{E} \quad \text{electron force balance}$$

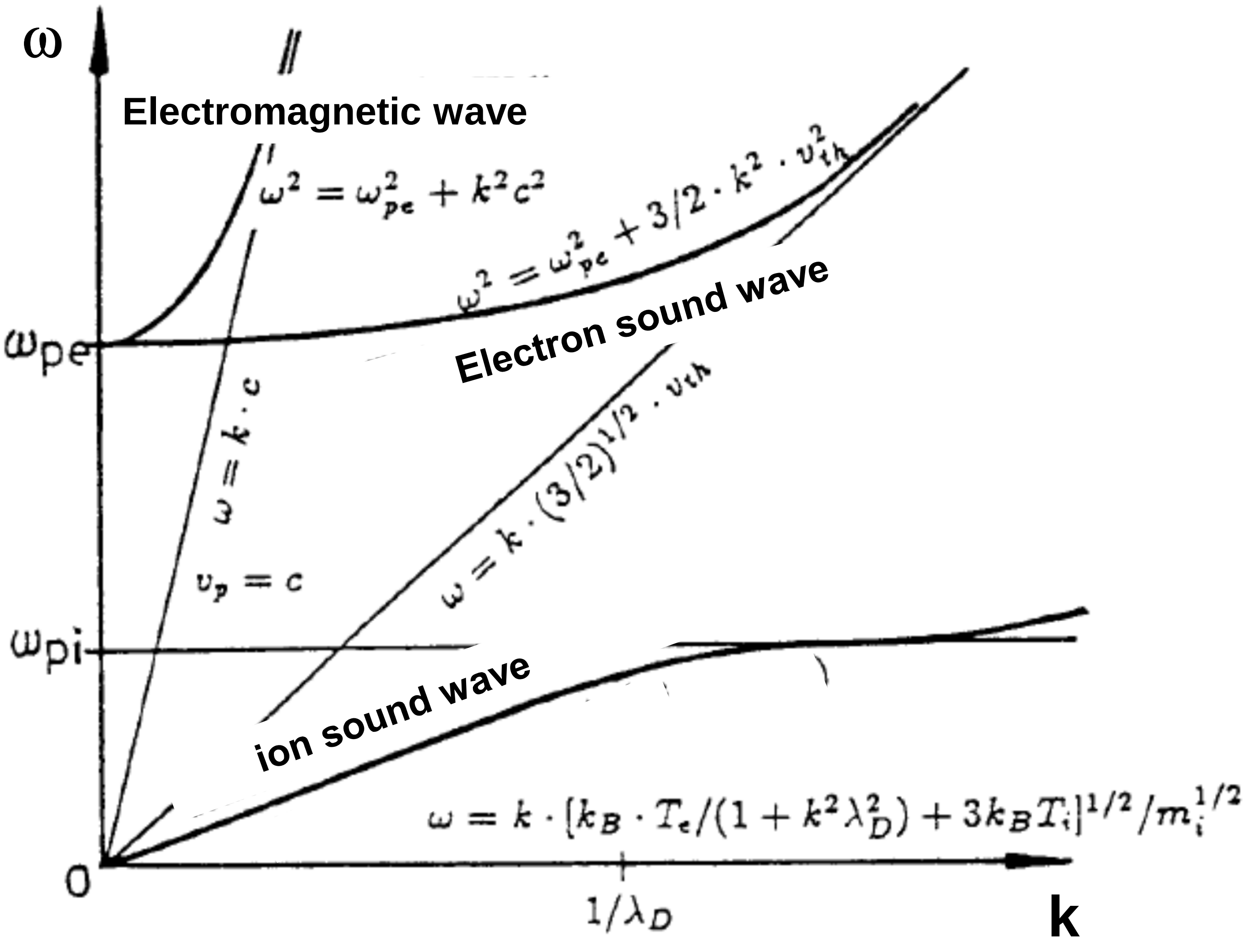
After linearisation and Fouriertransform, a 9 x 9 Matrix system is obtained

Matrix is the equivalent of the ‚dielectric tensor'

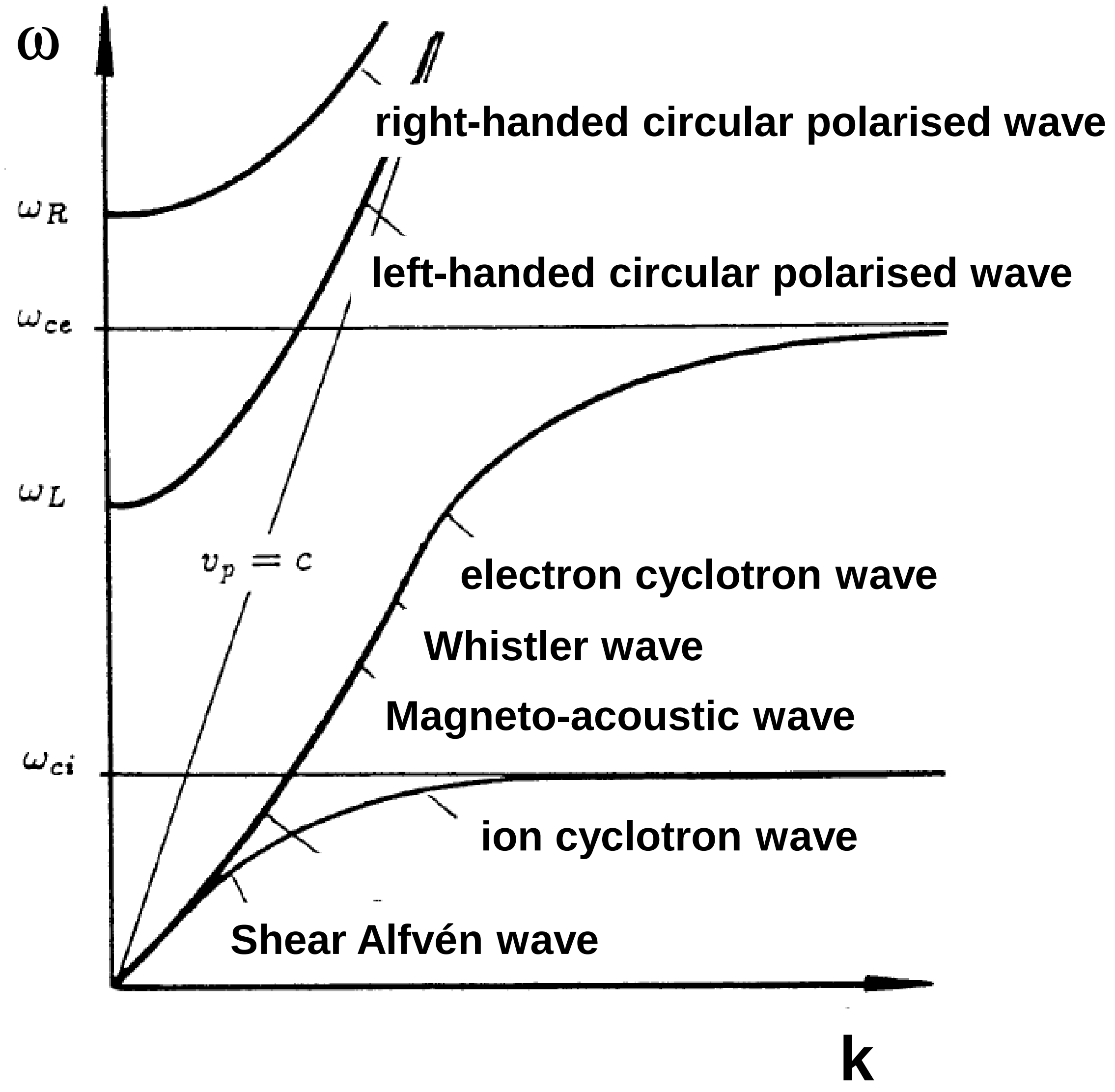
Solutions give dispersion relation $\omega = \omega(k)$

- neutral gas: e-m waves and sound waves uncoupled
- plasma: sound waves couples to electrostatic wave (charge density)

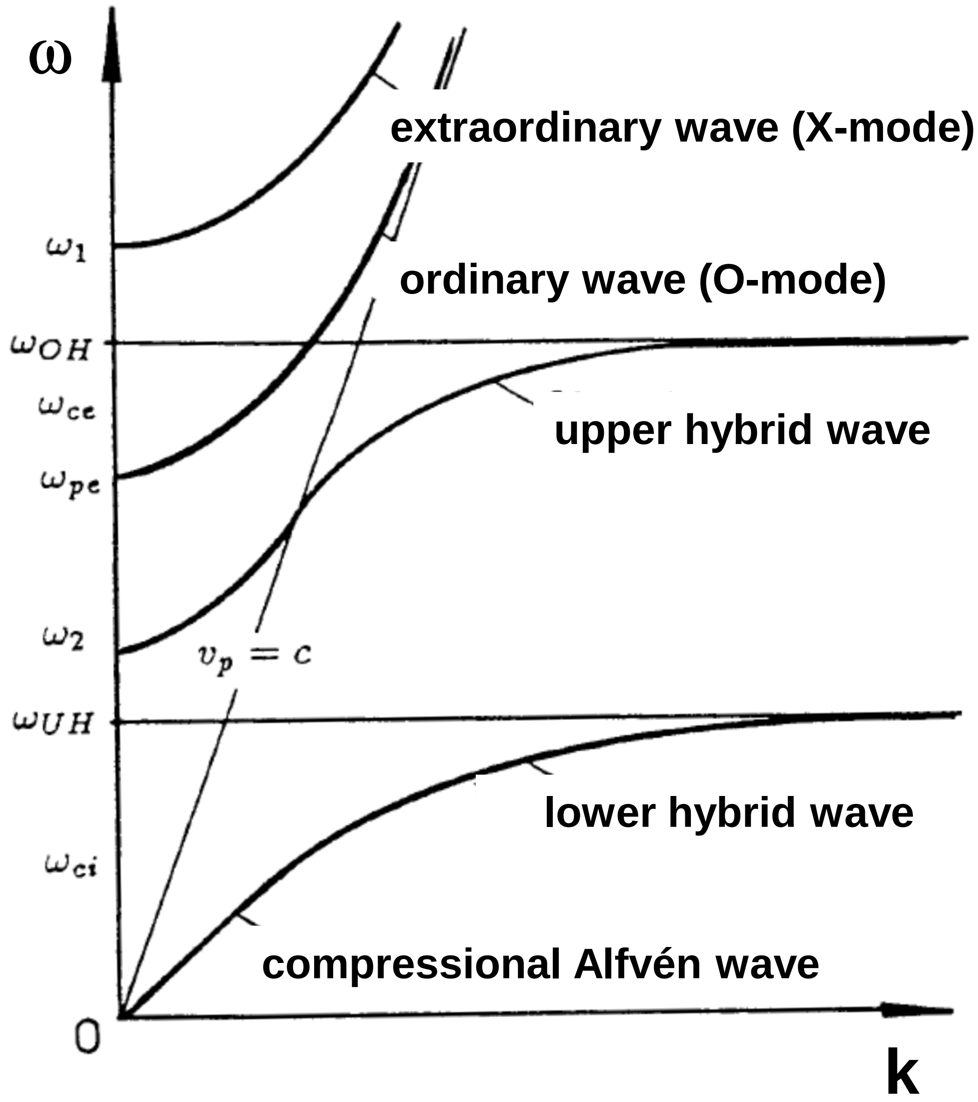
Waves in an Unmagnetised Plasma



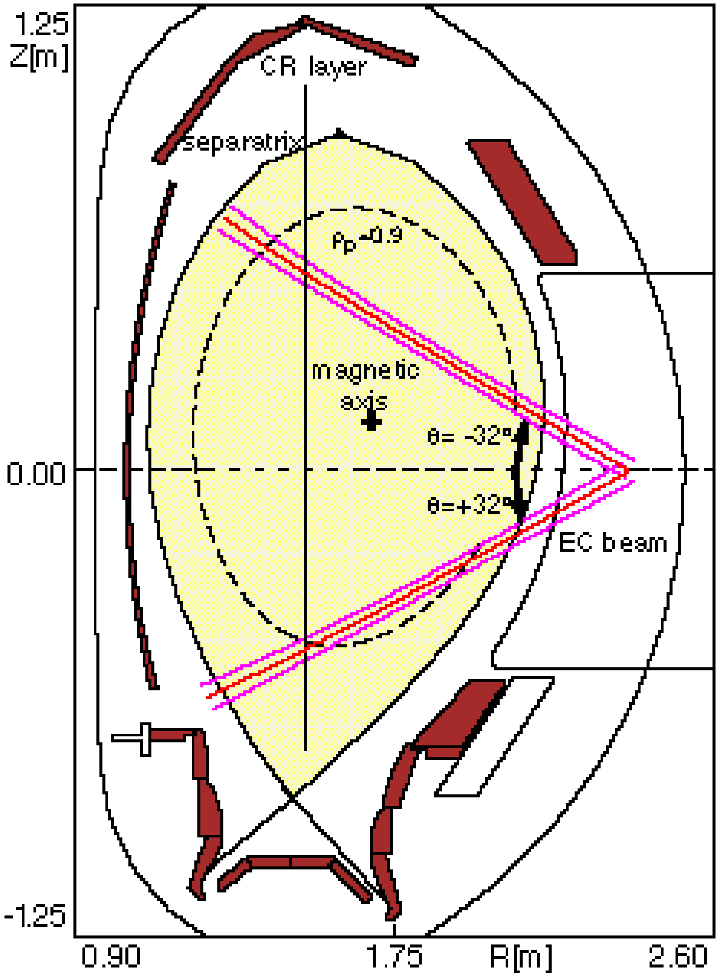
Waves in a Magnetised Plasma: propagation || to B

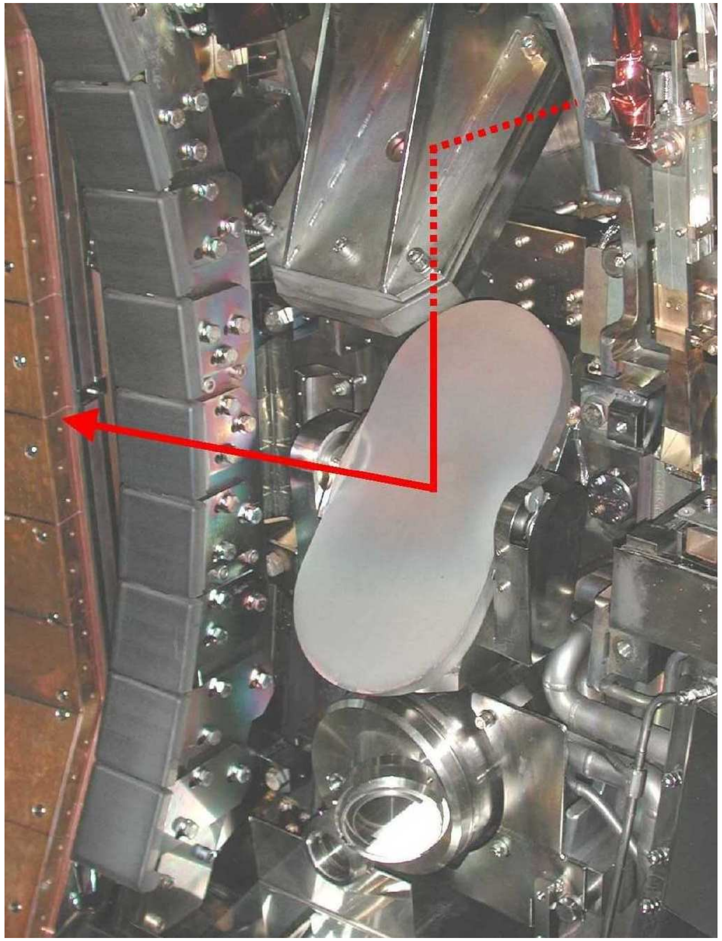


Waves in a Magnetised Plasma: propagation ⊥ to B



Electron Cyclotron Resonance Heating (ECRH)

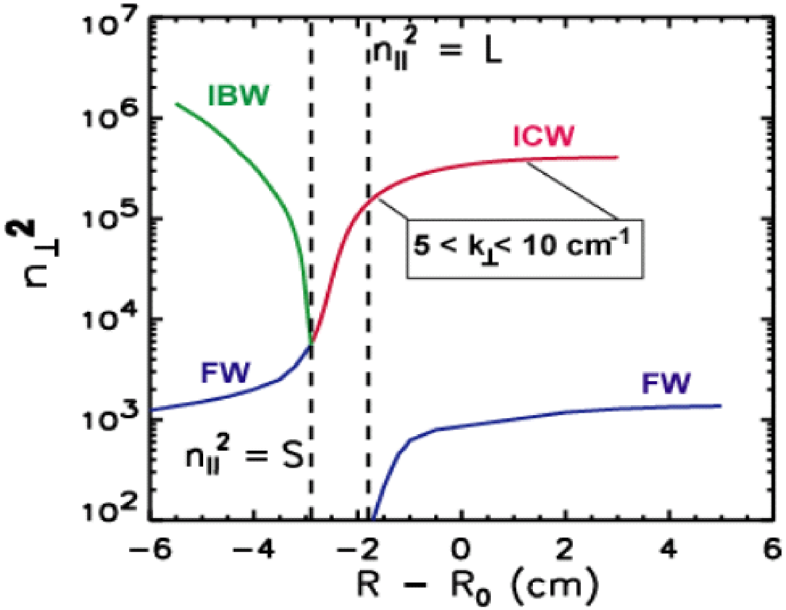


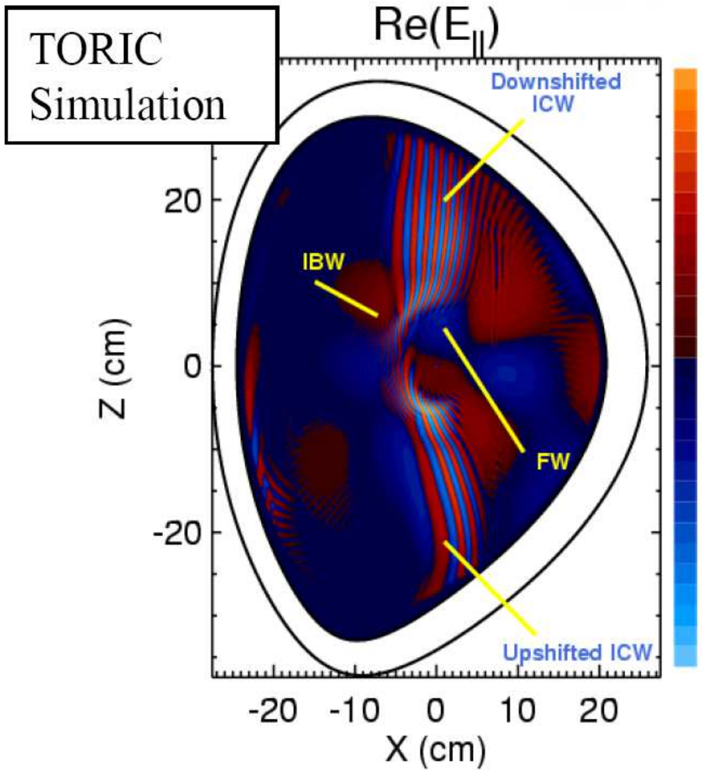


Microwave beam absorbed at $\omega=\omega_{ce}$ – good localisation and control

Example: ion Bernstein wave, electrostatic ion cyclotron wave

- In the vicinity of the ion-ion hybrid layer, mode conversion to shorter wavelength waves occurs.





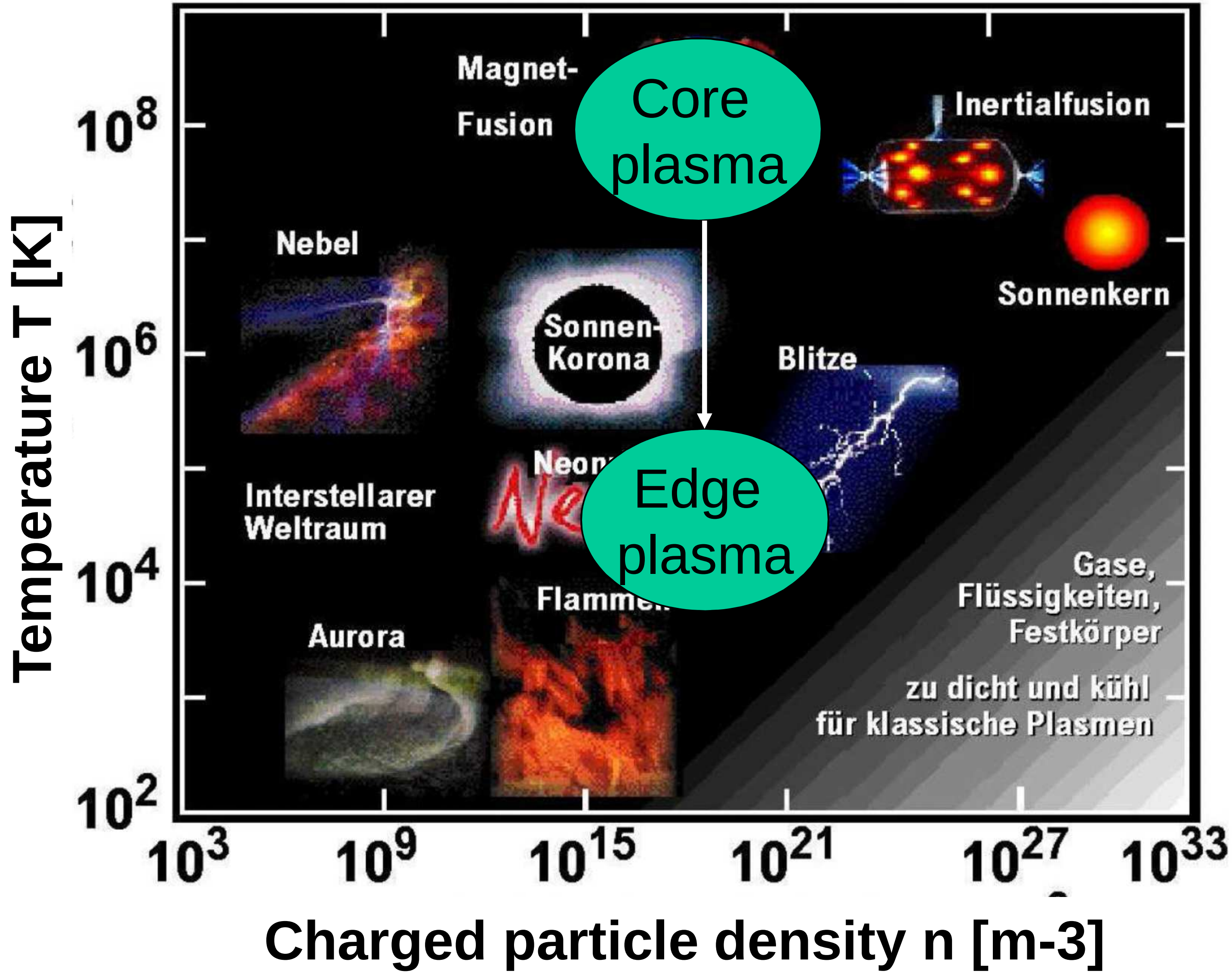
IBW : Ion Bernstein Wave

Propagates towards the high field side

ICW : Ion Cyclotron Wave

Propagates towards the low field side

‚The Plasma Universe‘



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