

Fast particles in tokamak plasmas

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Thanks to many contributors, especially Simon Pinches and Jon Graves

- Particle orbits in tokamaks
- Sources of energetic particles
- Measuring fast ions
- Interaction of energetic particles with instabilities
 - Resonant interaction of fast ions with Alfvén instabilities
 - Effect of fast ions on instabilities in thermal plasma
- Redistribution and loss of fast ions
 - Losses from instabilities and 3D fields
 - Effects on driven current
- Controlling and utilising fast ions



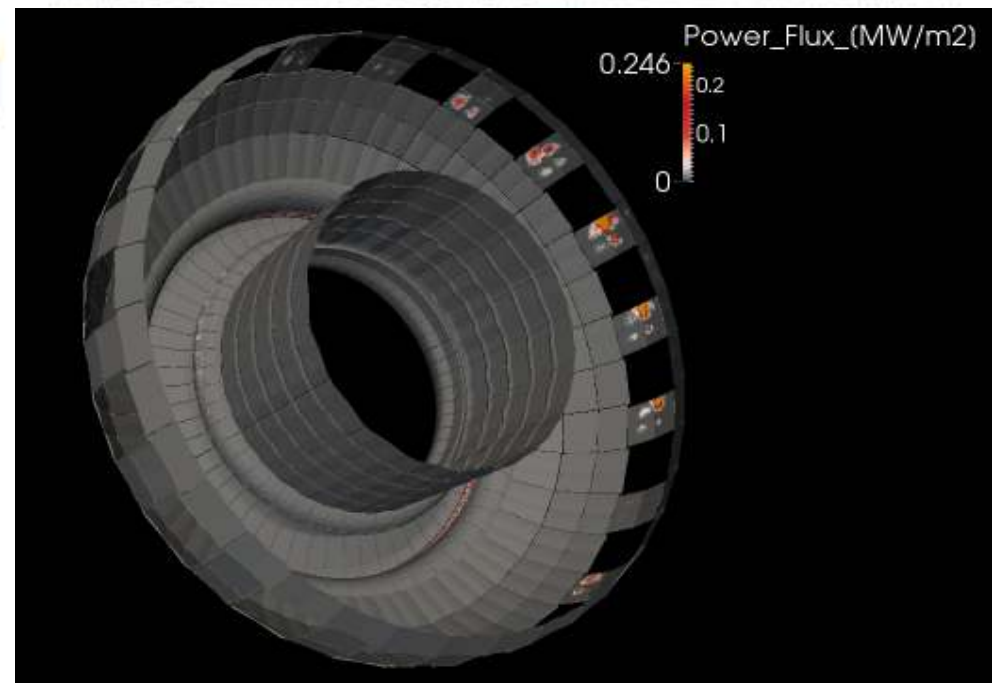
Why do we care about fast ions?

- Loss of bulk plasma heating
 - Unacceptable for an efficient power plant
 - May lead to ignition problems
- Damage to first wall
 - Can only tolerate fast ion losses of a few % in a reactor



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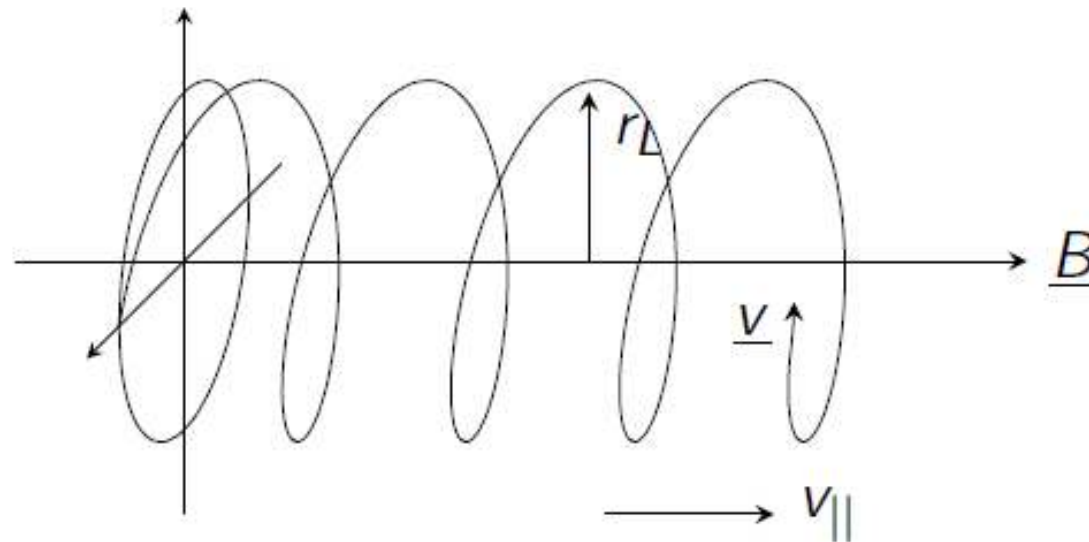


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Motion in a magnetic field

In magnetic fields, charged particles follow helical paths



Particles are free to move along magnetic fields, but are constrained in the perpendicular direction by the Lorentz force:

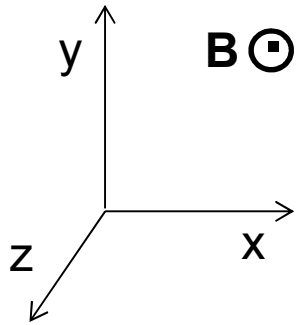
$$\underline{F} = q\underline{v} \times \underline{B}$$

- The equation of motion of a particle of mass, M , charge, e , velocity, \mathbf{v} , in an electric field, \mathbf{E} , and a magnetic field, \mathbf{B} , is

$$M \frac{d\mathbf{v}}{dt} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

- In the presence of uniform E-field and no B-field the ions accelerate in direction of \mathbf{E} , whilst the electrons accelerate in opposite direction to \mathbf{E}





- In a uniform B-field and no E-field...

- $m \frac{dv_z}{dt} = 0 \quad m \frac{dv_x}{dt} = ev_y B \quad m \frac{dv_y}{dt} = -ev_x B$

- Taking derivatives...

- $\frac{d^2 v_x}{dt^2} = \frac{eB}{m} \frac{dv_y}{dt} = -\left(\frac{eB}{m}\right)^2 v_x; \quad \frac{d^2 v_y}{dt^2} = -\frac{eB}{m} \frac{dv_x}{dt} = -\left(\frac{eB}{m}\right)^2 v_y$

- This is the simple harmonic oscillator equation

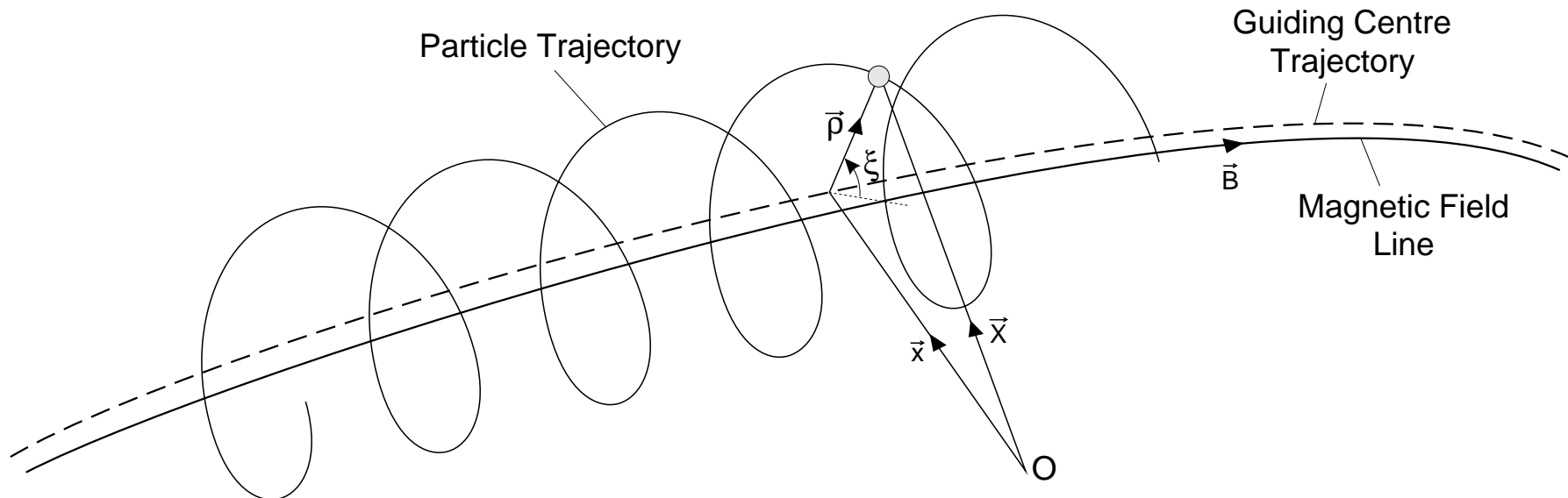
- Integrating gives...

- $x = x_0 + \frac{v_{\perp}}{\Omega_c} \sin(\Omega_c t + \phi) \quad ; \quad y = y_0 + \frac{v_{\perp}}{\Omega_c} \cos(\Omega_c t + \phi)$

- Which describes circular motion about (x_0, y_0) with frequency $\Omega_c = eB/m$ and “Larmor radius”, $\rho = v_{\perp}/\Omega_c$



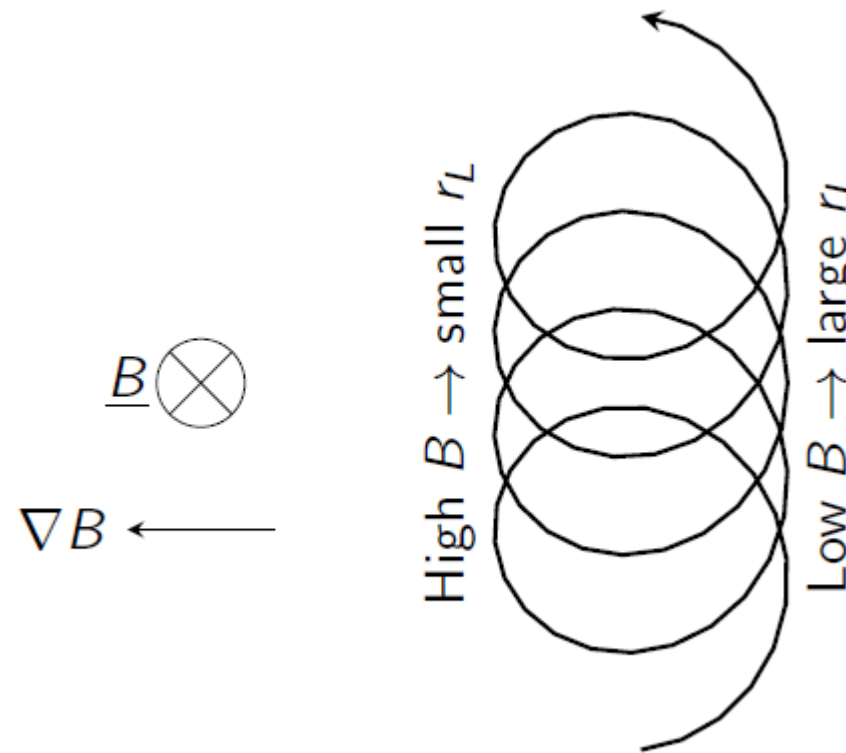
- “Cyclotron frequency”: $\Omega_c = \frac{eB}{M}$
- “Larmor radius”: $\rho = \frac{Mv_{\perp}}{eB}$



- If a system performs periodic oscillations, invariants of the motion can be found
- The first adiabatic invariant (ie the periodicity remains the same in the presence of a perturbation) is the magnetic moment

$$\mu = \frac{Mv_{\perp}^2}{2B}$$

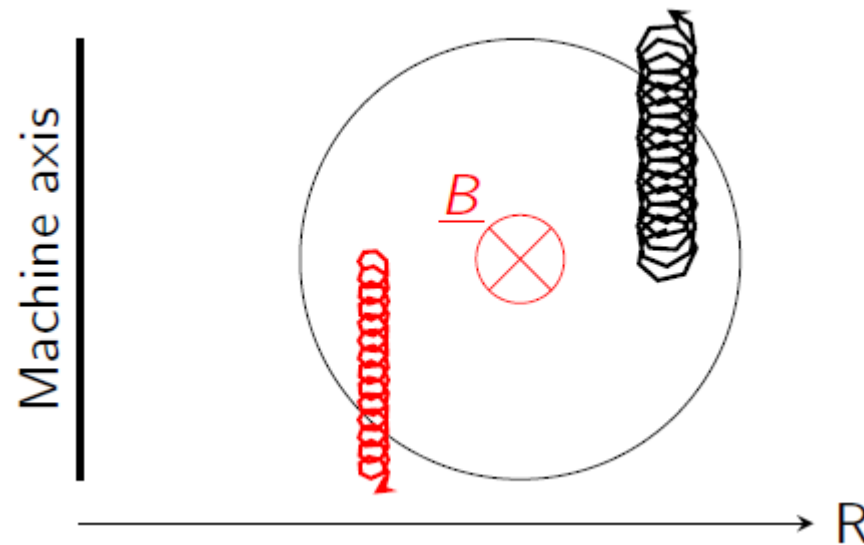
- In a tokamak, the magnetic field is not uniform
- The B-field is stronger near the centre than the outside, producing a particle drift in the direction perpendicular to the magnetic field and its gradient



- The ∇B drift is given by

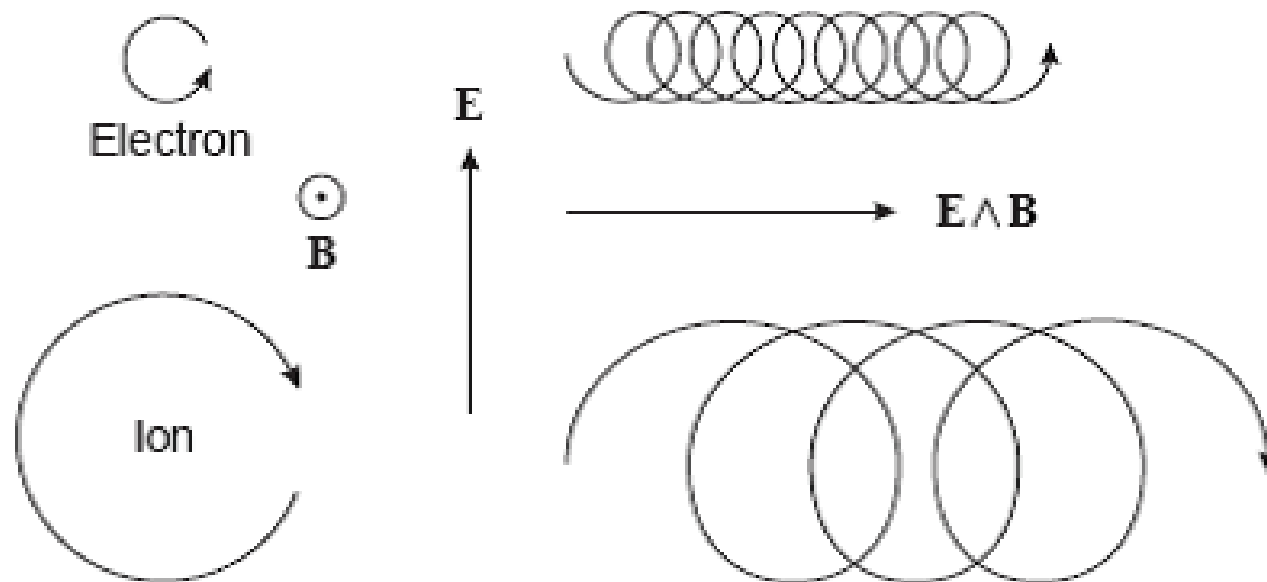
$$v_{\nabla B} = \frac{1}{2} \frac{M v_{\perp}^2}{e B} \frac{\mathbf{B} \times \nabla B}{B^2}$$

- It depends on charge, so ions and electrons separate



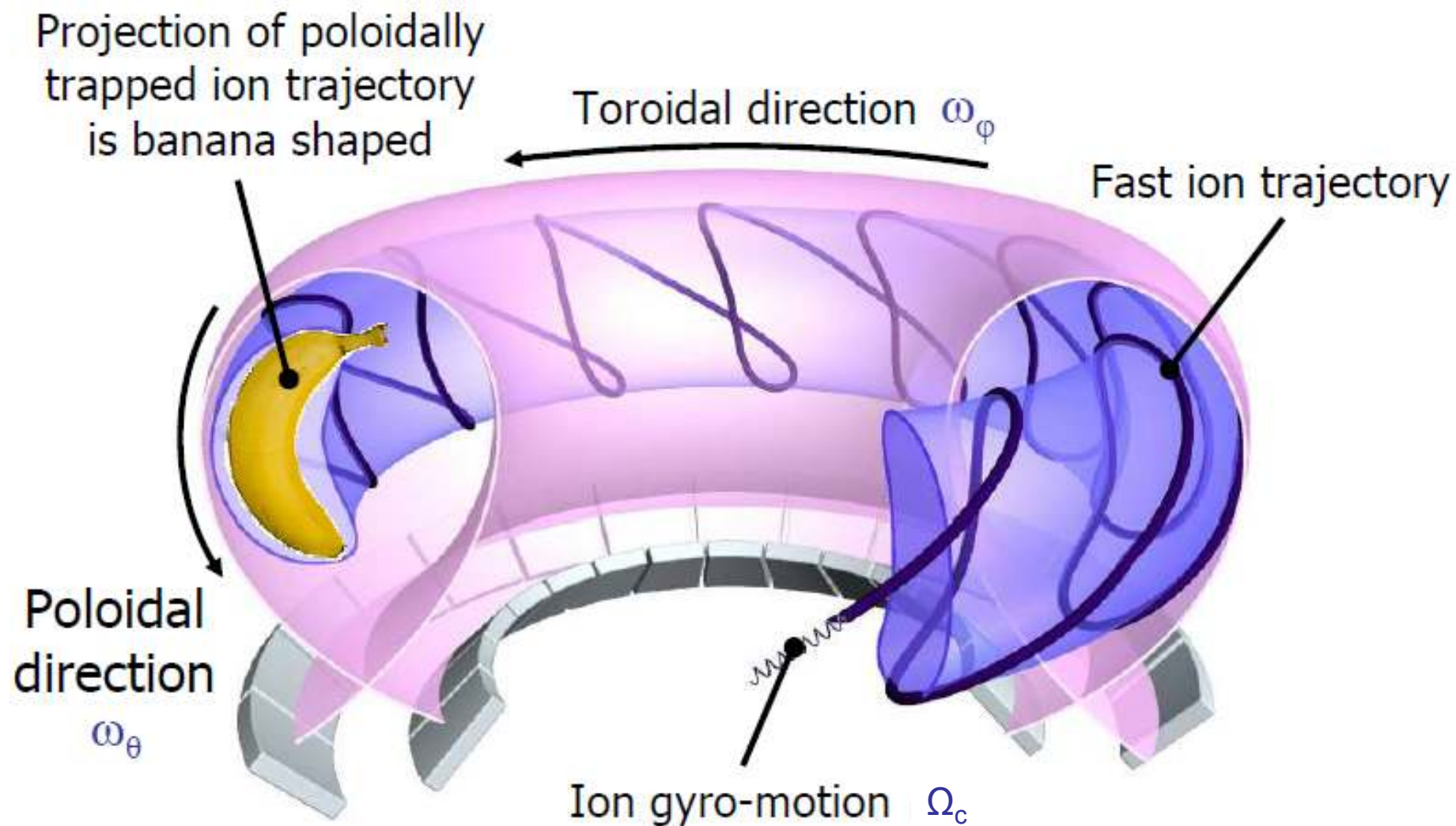
- Charge separation creates a vertical electric field

- The electric field accelerates (decelerates) positive ions when moving in the same (opposite) direction as the E-field
- As velocity increases, the Larmor radius increases, resulting in a drift of the guiding centre



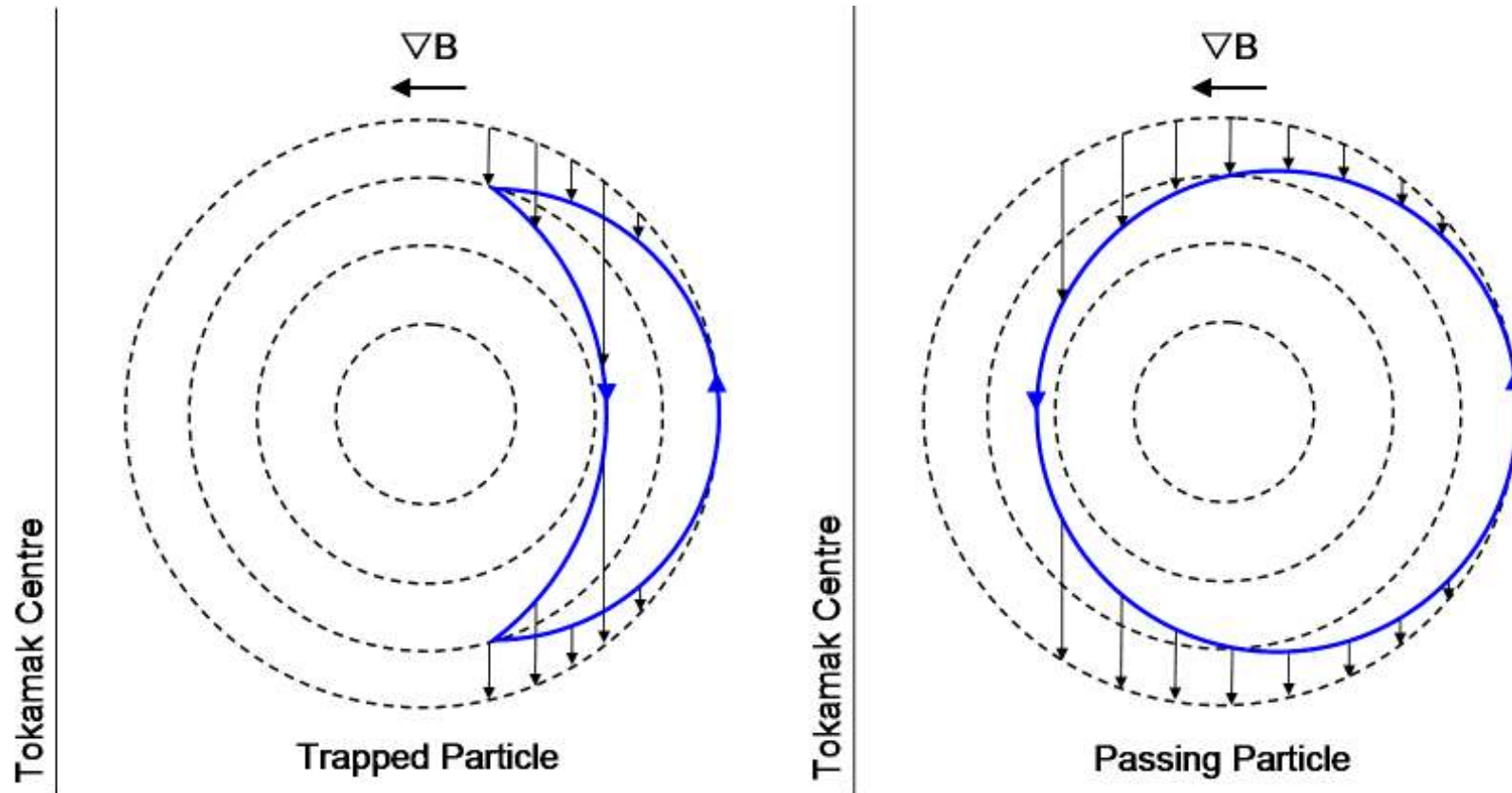
- To conserve magnetic moment, as particles move to region of higher B-field, their perpendicular velocity increases
- To conserve energy, the parallel velocity decreases and in some cases reaches zero and the particle is reflected – these are called ‘trapped’ particles
- Trapped particles bounce back and forth between mirror points, all the while experiencing ∇B drift from their surface, earning their trajectory the name “banana orbits”

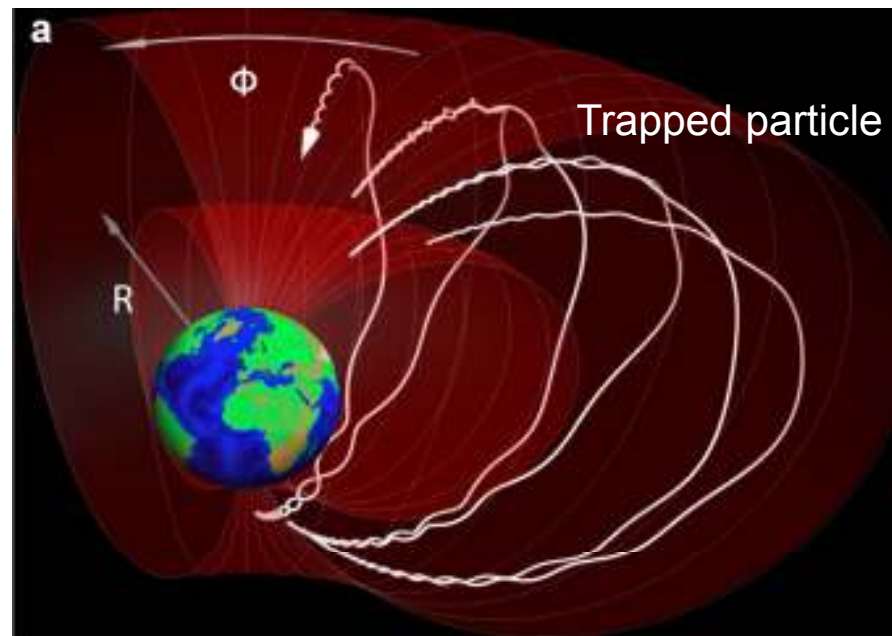




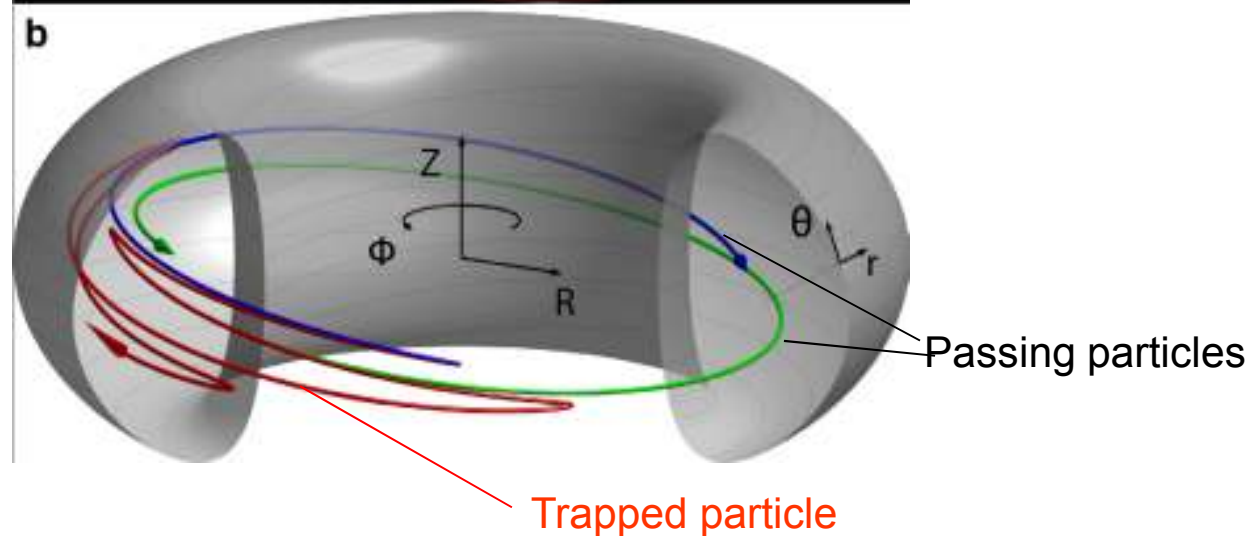
Various natural frequencies associated with particle motion

- All particles experience vertical ∇B drift but don't go anywhere!





In a tokamak,
two kinds of confined
particles exist:
-trapped
-passing

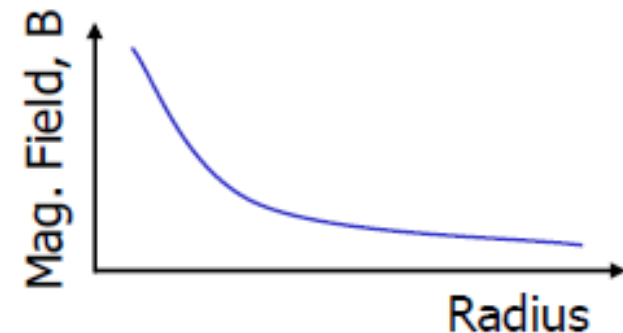
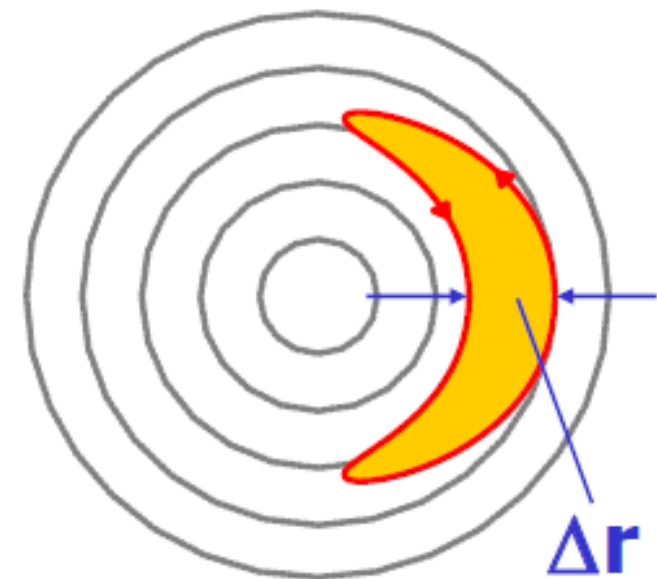


Source: Graves,
Nature Commun. 2012

- **Require orbit width less than minor radius**

- $\Delta r \ll a \Rightarrow$ *Build big devices*
- $\Delta r = mv_{||}/eB_{\theta}$
 $= 2\pi a mv_{||}/(e\mu_0 I_p)$
- $\Rightarrow I_p > 1.5 \text{ MA}$
- \Rightarrow *Build with high current!*

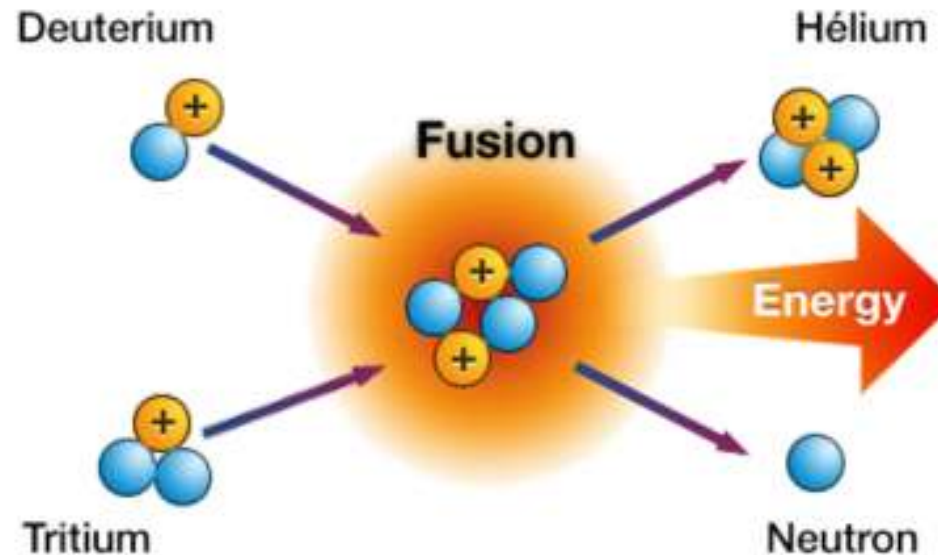
Poloidal cross-section



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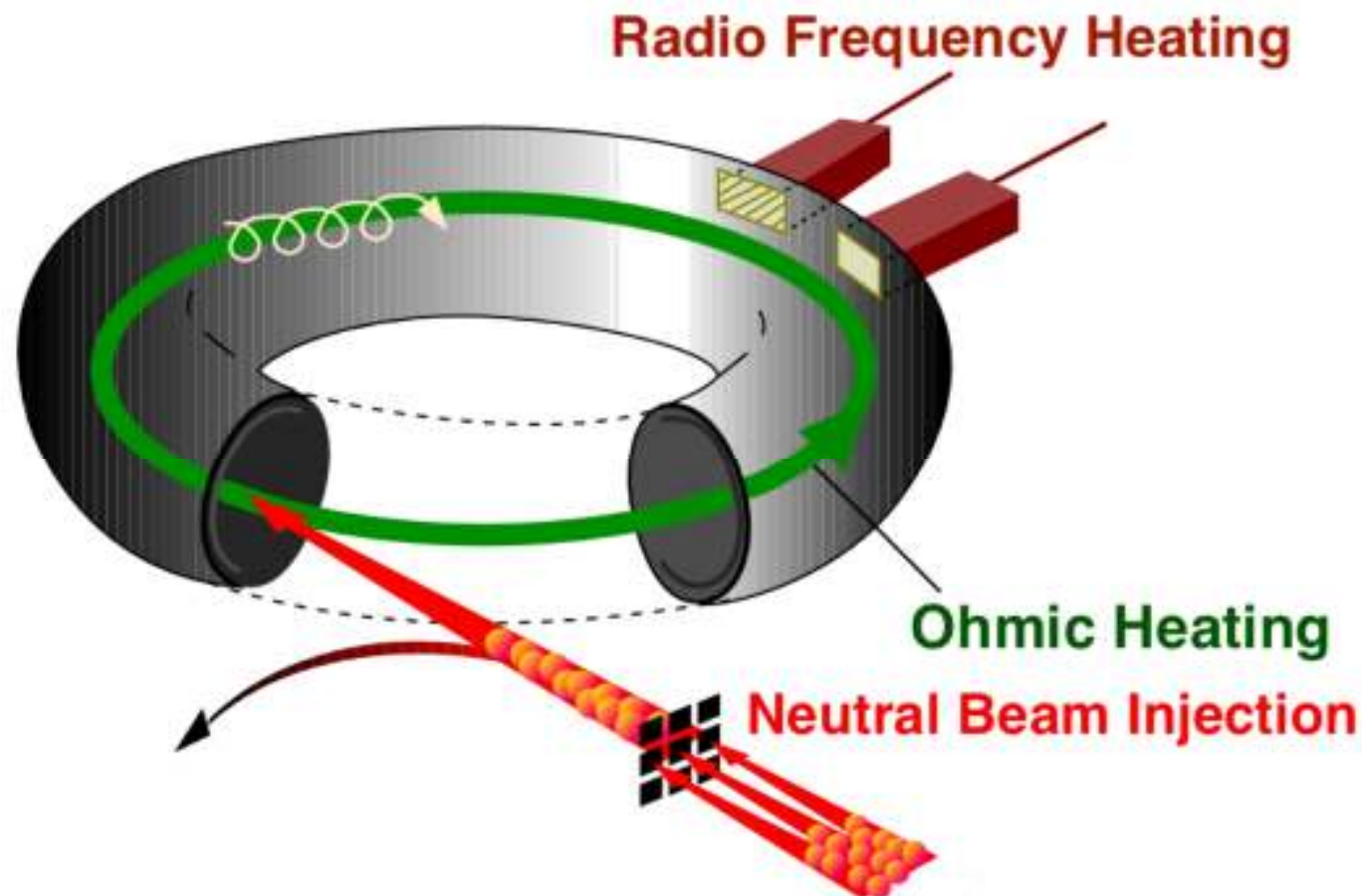


- In a burning plasma, the primary source of fast particles is from the fusion reactions directly, which produce 3.52MeV alpha particles



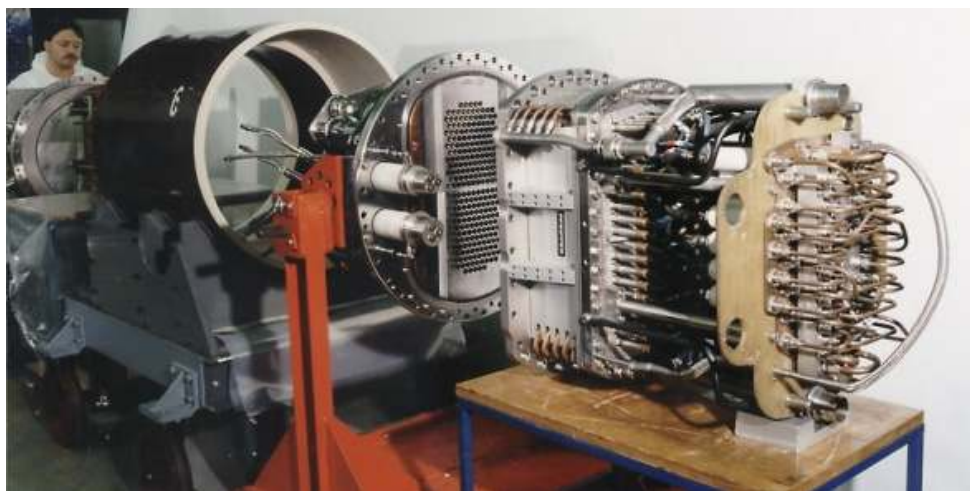
Heating the plasma

- However we need to heat the plasma to hot enough temperatures to maximise the reaction rate

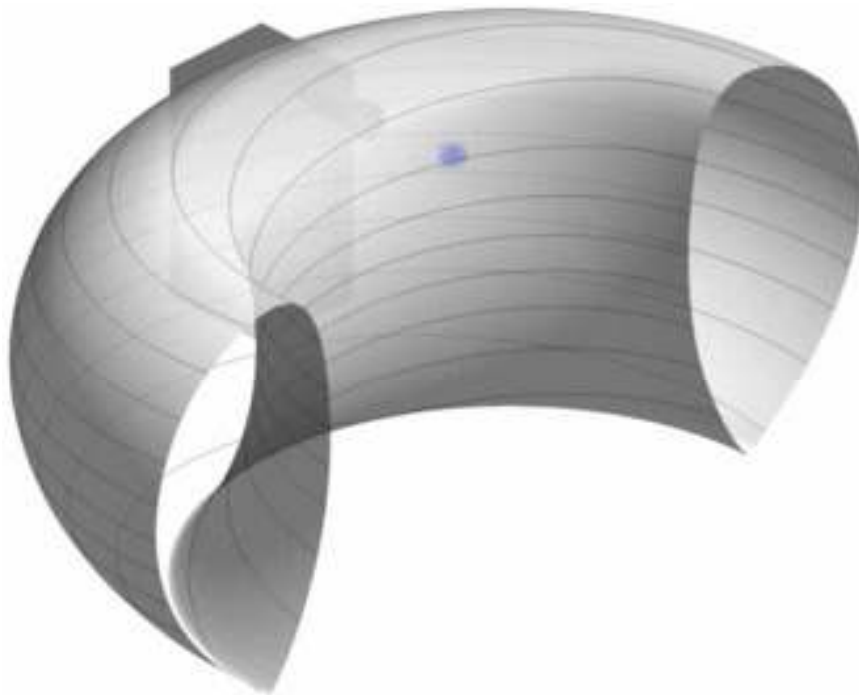


- Energetic neutral particles are fired in a beam into the plasma, carrying a large uni-directional kinetic energy
- In the plasma, beam atoms lose electrons due to collisions, i.e. they get ionised and captured by the B-field
- These new ions are much faster than average plasma particles. Ion-ion, ion-electron and electron-electron collisions mean the group velocity of beam atoms increases mean velocity of bulk plasma
- In JET, the neutral particles are $\sim 120\text{keV}$ and total up to 30MW. In ITER they will be up to 1MeV

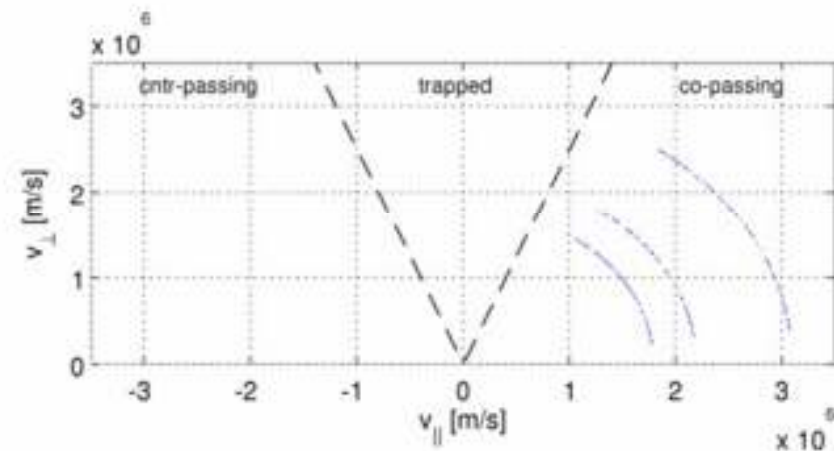




- Simulation of fast ions born due to NBI:



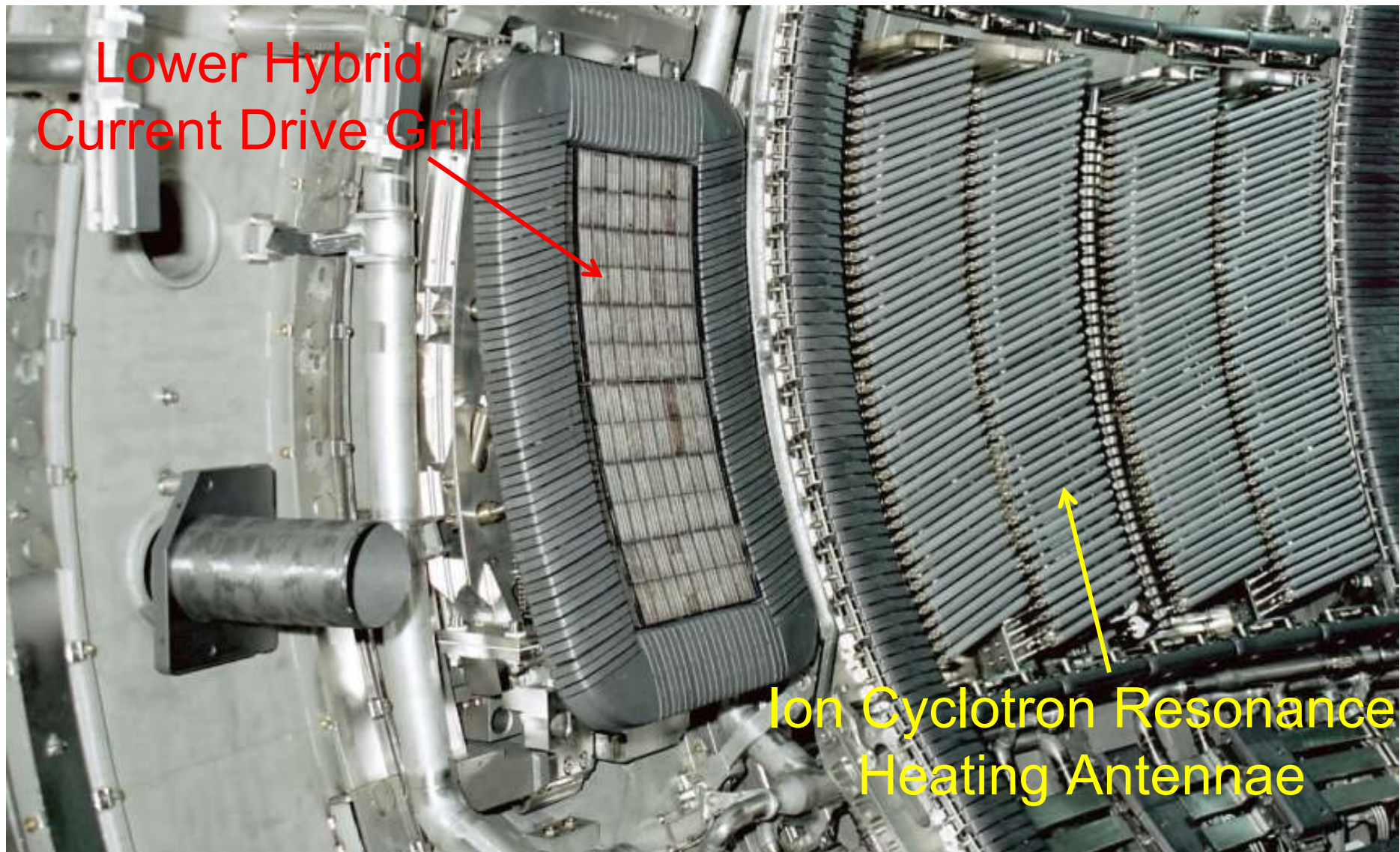
Mattia Albergante



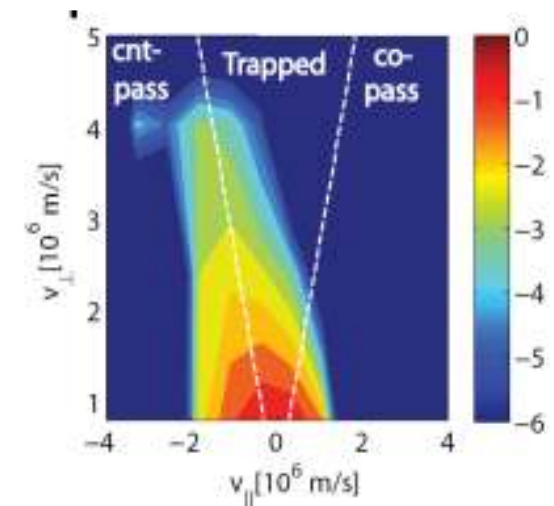
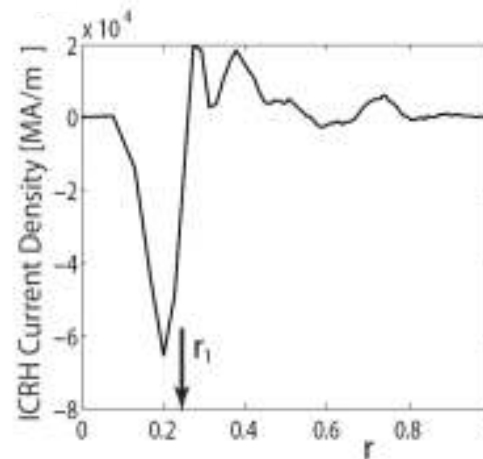
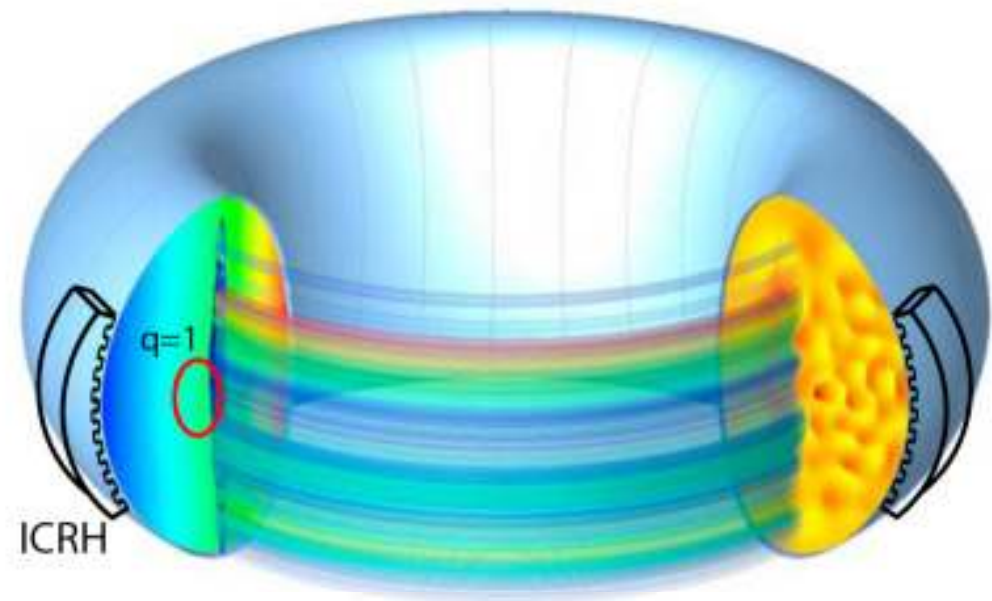
- Plasmas can host sound, electrostatic, magnetic and electromagnetic waves.
- Depending on local parameters, plasma waves can propagate, get dumped (absorbed), be reflected or even converted to different plasma waves
- Wave absorption is extremely efficient if the wave frequency is resonant with some of the fundamental oscillations of the medium
- Get effective absorption at ion/electron cyclotron resonance layers, which are largely determined by magnetic field (remember $\Omega_c = \frac{eB}{M}$)



Radio Frequency Antennae



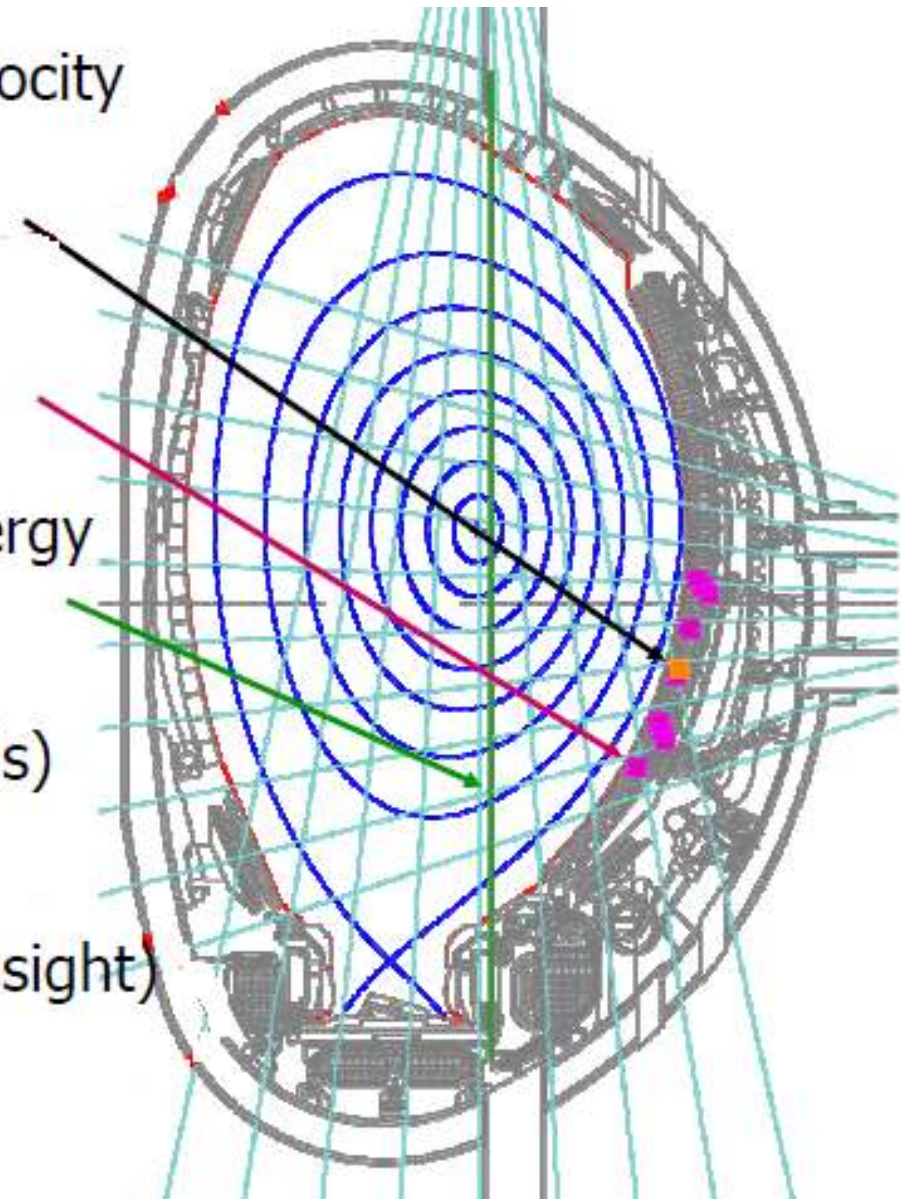
- Simulation of fast ions born due to ion cyclotron resonance heating:



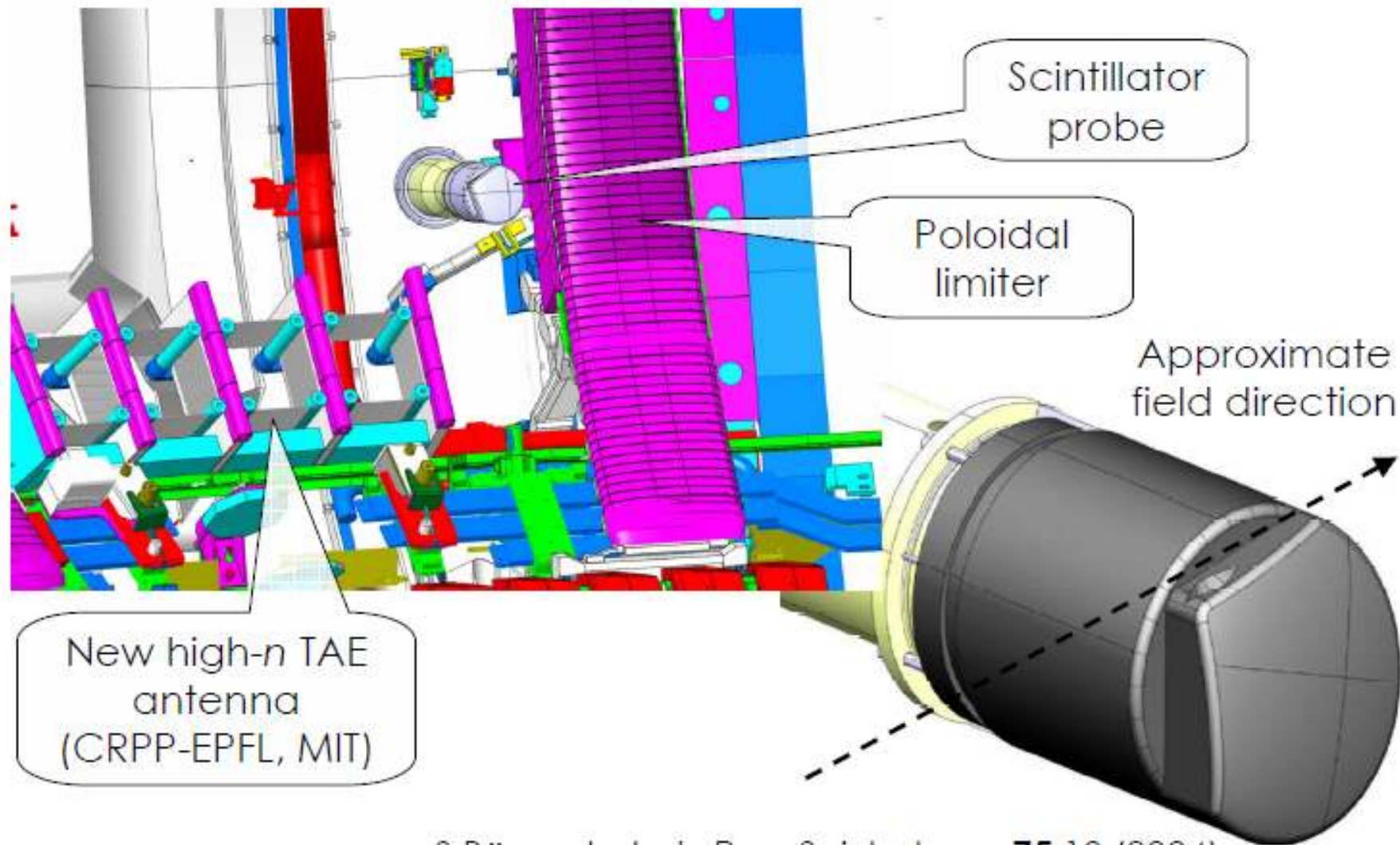
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- **Scintillator probe** provides velocity distribution of lost fast ions
- **Faraday cups** give spatial and energy distribution of lost ions
- **Neutral Particle Analyser** energy distribution of confined fast ions
- **γ -ray tomography** (19 channels) and
 γ -ray spectroscopy (2 lines of sight) for energy distribution

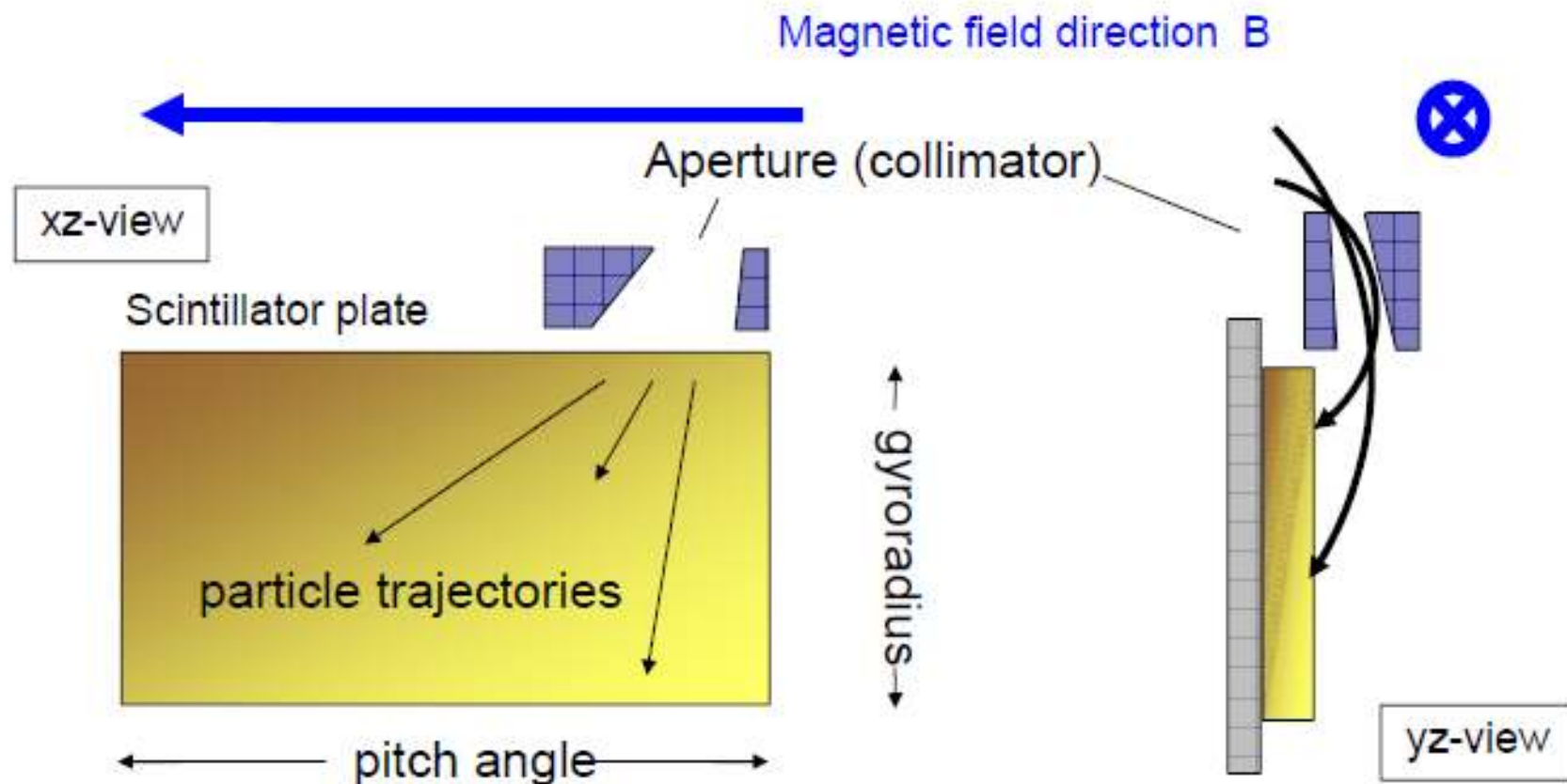


Example: Scintillator Probe



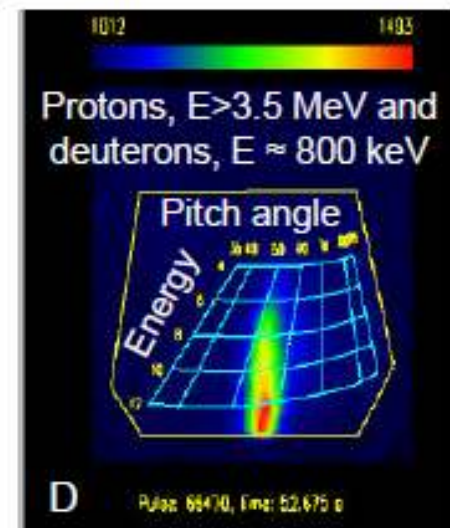
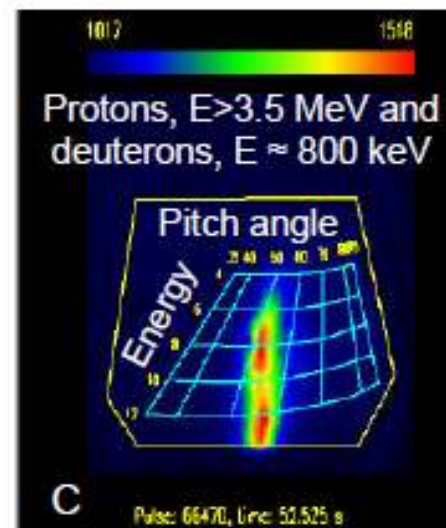
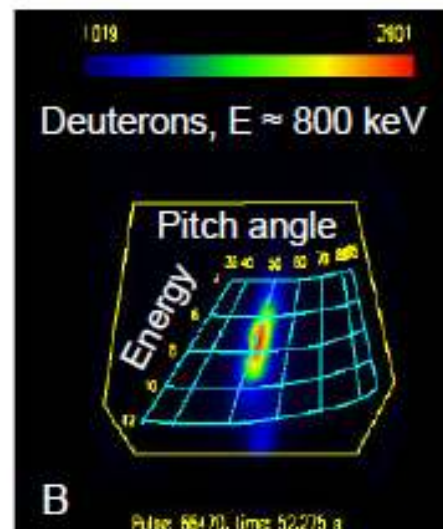
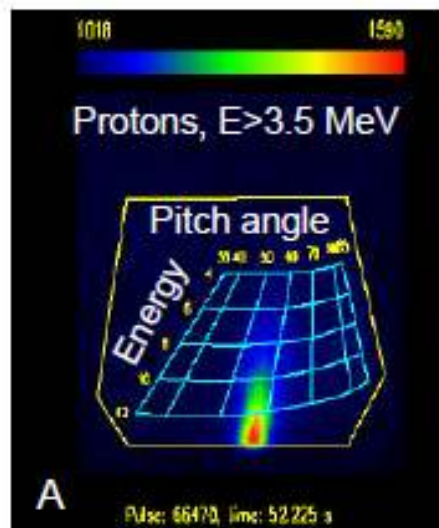
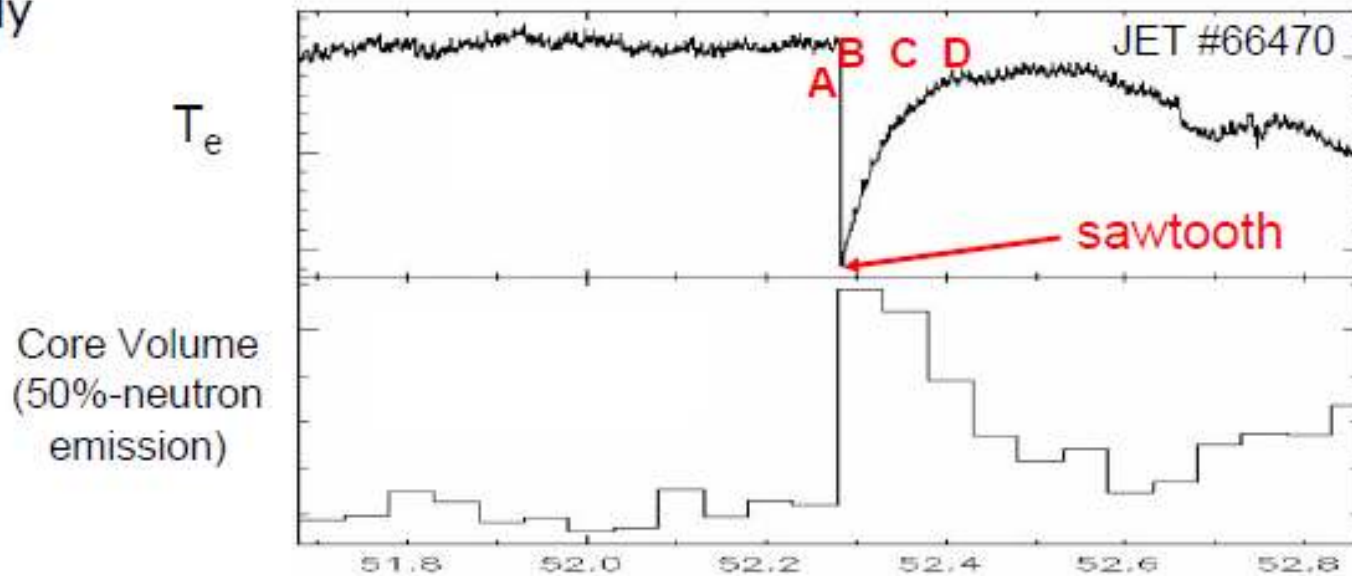
S Bäuml et al., Rev. Sci. Instrum. **75** 10 (2004)

Probe head collimator and magnetic field form a magnetic spectrometer



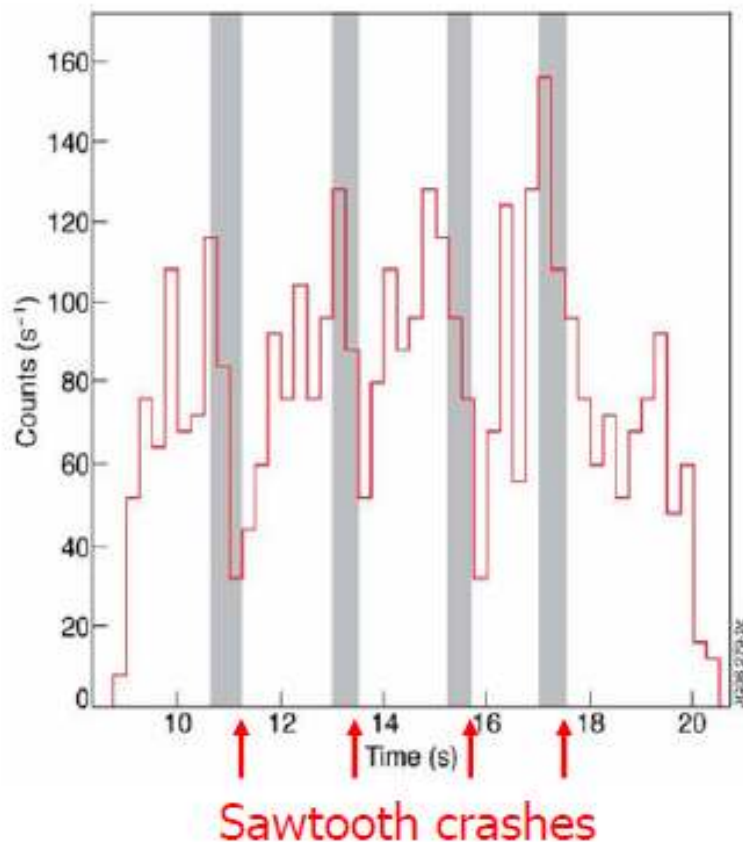
Scintillator probe converts pitch-angle / energy distribution into 2D picture

V. Kiptily

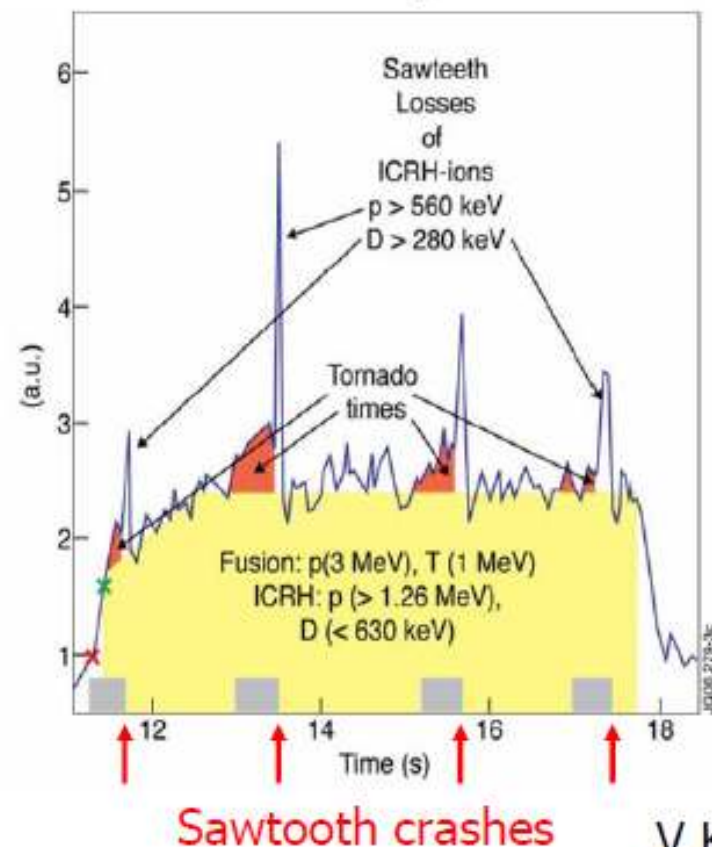


- Tornado modes (TAEs within $q=1$) redistribute fast ions, which then leads to sawtooth crashes

3.1-MeV γ -ray emission from $^{12}\text{C}(d,p\gamma)^{13}\text{C}$;
Deuterons with $E > 500$ keV



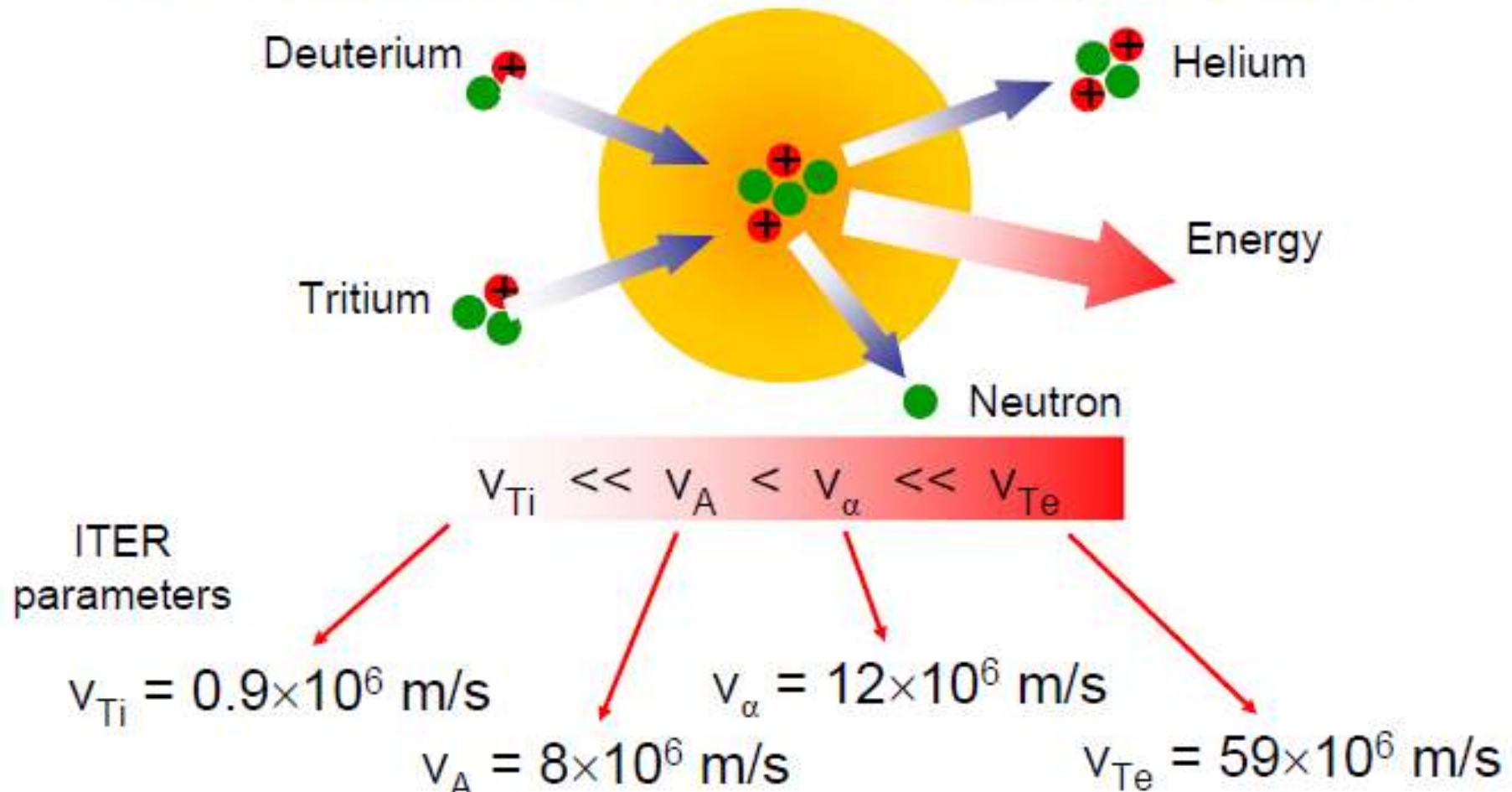
Scintillator probe



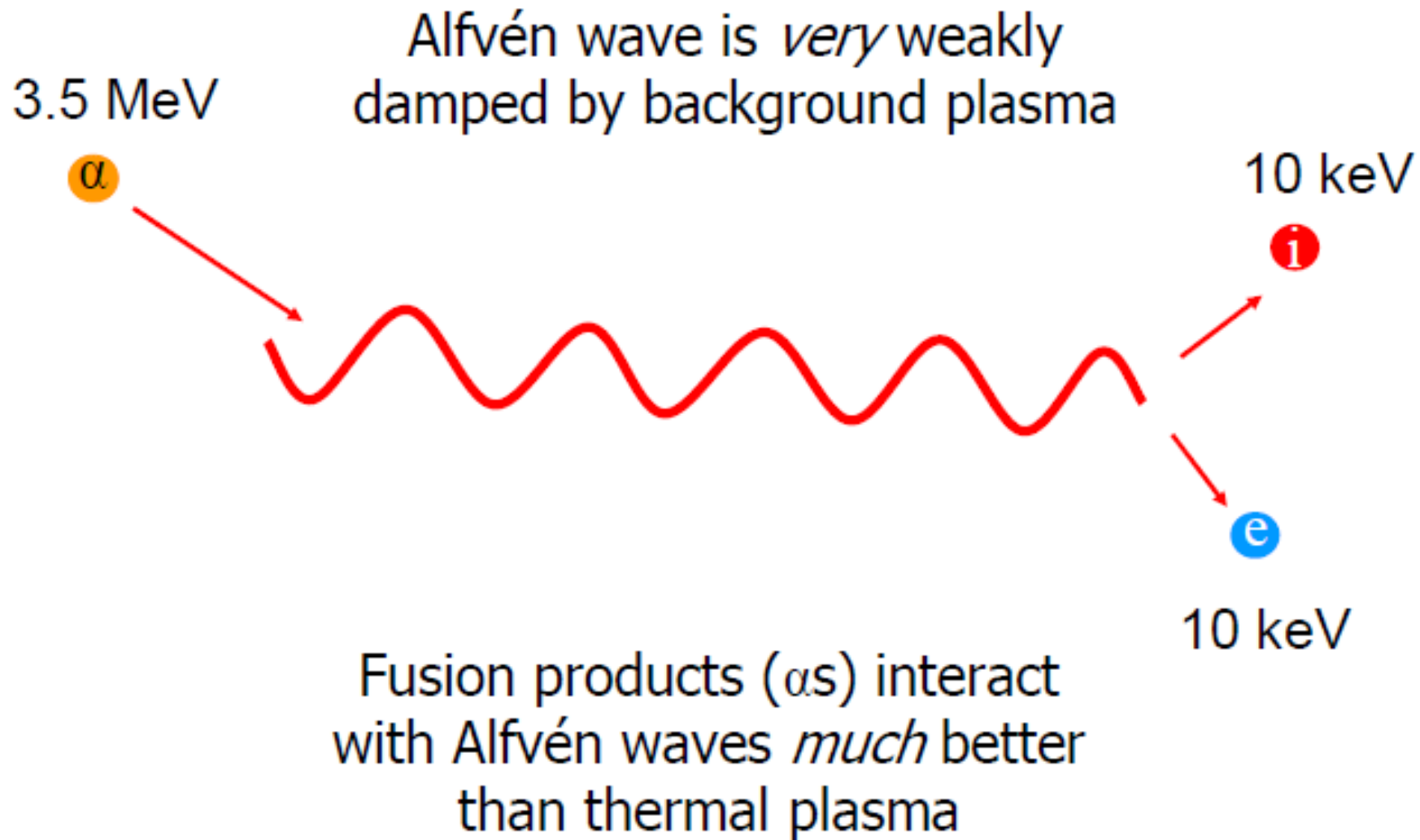
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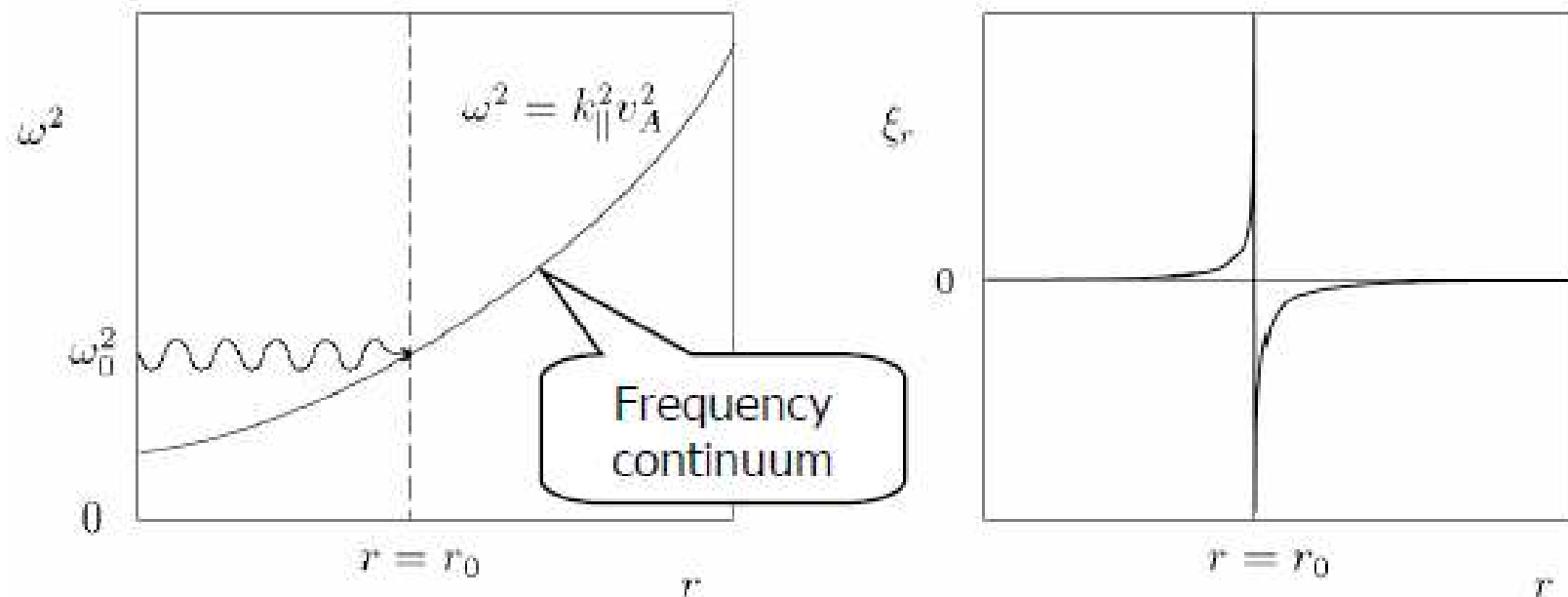
- New physics element in burning plasmas:
 - Plasma is self-heated by fusion alpha particles



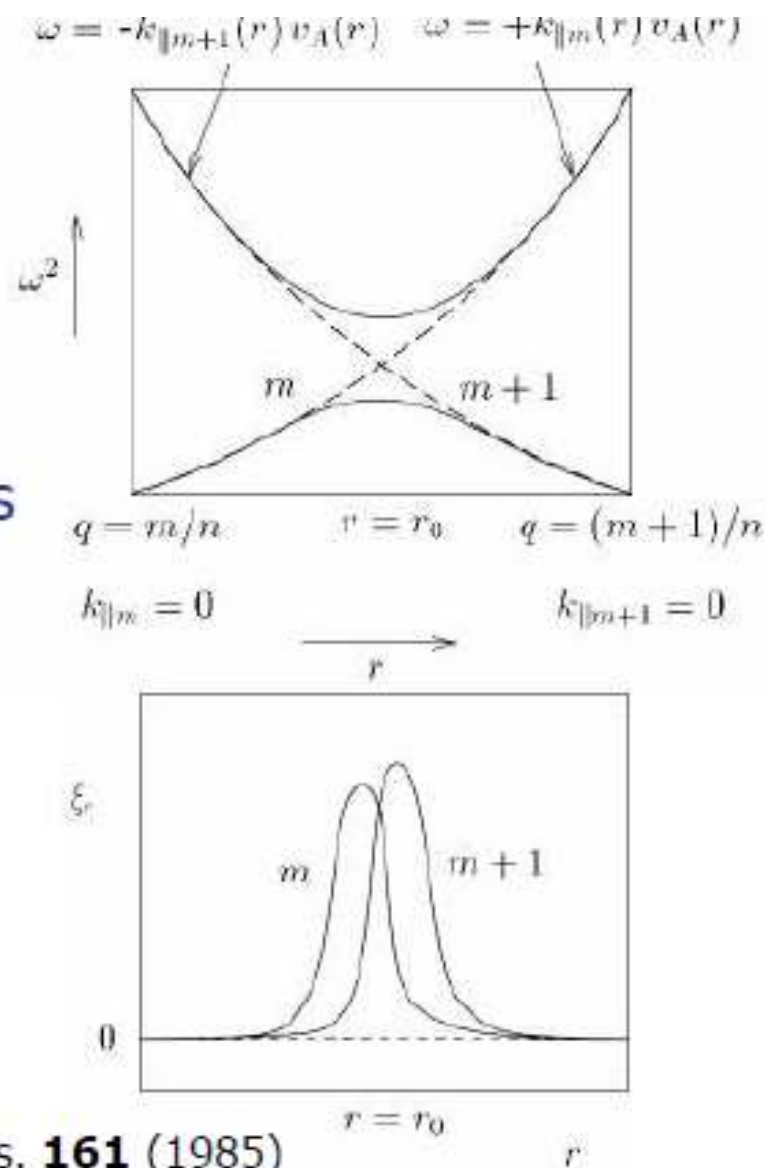
Alfven waves and alphas



- Analogous to waves on a string
 - $v_A = B/\sqrt{(\mu_0 m_i n_i)}$
 - $\omega^2 = \omega_A^2(r) \equiv k_{\parallel}^2 v_A^2(r)$
 - Form continuum of waves in inhomogeneous plasma
 - Damped due to phase mixing with neighbouring waves



- Tokamak plasma:
 - Fourier decomposition:
 - $A \sim \exp[i(n\phi - m\theta - \omega t)]$
 - $B \approx B_0 R_0 / R \approx B_0 (1 - r/R_0 \cos \theta)$
 - Neighbouring poloidal harmonics couple due to toroidicity
 - Gaps in frequency continuum
 - Toroidal Alfvén Eigenmodes (TAE) exist in frequency gap
 - Weakly damped
 - $f_{\text{TAE}} \sim v_A / (2qR)$



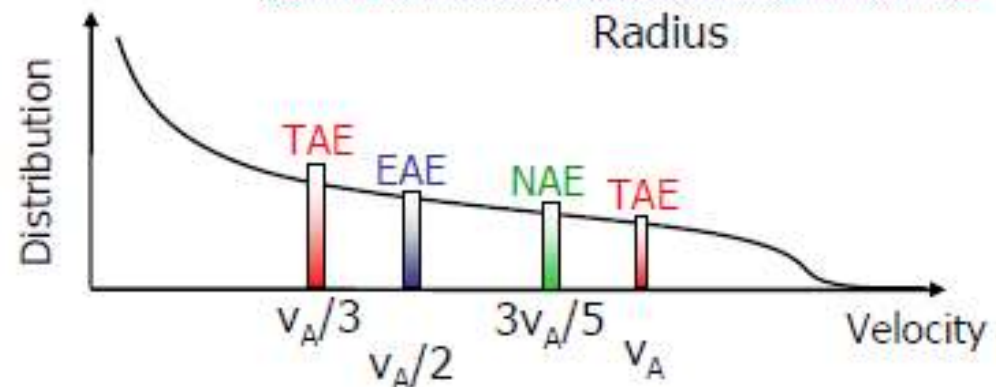
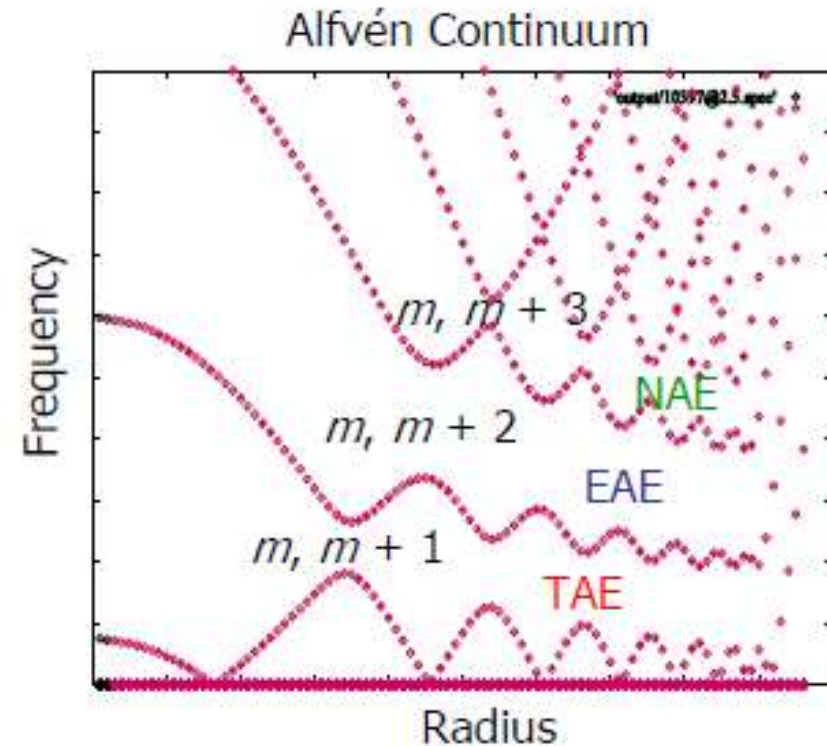
C. Z. Cheng, Liu Chen and M. S. Chance, Ann. Phys. **161** (1985)

- Exist in frequency gaps
- Comprise of two primary harmonics, m and $m + L$
 - **Wave-particle resonance condition:**

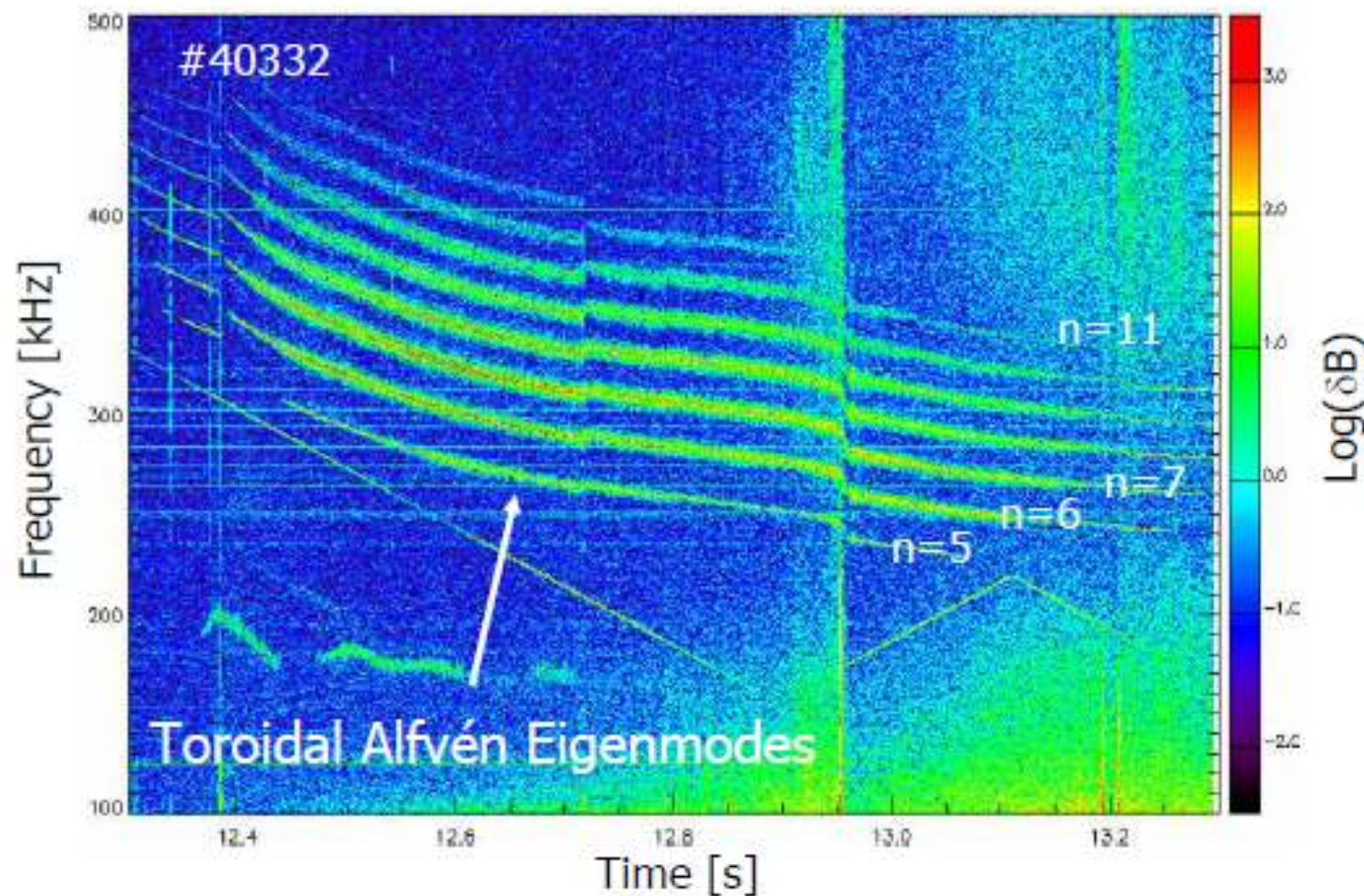
$$\omega - n \omega_\phi + (m \pm 1) \omega_\theta = 0$$

$$v_{||} = \pm L / (2 \pm L) v_A$$

- **TAE:** $L = 1$
- **EAE:** $L = 2$
- **NAE:** $L = 3$

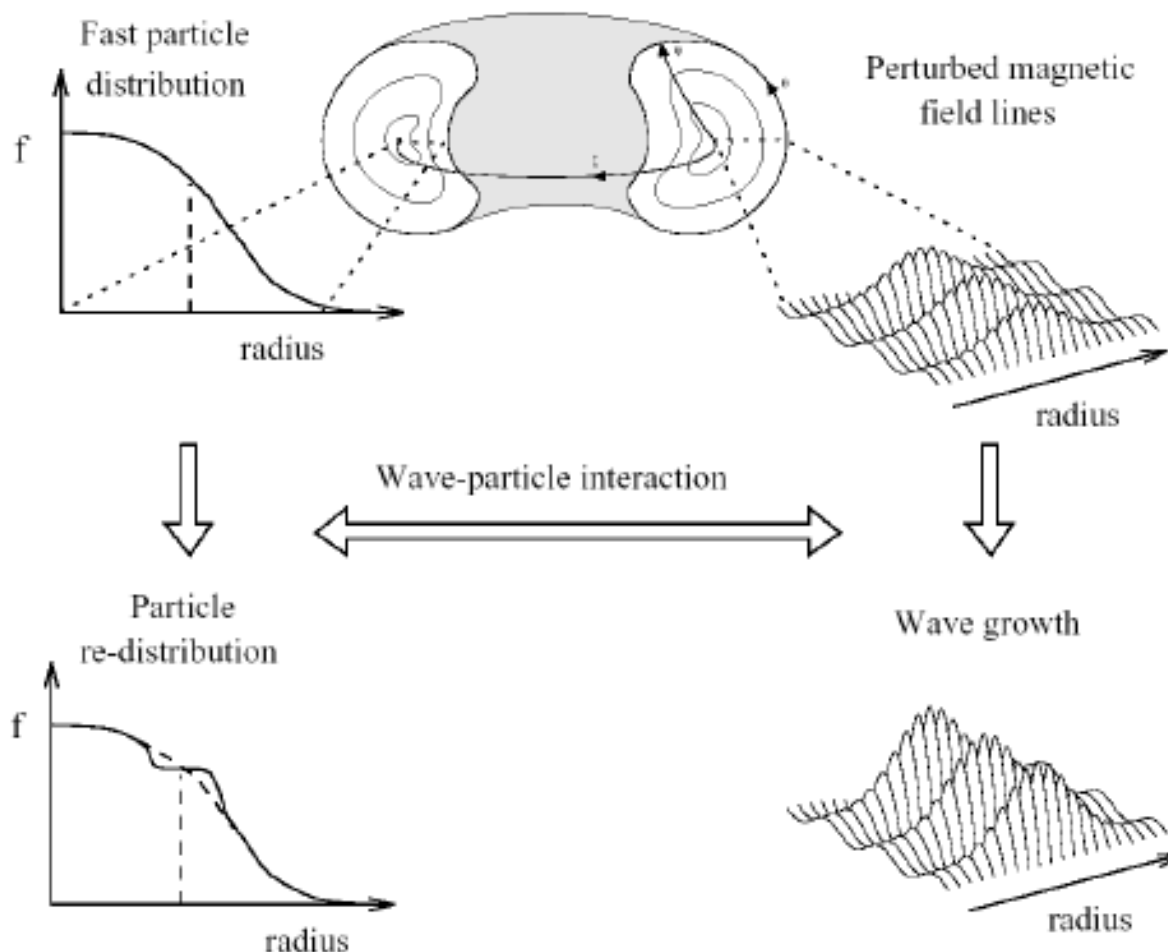


Example: TAE driven by ICRH in JET



- TAE have constant amplitude and fine frequency splitting
 \Rightarrow **Nonlinear effect**

- **Linearly unstable AE grows and saturates**
 - Nonlinear wave-particle interaction
 - Wave redistributes fast ions and removes drive



- **Collective instabilities**

- Fast particle gradients act as source of free energy

- Non-Maxwellian distribution

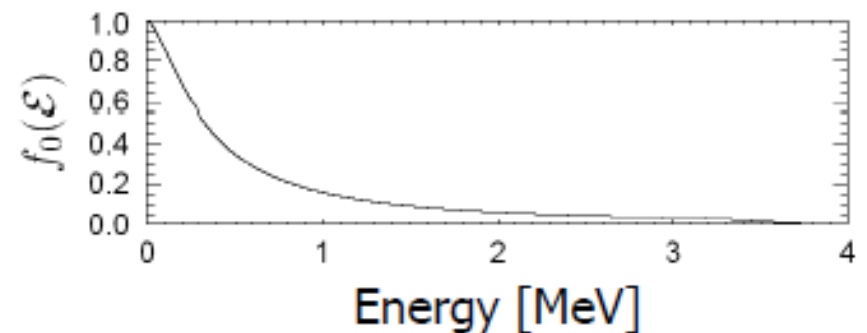
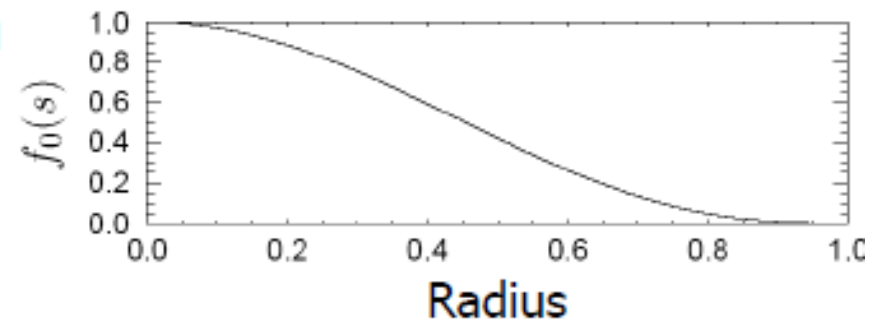
- $\gamma \sim \omega \frac{\partial f}{\partial E} - n \frac{\partial f}{\partial P_\phi}$

- Negative radial gradient

\Rightarrow *Drive* ($n > 0$)

- Negative energy gradient

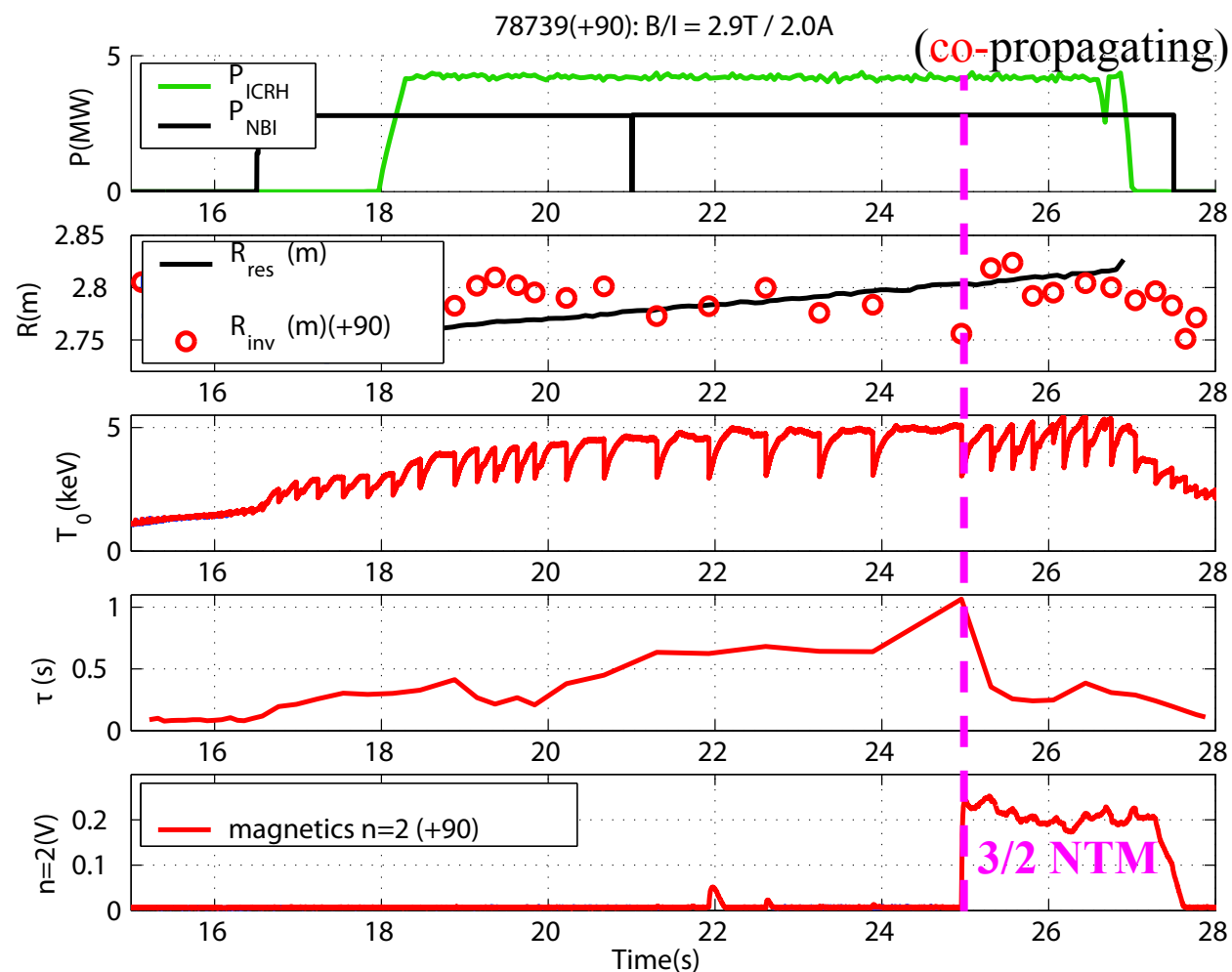
\Rightarrow *Damping*



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- Fast ion populations inside $q=1$ leads to very long sawtooth period, which increases likelihood of triggering NTMs

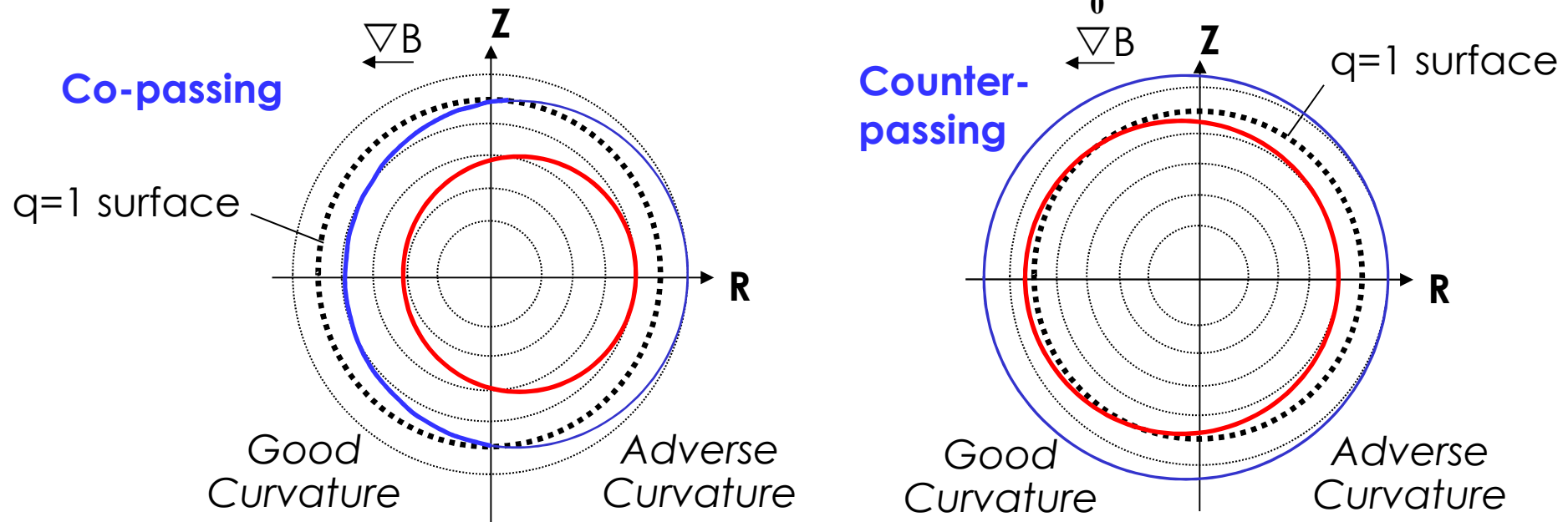


L-mode
 $\beta_N = 0.8$

Graves et al, Nucl Fusion 2010; Nature Comms 2012

Passing Particle Stabilisation Mechanism

δW has a term dependent upon curvature: $\delta W \sim - \int_0^{r_1} (\xi \cdot \nabla \langle P_h \rangle) (\xi \cdot \kappa) dr$



Co-pass, $\langle P_h \rangle' |_{r_1} < 0 \rightarrow$ stabilising

Co-pass, $\langle P_h \rangle' |_{r_1} > 0 \rightarrow$ destabilising

Ctr-pass, $\langle P_h \rangle' |_{r_1} < 0 \rightarrow$ destabilising

Ctr-pass, $\langle P_h \rangle' |_{r_1} > 0 \rightarrow$ stabilising

Graves, Phys Rev Lett, 2004

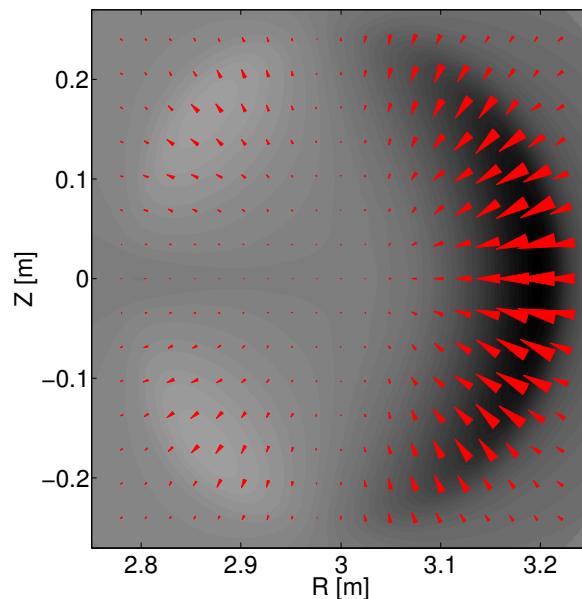
Effect of Passing Energetic Ions

- Highly localised outward pointing force (destabilisation) on both sides of the torus for a case with:

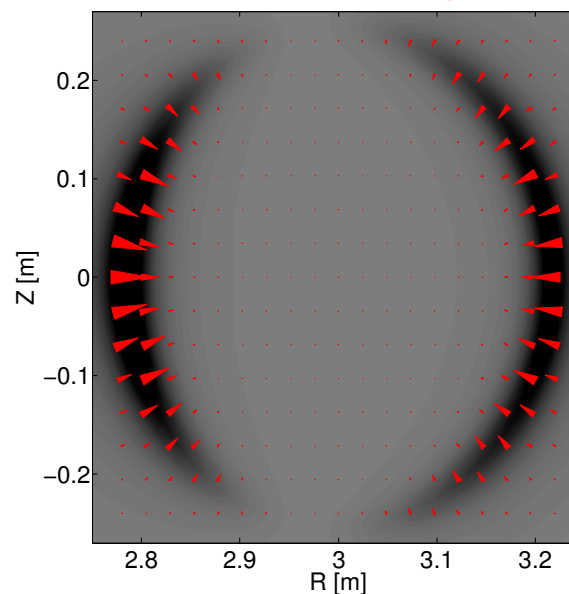
$$F_h(v_{\parallel}^+) < F_h(v_{\parallel}^-) \text{ and } \nabla F_h|_{r_1} < 0$$

- Obtain an inward pointing force (stabilisation) if one of the two conditions above are reversed
- Effect increases with orbit width

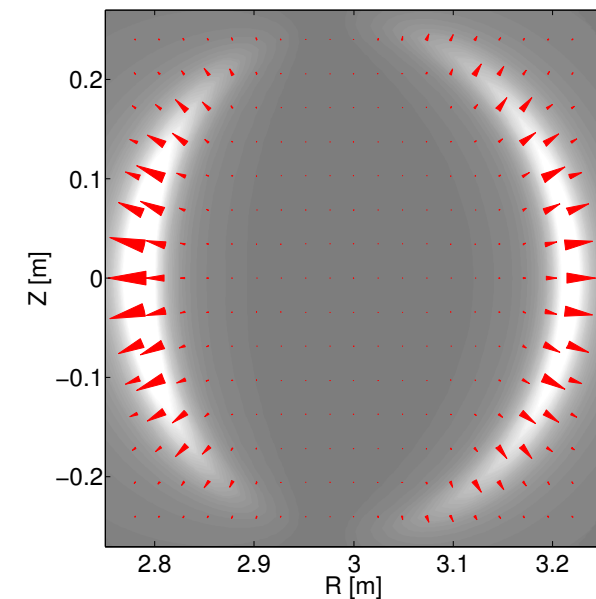
Trapped



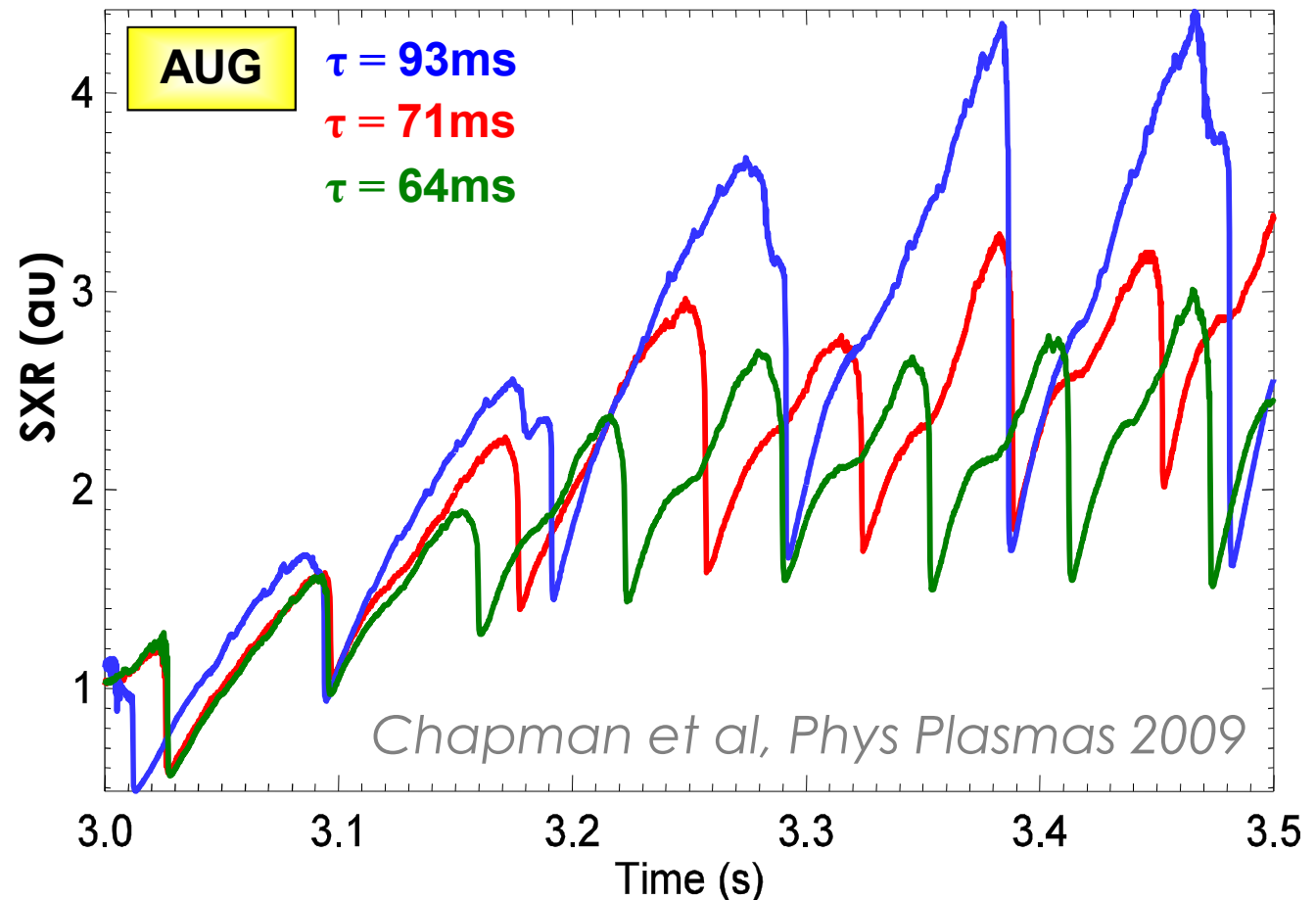
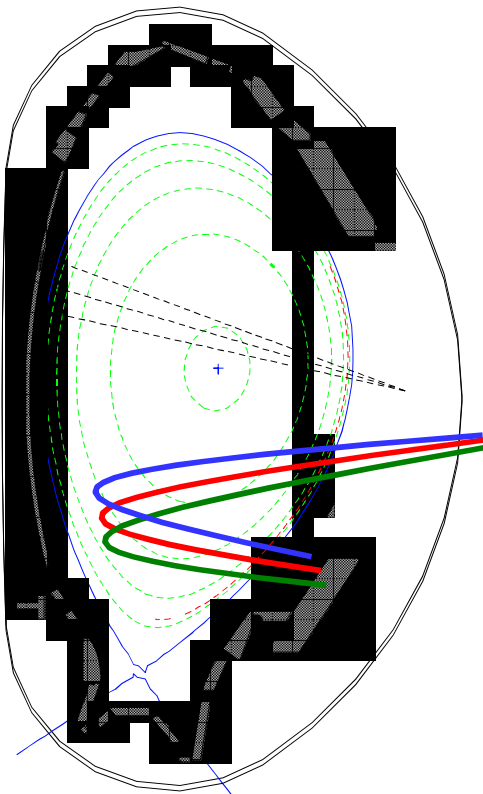
Co Passing



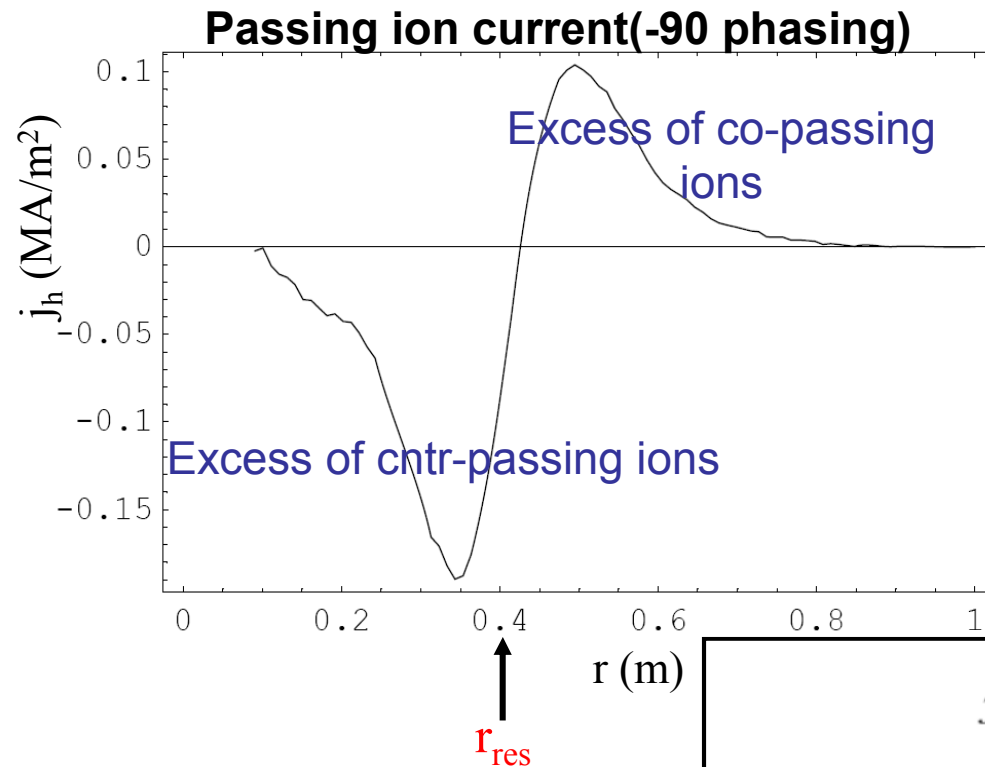
Counter Passing



- Off-axis NBI affects sawtooth behaviour
 - As f_h moves further outside $q=1$, τ_s decreases by $\sim 50\%$
 - Modelling again shows passing ions dominate over NBCD



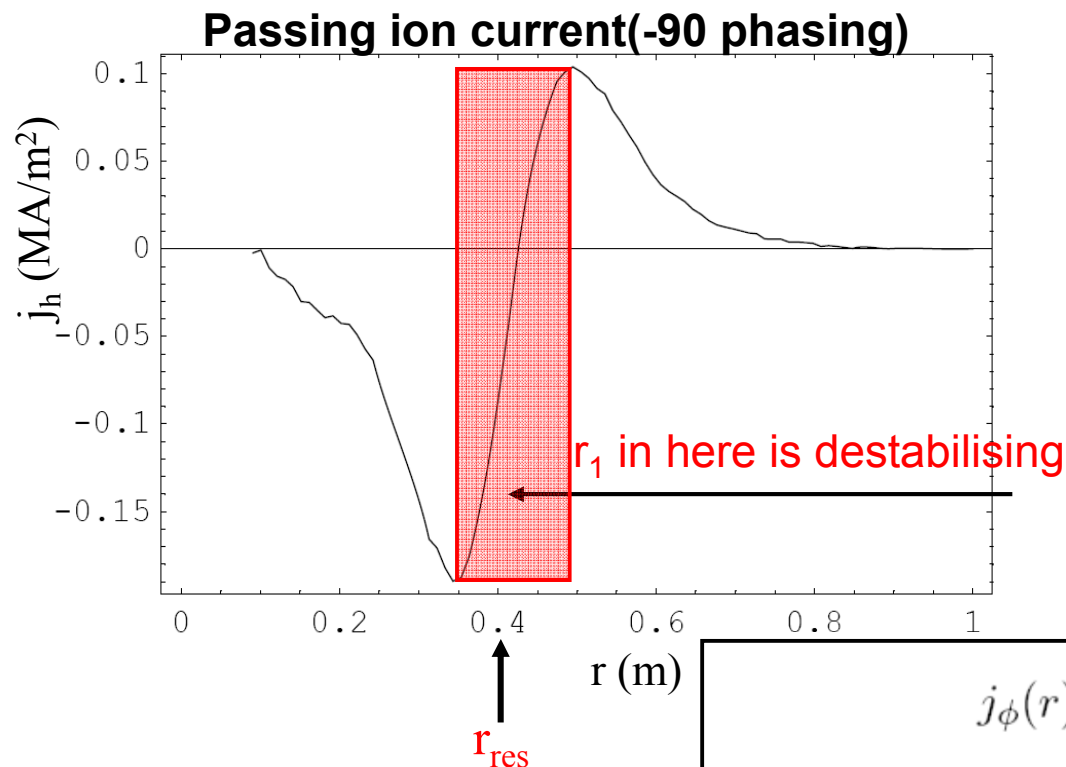
- Theory extended for toroidally propagating ICRF waves
- Parallel velocity asymmetry in F_h seen e.g. in the ICRH current from SELFO



$$j_{\phi}(r) \approx eZ_h \int dv^3 \sum_{\sigma} v_{\parallel} F_h$$

Graves et al, PRL, 2009

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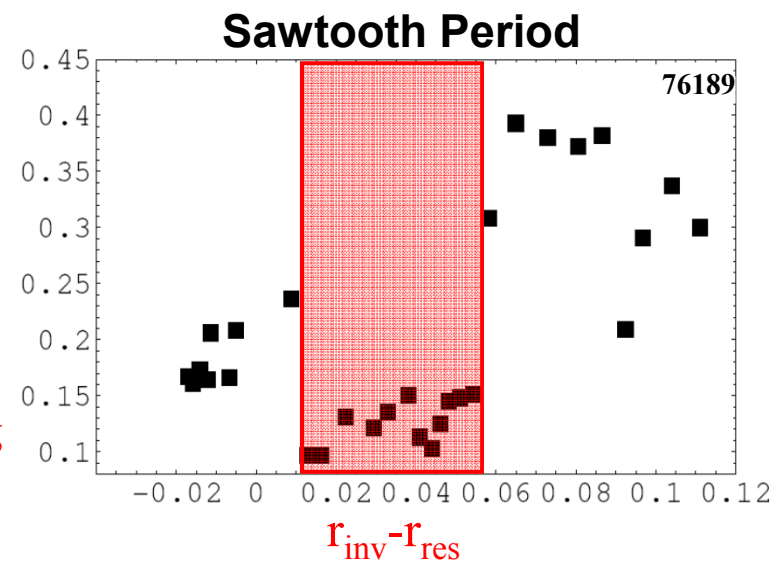
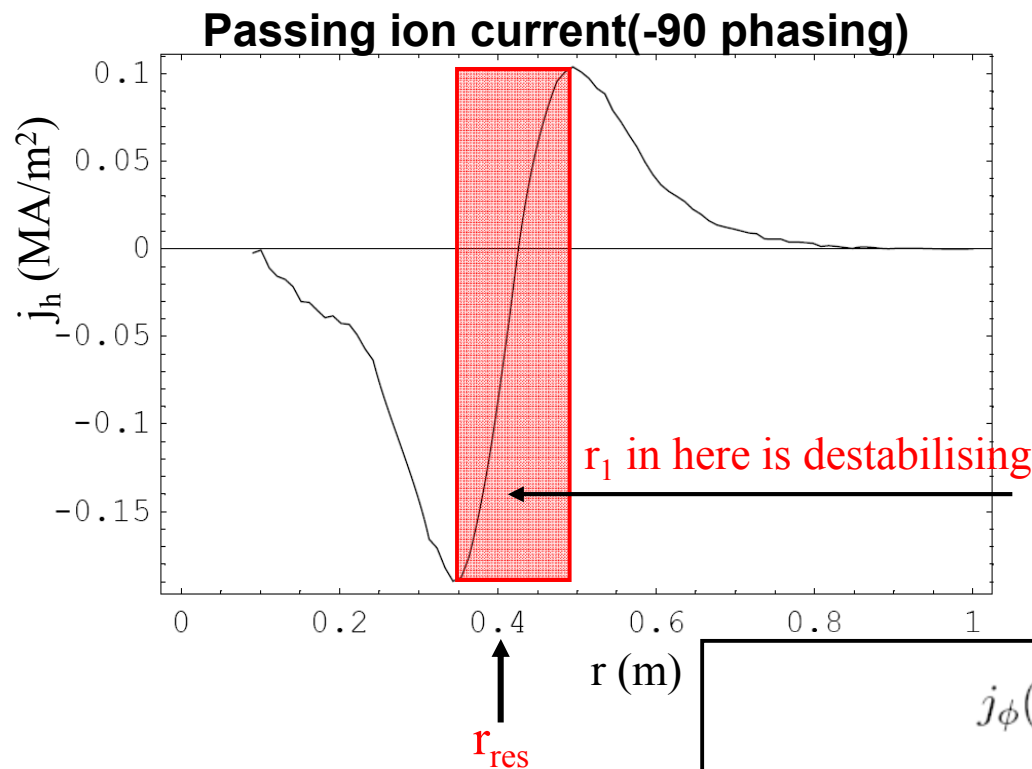


Graves et al, PRL, 2009

$$j_\phi(r) \approx eZ_h \int dv^3 \sum_\sigma v_\parallel F_h$$

$$\delta W \propto - \int dv^3 \sum_\sigma \left(\frac{v_\perp^2}{2} + v_\parallel^2 \right) \Delta_r \frac{\partial F_h}{\partial r} \Big|_{r_1} \sim - T_h \frac{dj_\phi}{dr} \Big|_{r_1}$$

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Graves et al, PRL, 2009

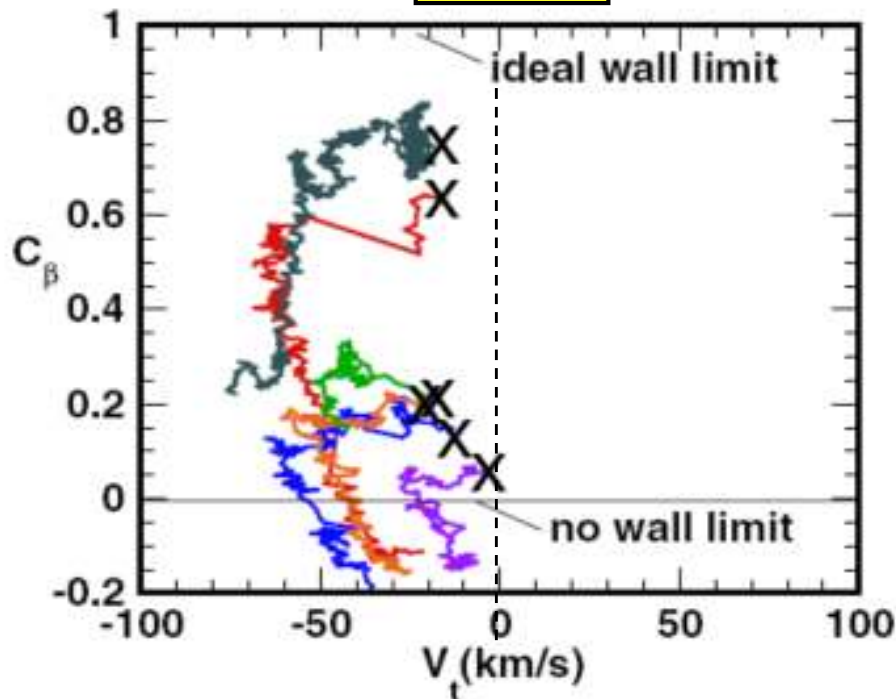
$$j_\phi(r) \approx eZ_h \int dv^3 \sum_{\sigma} v_{\parallel} F_h$$

$$\delta W \propto - \int dv^3 \sum_{\sigma} \left(\frac{v_{\perp}^2}{2} + v_{\parallel}^2 \right) \Delta_r \frac{\partial F_h}{\partial r} \Big|_{r_1} \sim - T_h \frac{dj_\phi}{dr} \Big|_{r_1}$$

Passive stabilisation of RWM at low rotation

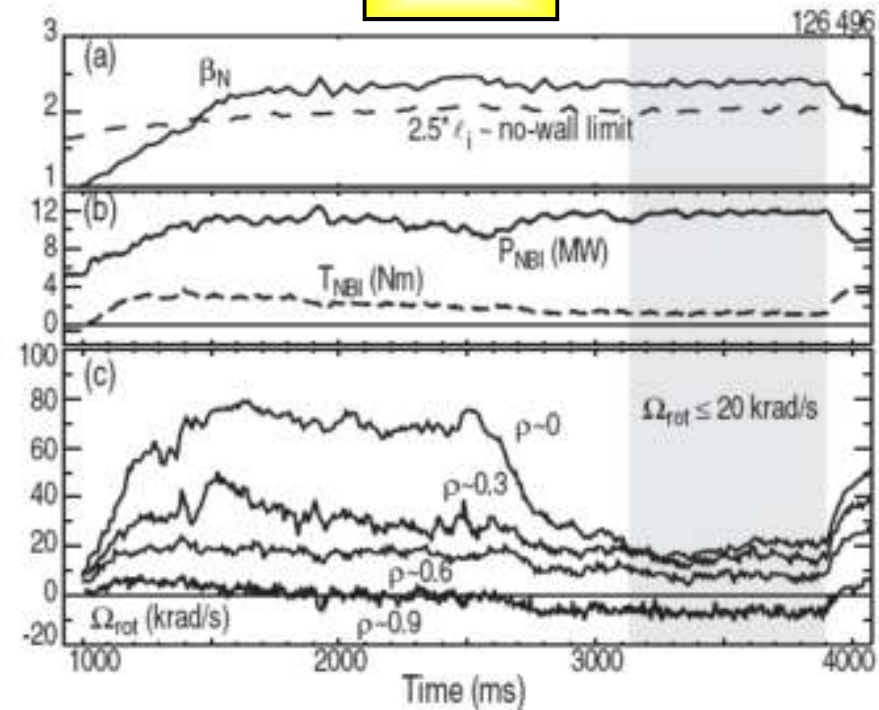
- Previous experiments explained RWM stability by rotation damping
- Recently DIII-D, JT-60U & NSTX operated with $\beta > \beta^\infty$ despite low v_ϕ

JT-60U



Takechi et al, PRL, 98, 055002

DIII-D



Reimerdes et al, PRL, 98, 055001

- Why do resonances occur?
 - Change in mode energy has a term:

$$\delta W_K \sim \sum_{l=-\infty}^{\infty} \frac{\omega - \omega_{E \times B} - n \omega_*$$

$$\omega - \omega_{E \times B} - n \langle \dot{\phi} \rangle - l \langle \dot{\theta} \rangle$$

- When denominator tends to zero, get a large contribution to δW_K
- Resonance can occur in very different frequency regimes:

Transit Frequency $\omega_t \sim (v_{th} / R)$

Bounce Frequency $\omega_b \sim \sqrt{r / R} (v_{th} / R)$

Precession Drift Frequency $\omega_d \sim \rho_L / r (v_{th} / R)$

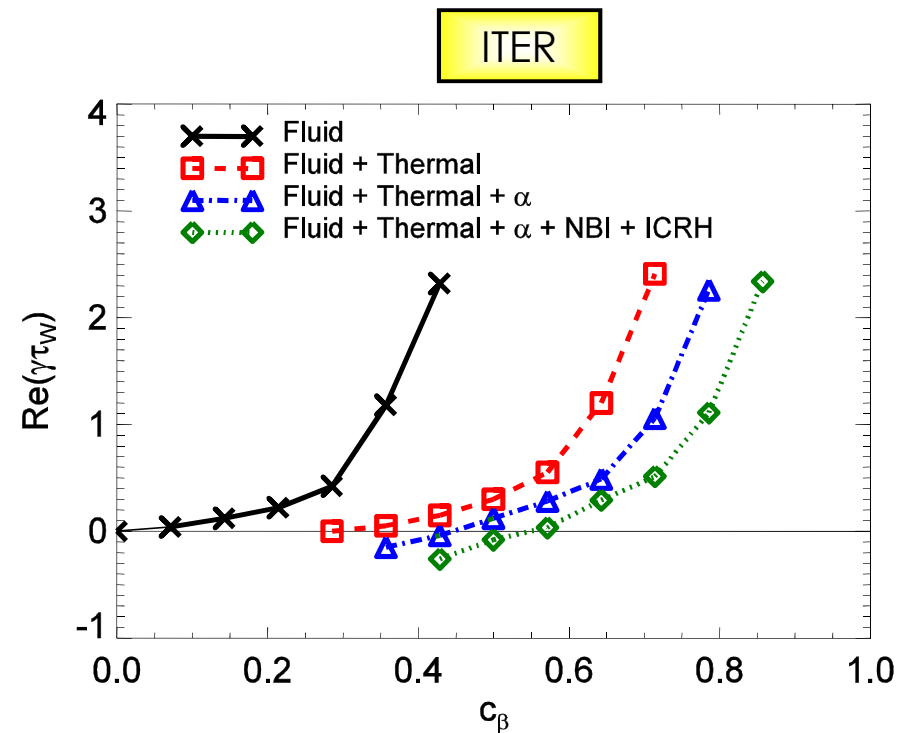
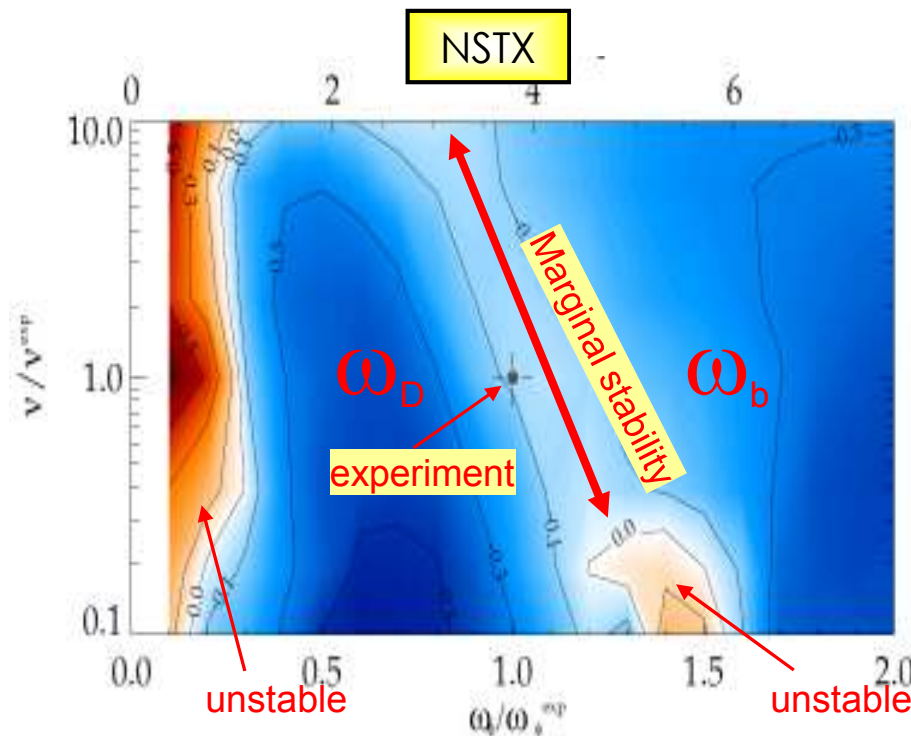
$$\omega_d \ll \omega_b < \omega_t$$



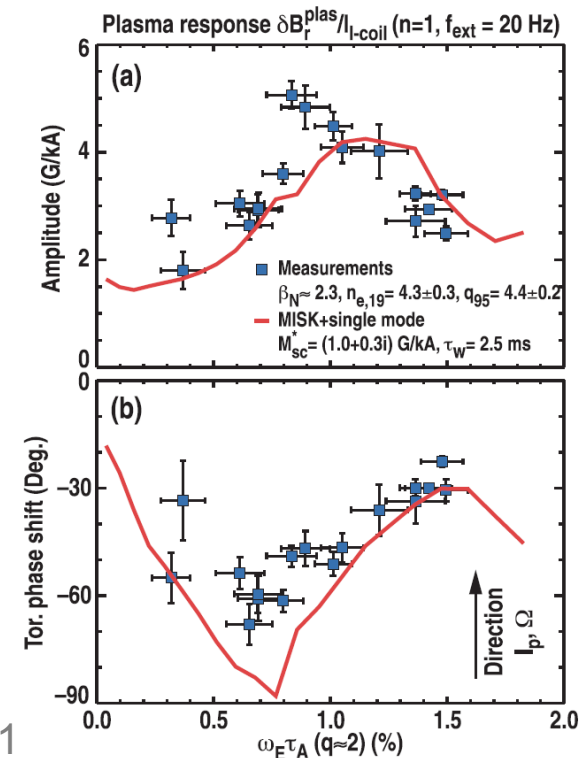
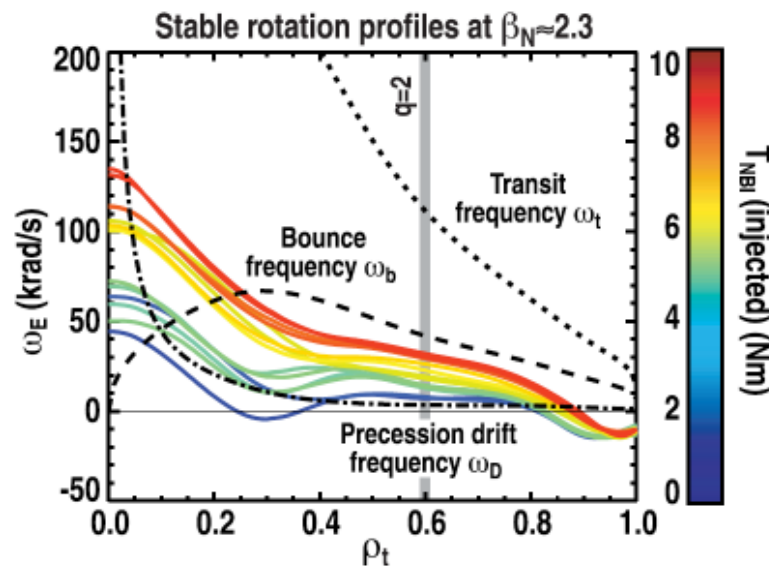
Kinetic Damping of RWM

- Stability depends on rotation influence of resonance with ω_b or ω_d
- Destabilisation between precession drift resonance at **low** V_ϕ , bounce resonance at **high** V_ϕ

$$\delta W_K \sim \sum_{l=-\infty}^{\infty} \frac{\omega - \omega_{E \times B} - n \omega_*$$



- RFA measured in DIII-D by varying the rotation profile agrees with MISK simulations including kinetic effects, though only qualitatively, due to approximations in the code
- Improved stability at very low rotation suggests that it may not be an RWM that leads to previously measured low-rotation thresholds in DIII-D and JT-60U!



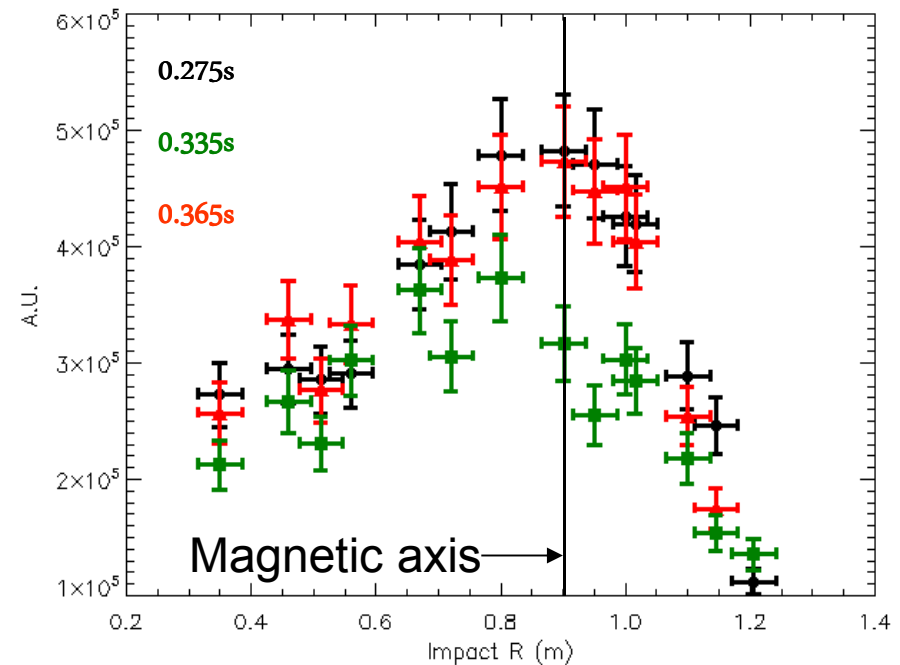
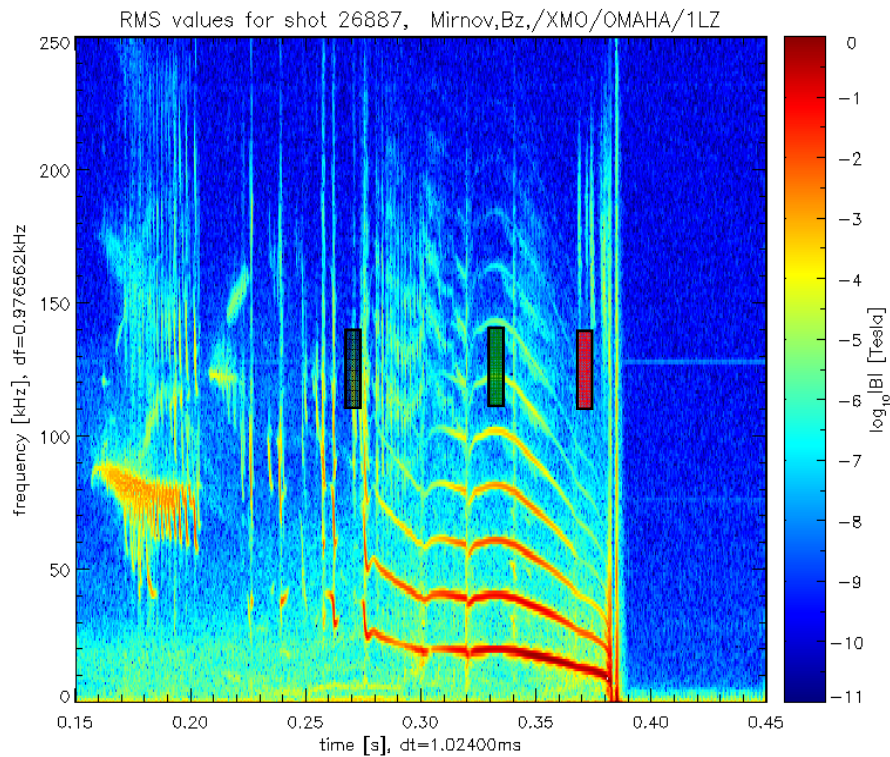
H. Reimerdes et al, PRL 2011, APS 2008 PO3.00011



- Particle orbits in tokamaks
- Sources of energetic particles
- Measuring fast ions
- Interaction of energetic particles with instabilities
 - Resonant interaction of fast ions with Alfvén instabilities
 - Effect of fast ions on instabilities in thermal plasma
- **Redistribution and loss of fast ions**
 - **Losses from instabilities and 3d fields**
 - Effects on driven current
- Controlling and utilising fast ions

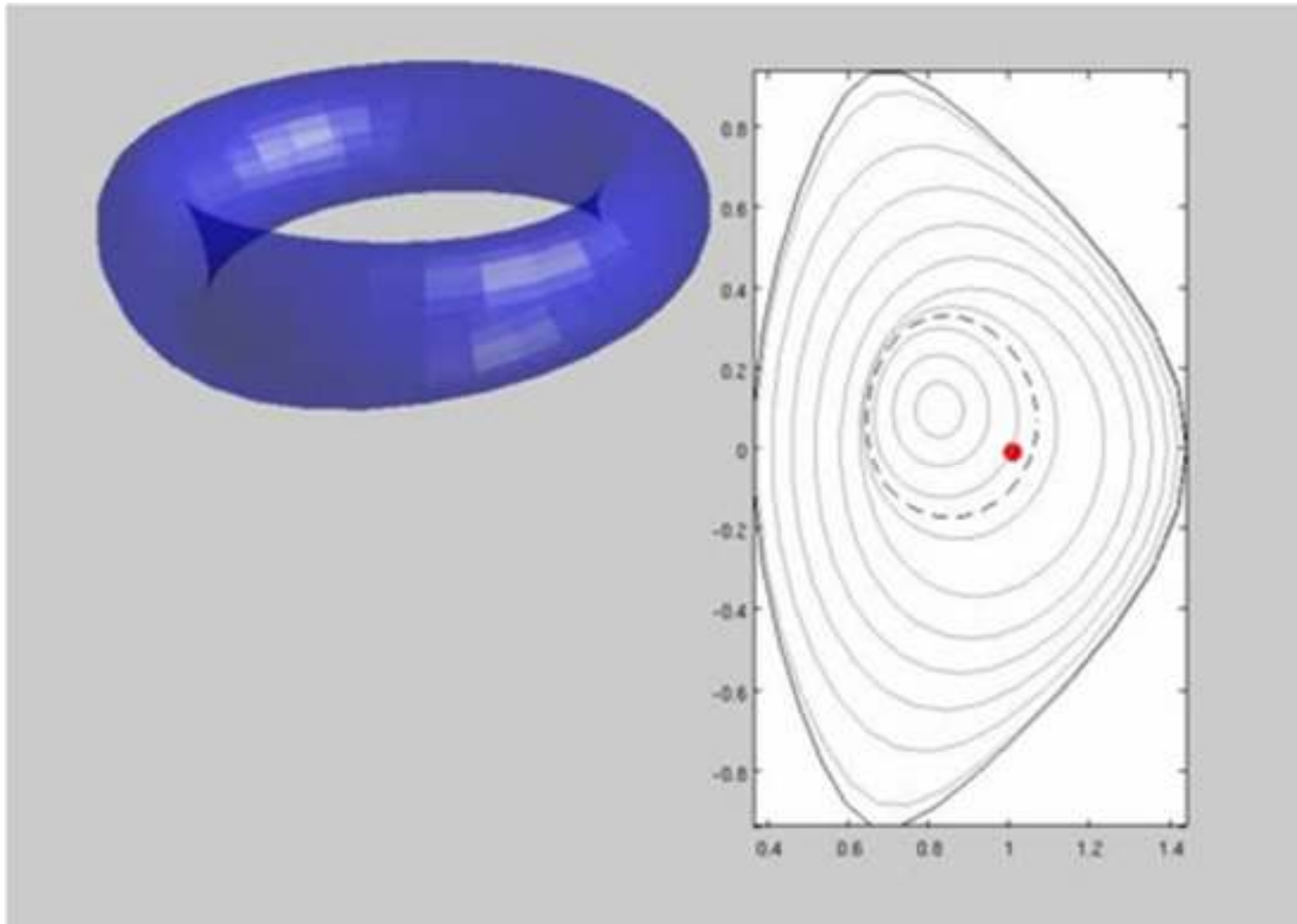


- Fast ion redistribution follows q-profile evolution
 - As q_{\min} approaches unity, LLM appears and fast ions are expelled from the plasma core (fast ions distribution represented by neutron emissivity)
 - As q_{\min} drops through unity, internal mode growth drops (alternatively, helical core amplitude decreases) and fast ions confined once more

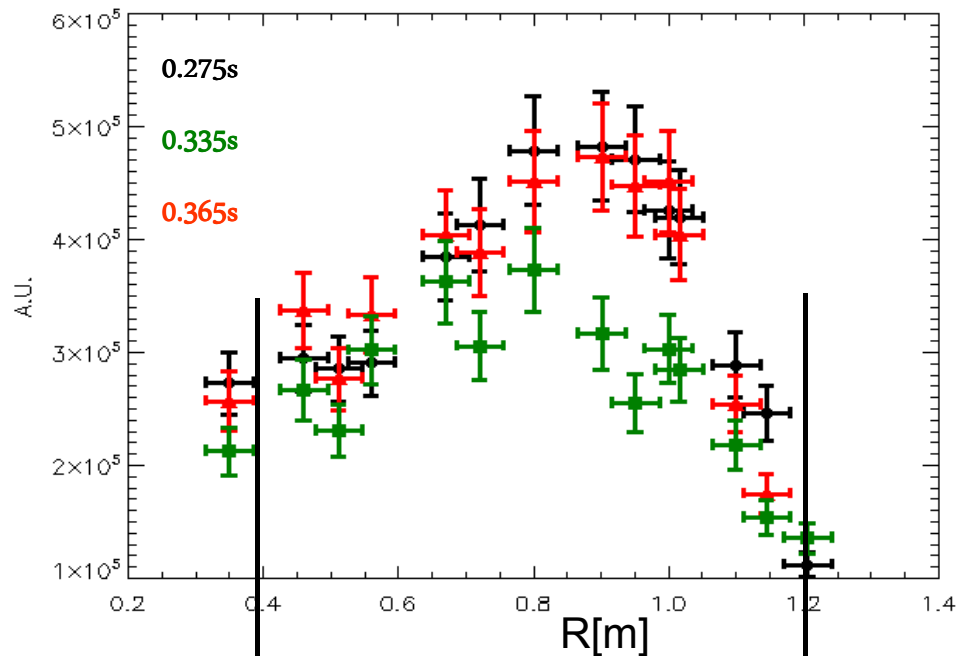


Redistribution due to helical modes

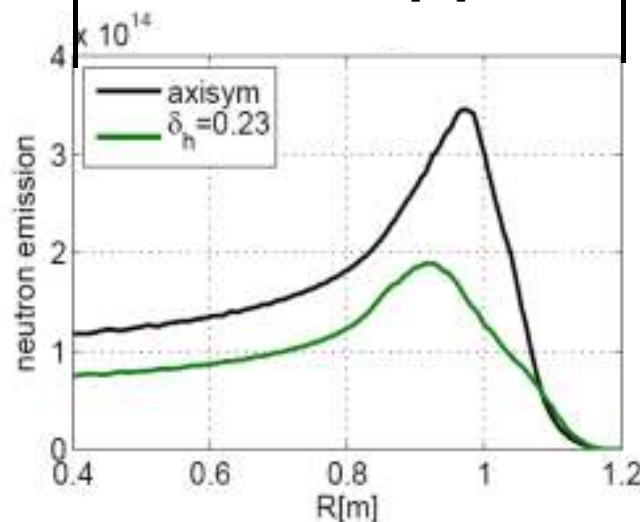
- Saturated $n=1$ mode leads to exotic particle orbits



Redistribution due to helical modes

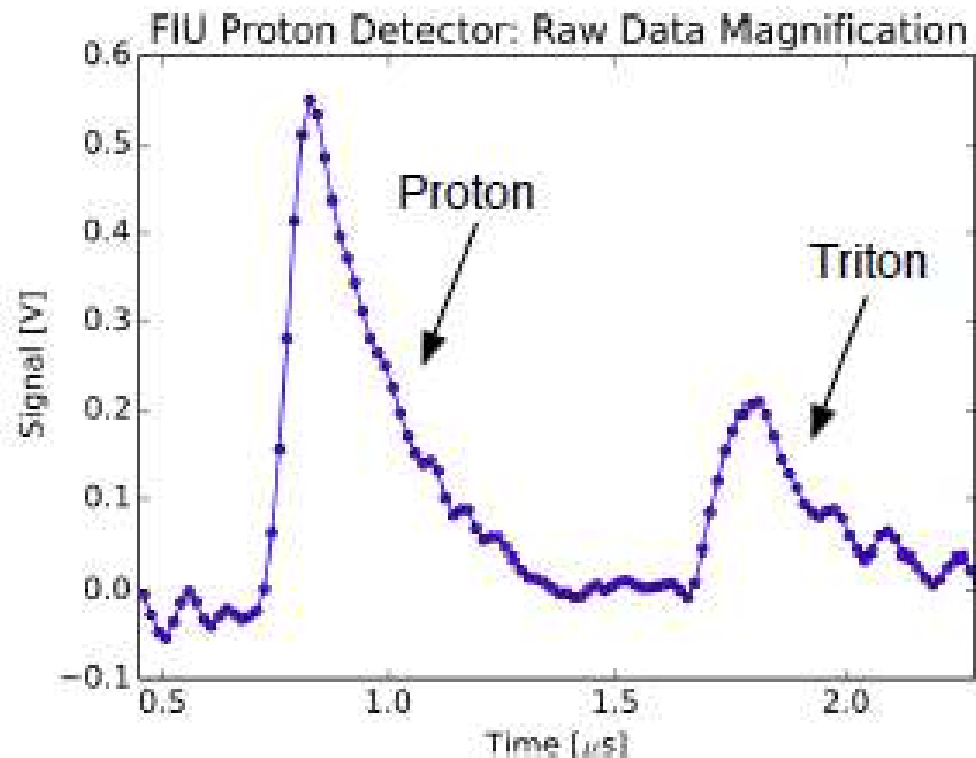
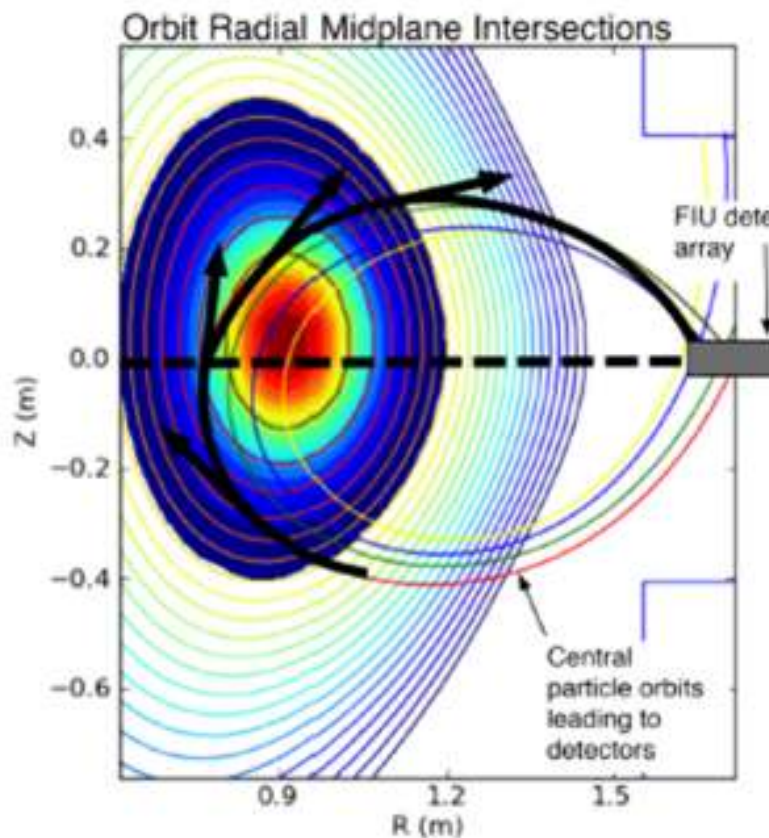


MAST neutron camera data



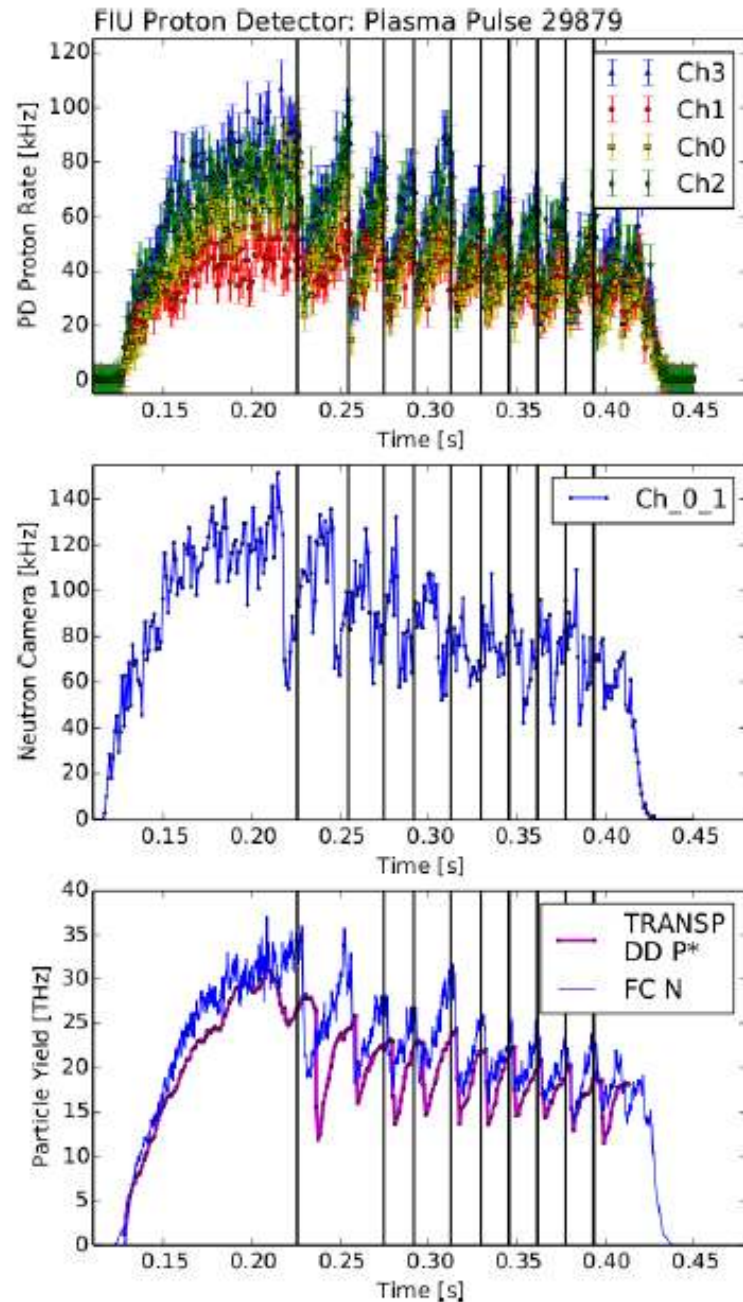
SCENIC modelling
(virtual diagnostic)

- Four channels detect particles at different angles, corresponding to birth positions in midplane; moved radially between shots
- Raw data consist of ~ 100 ns pulses produced by individual 3.0 MeV protons & 1.0 MeV tritons



RV Perez et al, HTPD, 2014

Proton losses due to sawteeth



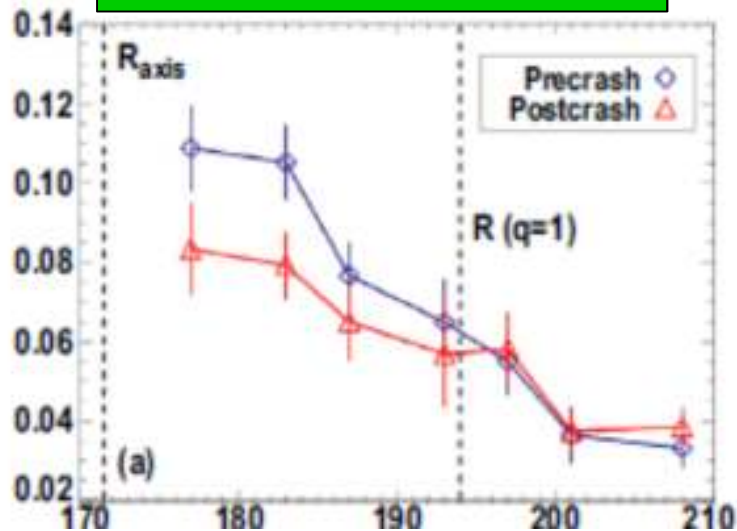
- First DD fusion proton data obtained with Proton detector up to 200kHz with 1ms time resolution
- Effect of sawteeth on fusion rate in sync with neutron camera and fission chamber data

RV Perez et al, HTPD, 2014



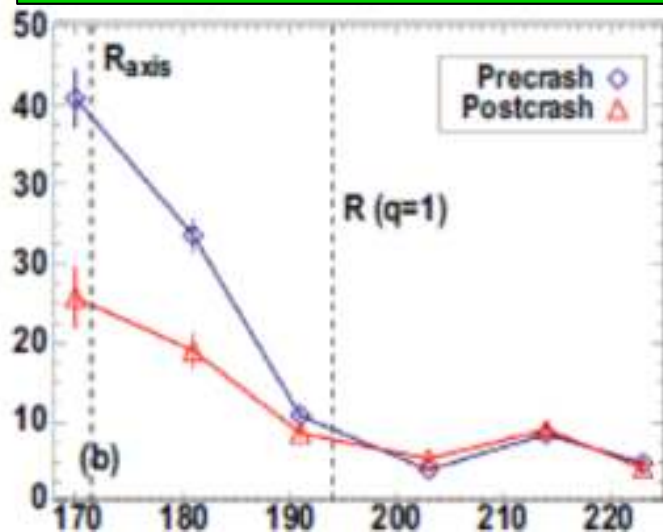
Redistribution due to sawteeth

Trapped ions (vert FIDA)

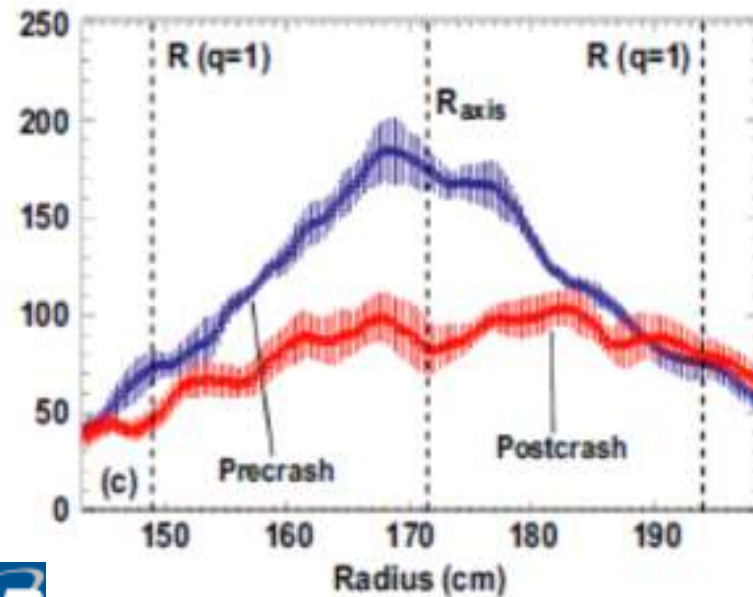


- Fast ions transported radially outwards by sawtooth crash
- Passing fast ions more affected than trapped fast ions

Co-passing ions (tang FIDA)



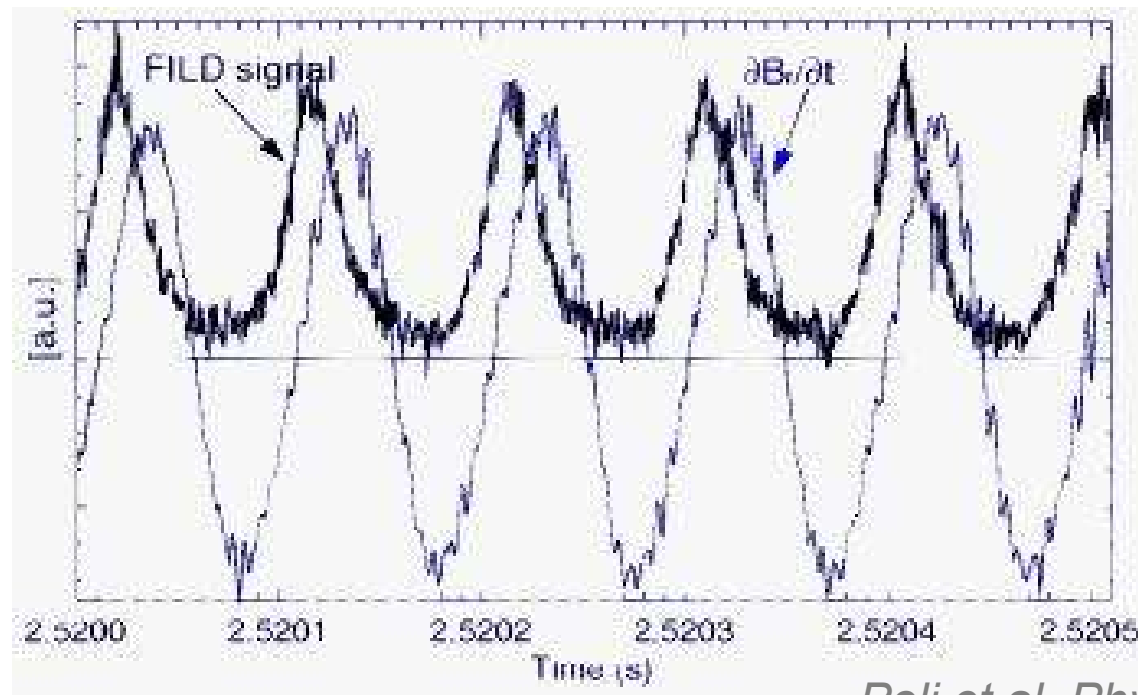
Counter-passing ions (FIDA imaging)



CM Muscatello et al, PPCF 2012



- NTMs also lead to enhanced fast ion losses
- Fast ions losses modulated at mode frequency
 - Losses of passing particles caused by drift island formation
 - Losses of trapped particles due to stochastic diffusion
- Insignificant change in wall loads predicted in ITER

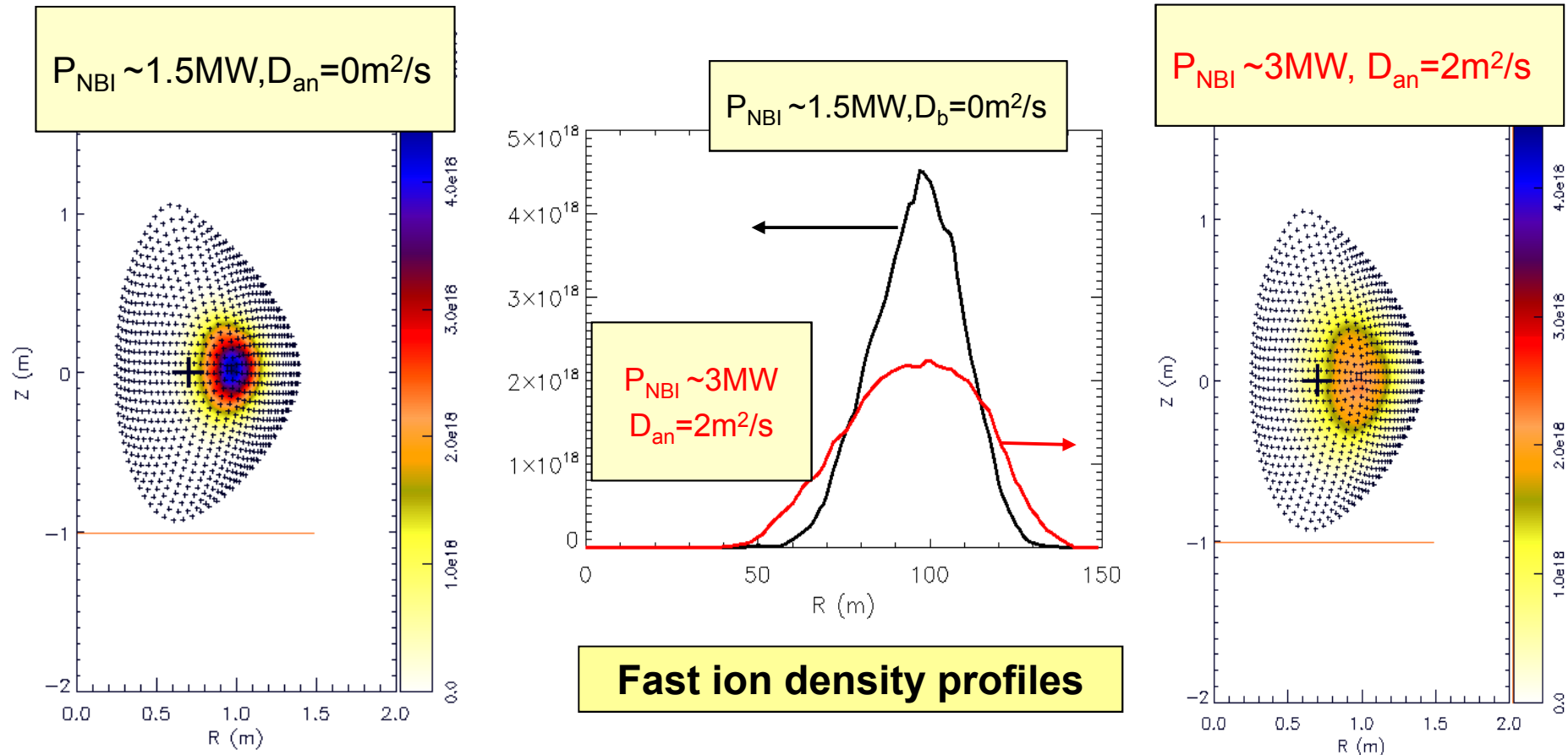


Poli et al, Phys. Plasmas 2008

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Impact of FI redistribution (on axis NBI)



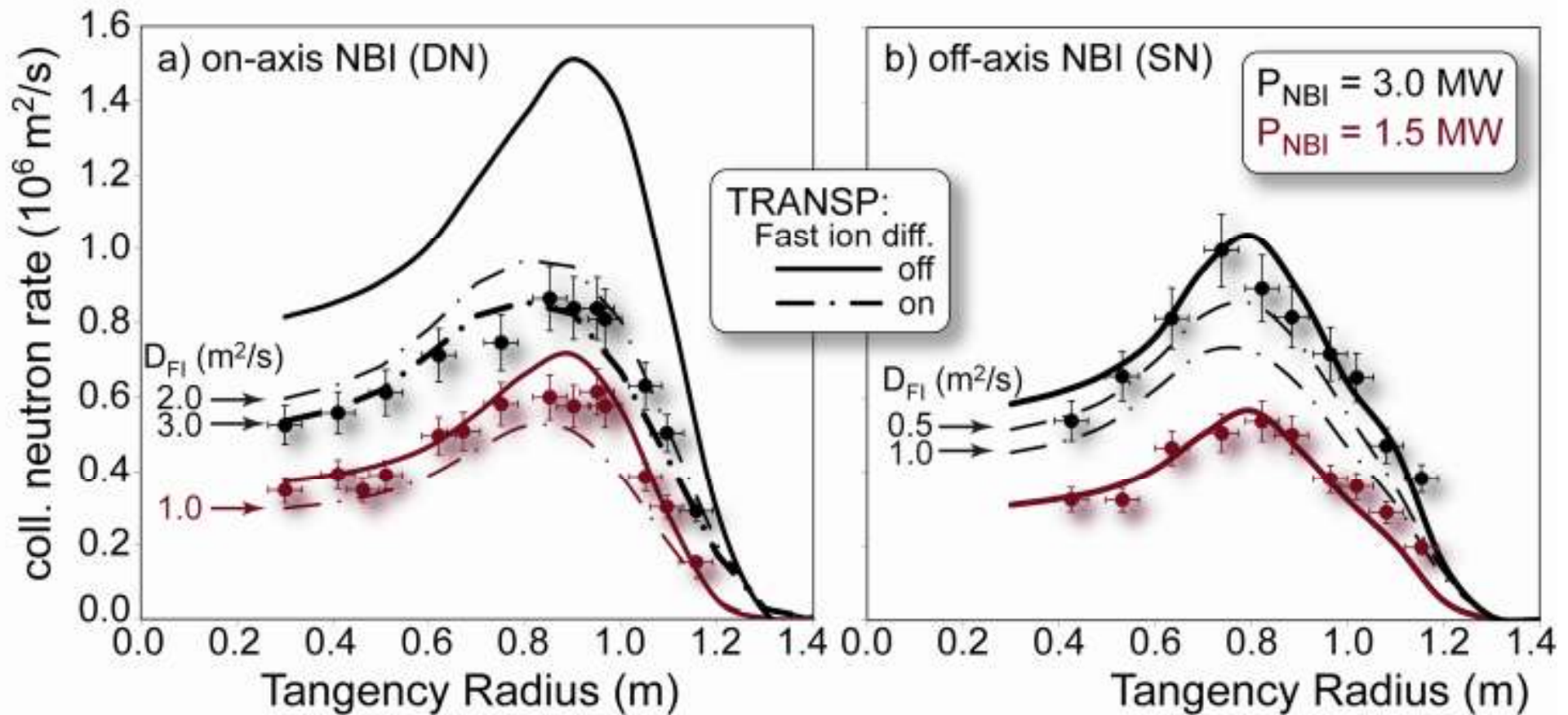
- Assuming ad hoc D_{an} , doubling beam power results in:
 - broader fast ion and current density profile
 - 50% increase in fast ion stored energy
 - reduced current drive efficiency by $\sim 20\%$

*Turnyanskiy et al,
Nucl Fusion, 2013*



Reduced redistribution for off-axis beams

- Doubling power from 1.5MW to 3MW for off-axis beam results in classical doubling of neutron emissivity
 - consistent with $D_{an} < 0.5 \text{ m}^2/\text{s}$



Turnyanskiy et al, Nucl Fusion, 2013

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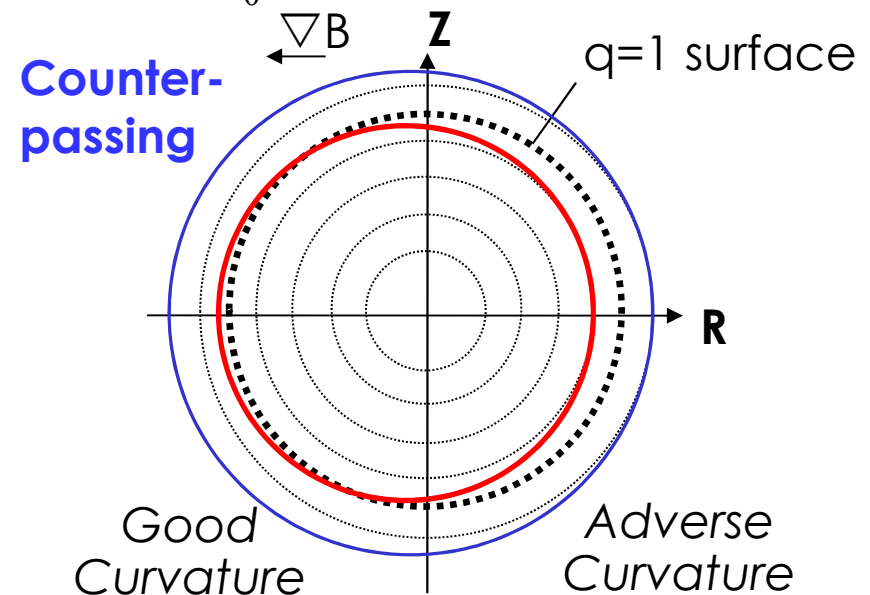
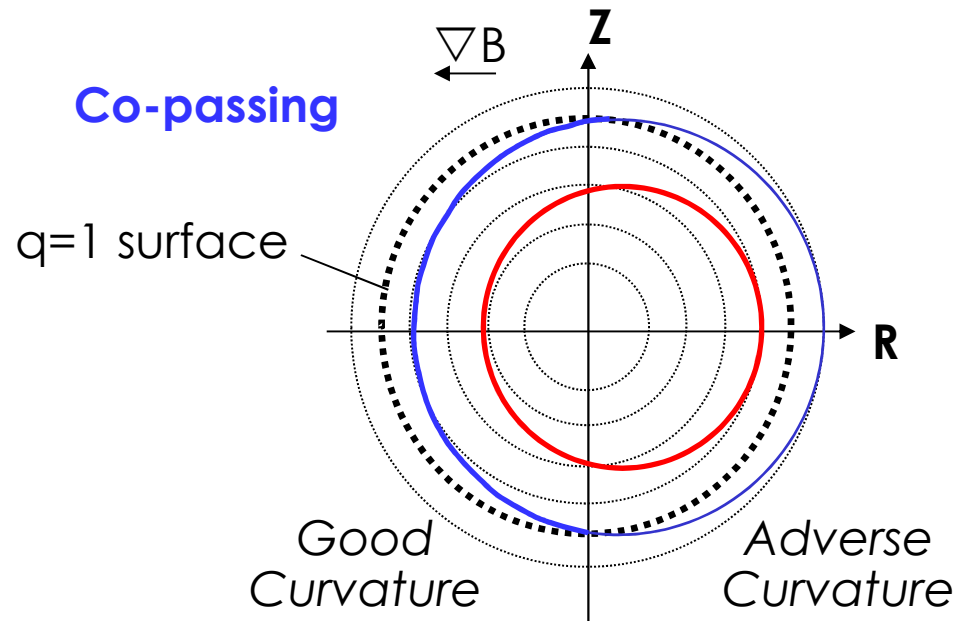


Reminder: Fast ion effect on sawteeth

δW has a term dependent upon curvature:

Graves, PRL, 2004

$$\delta W \sim - \int_0^{r_1} (\xi \cdot \nabla \langle P_h \rangle) (\xi \cdot \kappa) dr$$



Co-pass, $\langle P_h \rangle' |_{r_1} < 0 \rightarrow$ stabilising

Co-pass, $\langle P_h \rangle' |_{r_1} > 0 \rightarrow$ destabilising

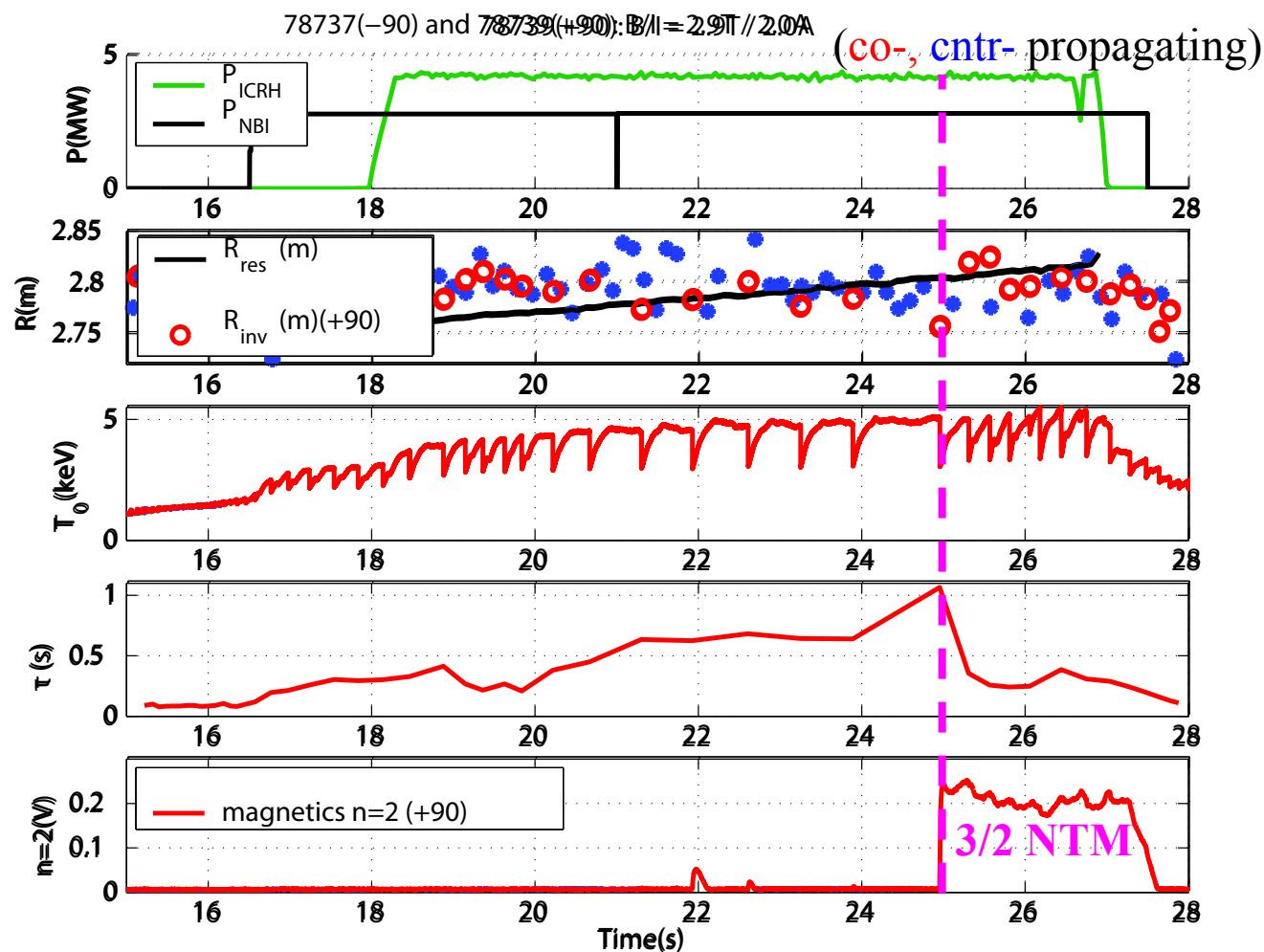
Ctr-pass, $\langle P_h \rangle' |_{r_1} < 0 \rightarrow$ destabilising

Ctr-pass, $\langle P_h \rangle' |_{r_1} > 0 \rightarrow$ stabilising

- Increasing effective orbit width leads to large passing ion effect
 - Large thermal velocity (ITER NNBI, ICRH)
 - Large fraction of barely passing ions (JET NBI, ICRH)



- Control demonstrated with ^3He minority with negligible ICCD
 - Also shown in H-mode with higher P_{aux}

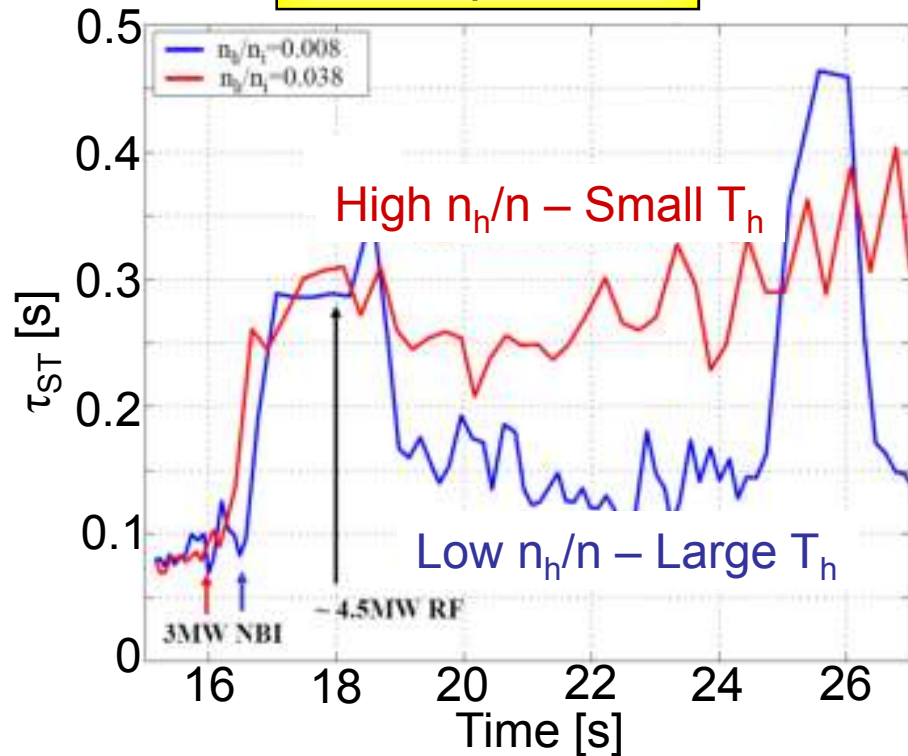


L-mode
 $\beta_N = 0.8$

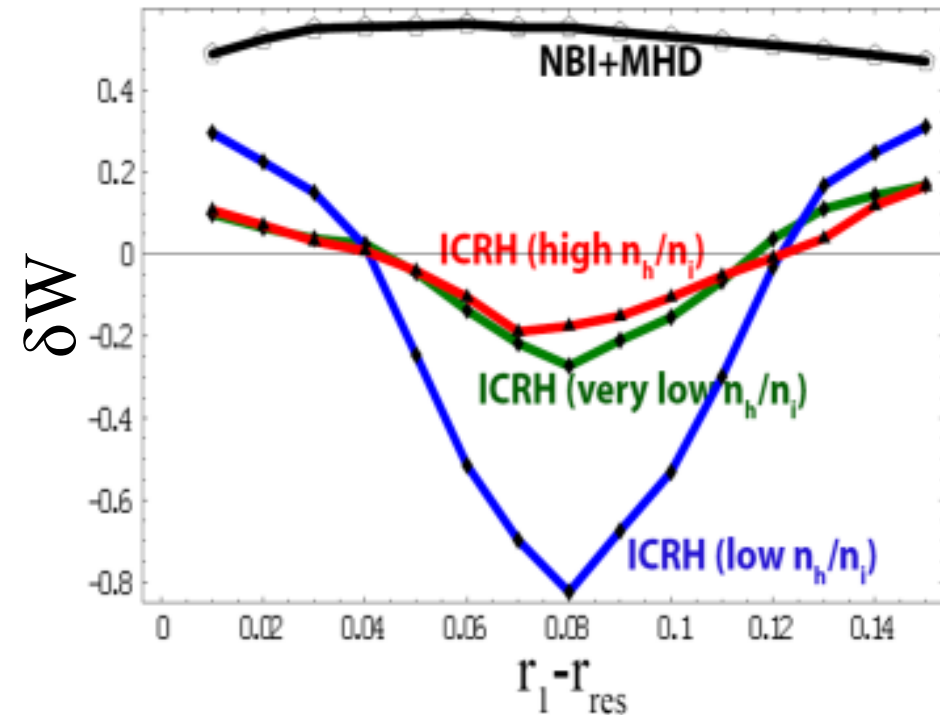
Graves et al, Nucl Fusion 2010; Nature Comms 2012

- Lower ^3He concentration \rightarrow larger $T_h \rightarrow$ shorter τ_{ST}
 - Too much $^3\text{He} \rightarrow$ tail energy too low
 - Too little $^3\text{He} \rightarrow$ broader f_h , more losses

JET Experiment

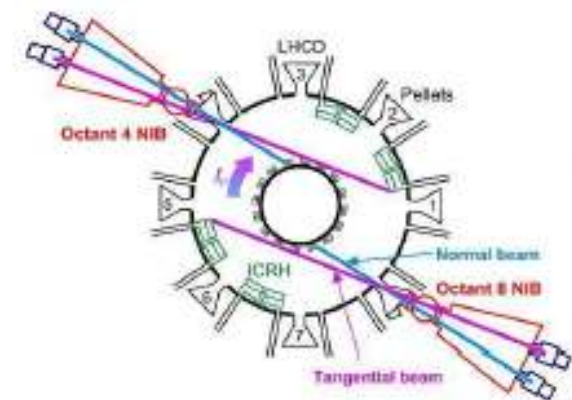


SELFO/HAGIS Modelling

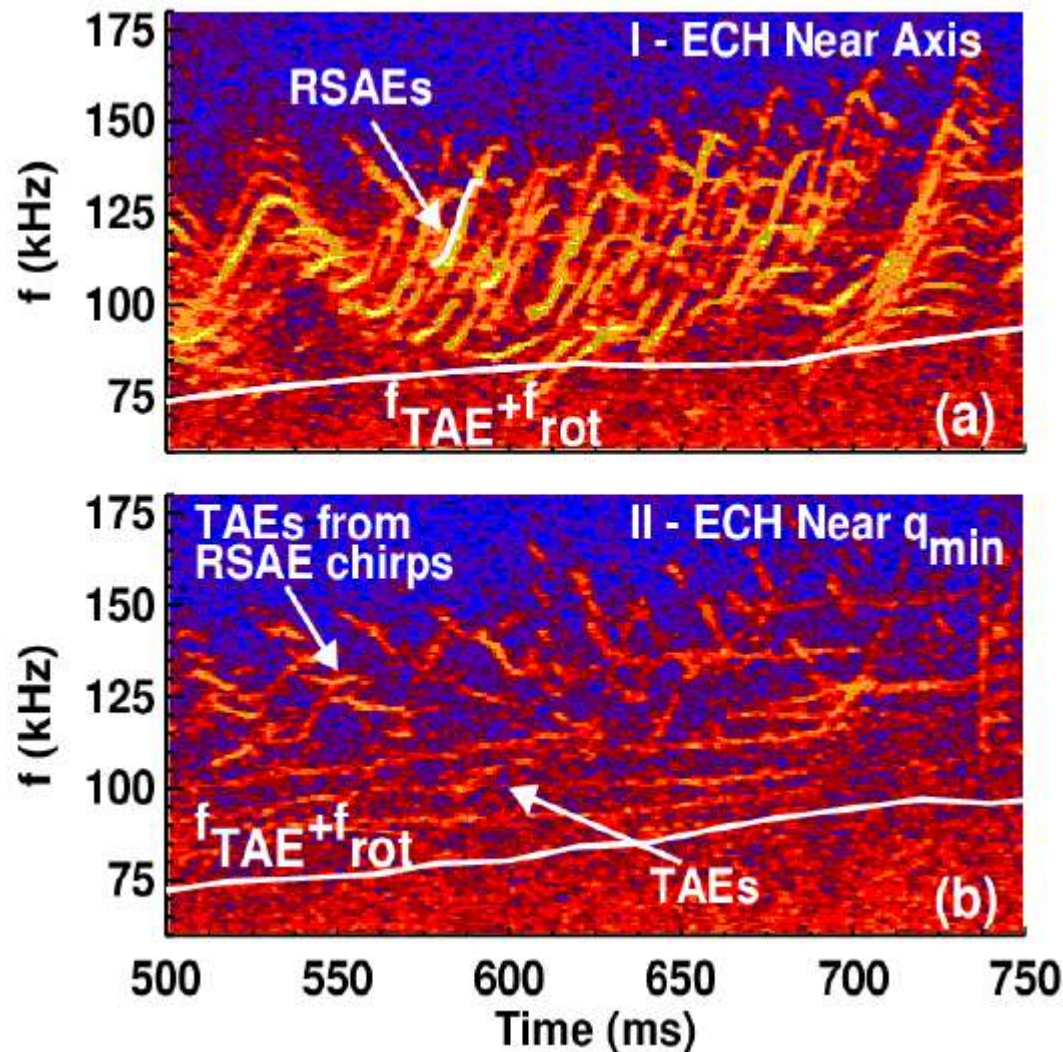


Graves et al, Nucl Fusion 2010

- Affect stability/existence of Alfvén Waves
 - Plasma conditions: density, safety factor, beta, isotope mix (mass density), magnetic field, introduce flow (rotation)
 - Monitor with magnetics/antenna
- Tailor fast particle distribution
 - Alphas: Fuelling
 - NBI: Beam geometry, injection energy
 - ICRF: Resonance layer
 - Field topology: Ripple, 3D field coils, aspect ratio



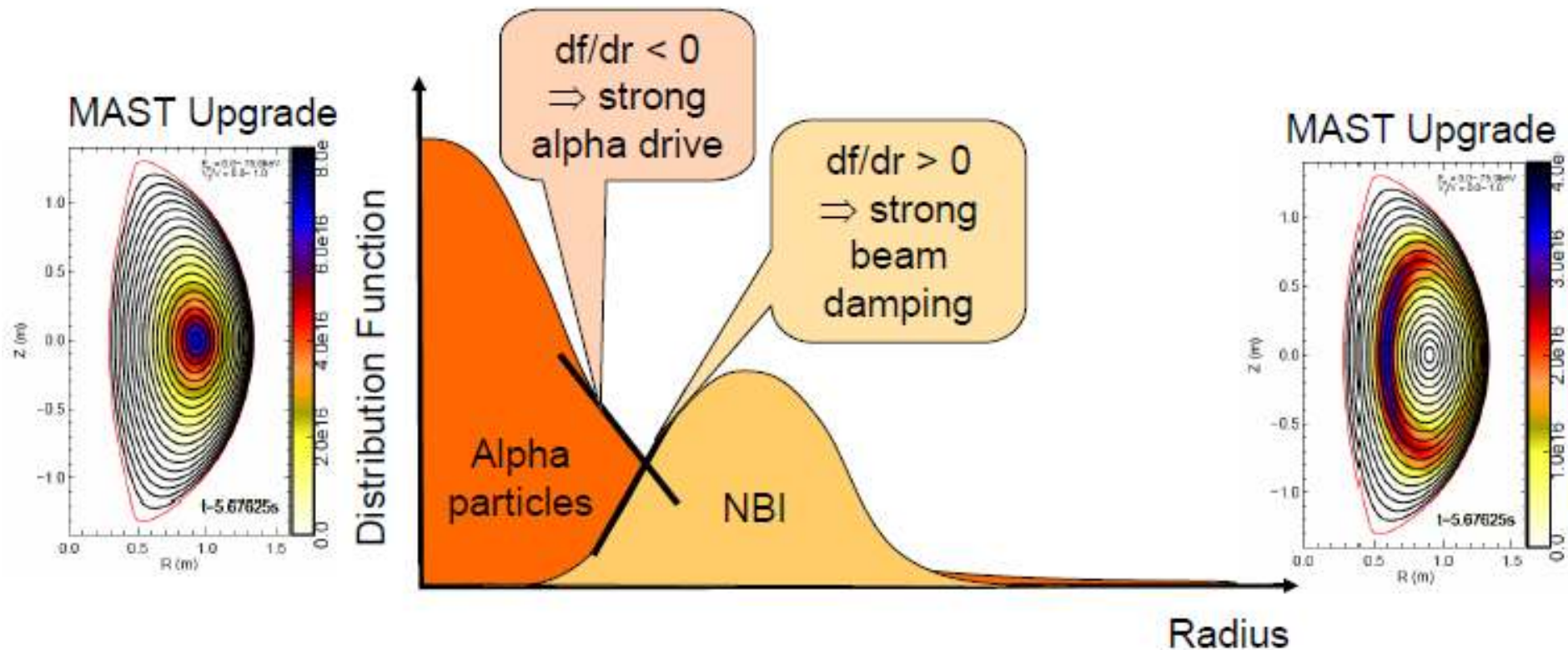
Controlling AEs with ECRH



- Applying ECRH near radial position of AEs shown to suppress the modes

Tailoring the fast ion distribution

- Alpha particles strongly peaked on-axis
- Use off-axis beams to change drive-damping balance?



- Our original concern was loss of heating and damage to the first wall. Is this a problem?
 - Physics of fast ion driven instabilities is well understood
 - Fast particles drive instabilities and are in turn redistributed and, in some cases, lost
 - Typically, saturation amplitudes and losses are low today
 - Burning plasmas may be a different story – more to do!
- How about effects on other MHD and current drive?
 - Theory and experiment agree that fast ions strongly influence other modes: sawteeth, RWMs, ...
 - Fast ion driven modes can strongly influence current profile
- What are prospects for controlling AEs?
 - Tailoring distribution/plasma conditions, and ECRH



Problems Solving



- Determine the condition for the pitch angle at the midplane ($v_{\parallel 0} / v_{\perp 0}$) for a particle to be in a trapped orbit
 - Hint: Use conservation of magnetic moment and energy to relate magnetic field at midplane and bounce point, then assume $B \sim 1/R$

