Max-Planck-Institut
für Plasmaphysik

IPP

DPG School ‚The Physics of ITER'
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Energy Transport,
Theory (and Experiment)

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Special acknowledgments for material and discussions

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Transport in tokamak plasmas

- The topic of “transport” is dedicated to the physical processes by which particles, momentum and energy are moved (transported) in a domain in the real space (or in phase space more in general)
- The general goal is to identify the relationship between the thermodynamic fluxes and the thermodynamic forces
- Thermodynamic fluxes are particle, momentum and energy (heat) fluxes. In present tokamaks they are mainly imposed by external means (particle sources, torques and heating powers)
- Thermodynamic forces are the spatial gratdients of the particle and momentum densities and of the pressures (temperatures). These describe how plasma reacts to the imposed fluxes, namely the radial profiles of density,rotation and temperature

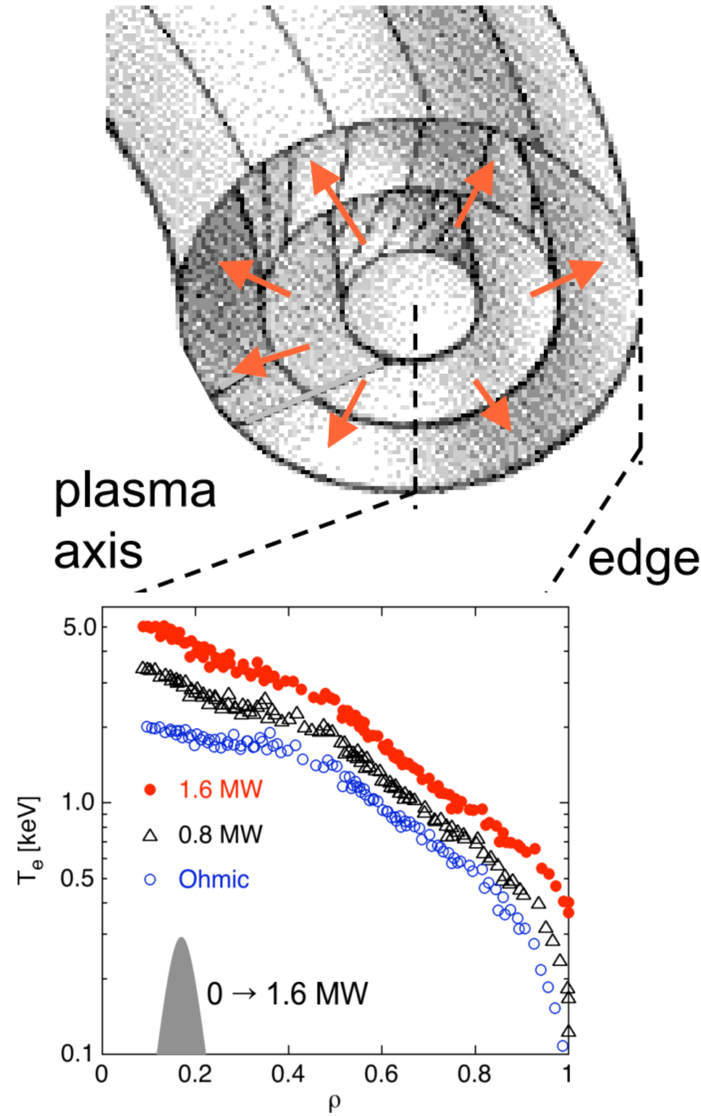
Transport in tokamak plasmas

- As we shall see, temperatures and densities are approx constant over flux surfaces

- Radial transport determines the profiles of temperatures and densities (their gradients = thermodynamic forces) in response to heat and particle sources (thermodynamic fluxes)

- Present lecture presents some general aspects and focuses on transport in the central region of the plasma (core)

- Edge region (pedestal) later today



Transport mechanisms in tokamak plasmas

- In tokamak plasmas, transport is produced by collisions and by turbulence
- Collisions provide a minimum (and unavoidable) level of transport
- However, in tokamaks, collisional transport is small. Already relatively weak heat sources increase the (temperature) gradients to levels at which they can trigger (micro-) instabilities, and develop turbulence, which drives the majority of transport
- Also MHD modes (macro-instabilities) (which can be triggered under certain conditions) modify the profiles. Therefore, they can be considered to produce additional transport. However, given the different spatial and time scales that they involve, they are presented in separate lectures (this afternoon)

The role of transport in tokamak plasmas

- **The minimization of transport is an essential element to achieve practical fusion energy collisions and turbulence**

- **In fact, the lowest is the transport (the highest is the confinement), the highest is the plasma energy for a given amount of heating**

- **Fusion power increases with plasma energy**

$$P_{fus} \sim n_D n_T \langle \sigma v \rangle \sim n_e^2 T^2 \sim W_{plasma}^2$$

- **A reactor must maximize the power multiplication factor Q (ignition), producing a large amount of fusion power**

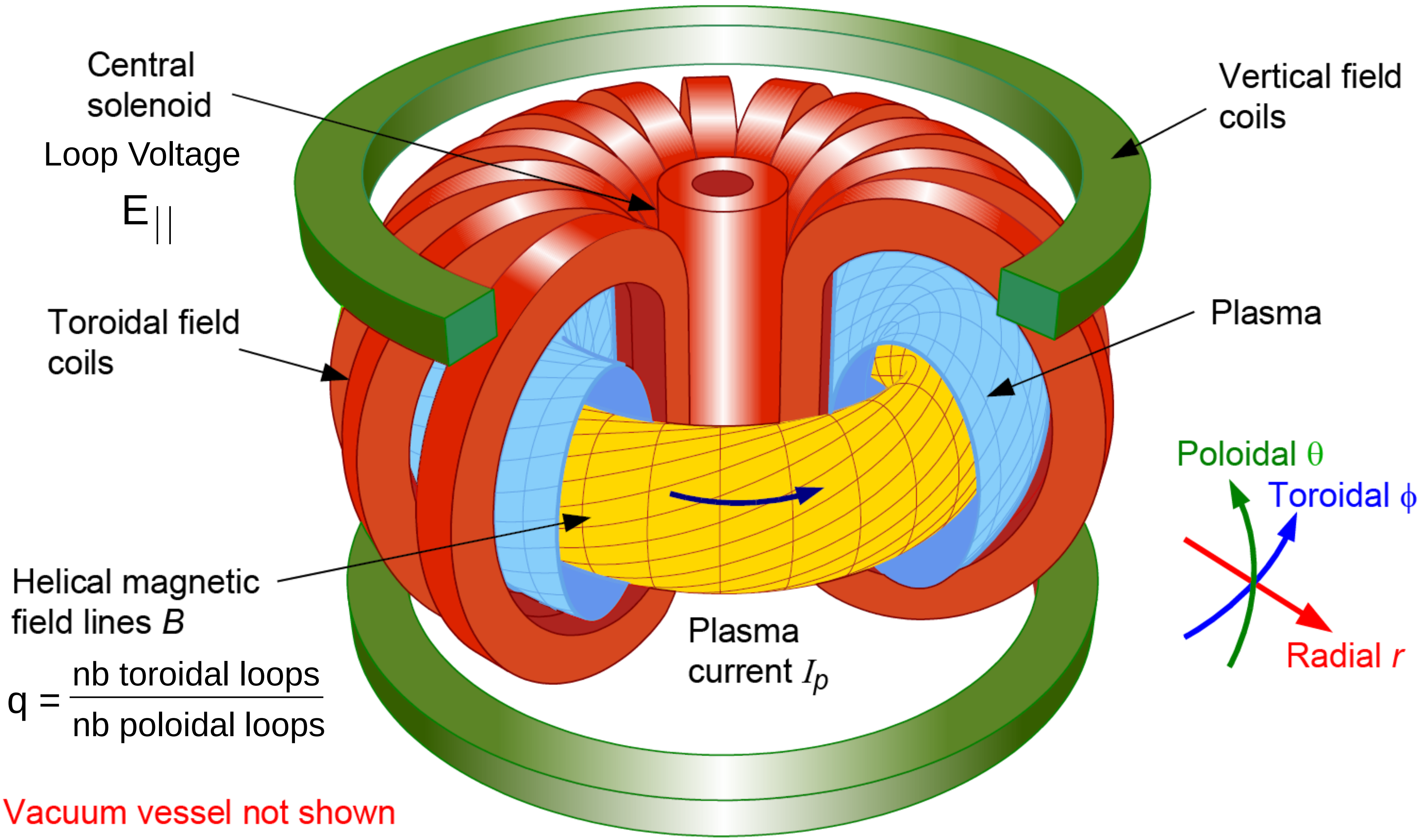
$$Q = P_{fus}/P_{ext}$$

- **This is why understanding, predicting, and controlling transport in plasmas is one of the highest priorities**

Outline

- Tokamak geometry, drifts and orbits, time scales
- Neoclassical and turbulent transport
- Turbulent transport
  - Linear micro-instabilities
  - Turbulence and transport, turbulence properties
  - Scaling of transport and confinement with plasma size
  - Mechanisms of turbulence reduction and suppression

Tokamak geometry plays an essential role



$$q = \frac{\text{nb toroidal loops}}{\text{nb poloidal loops}}$$

Vacuum vessel not shown

Gyromotion and drifts

➤ **Dominant motion is gyromotion (magnetized plasma)**

➤ **Any force F implies a drift of the particle in a direction perpendicular to both the field B and the force F**

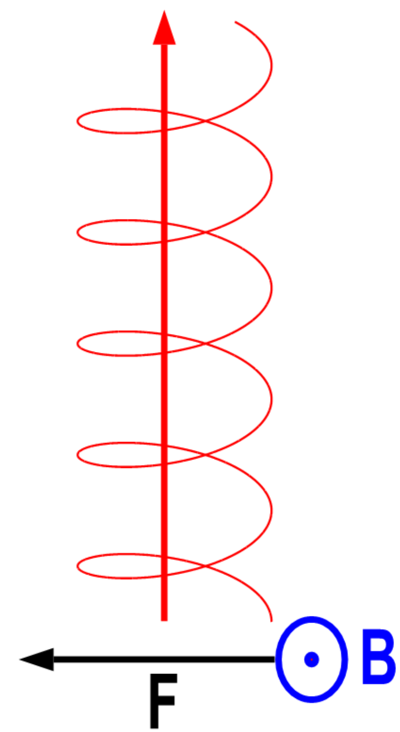
$$\mathbf{v} \simeq v_\parallel \mathbf{b} + \underbrace{\frac{m}{ZeB}\left(\frac{v_\perp^2}{2} + v_\parallel^2\right)\frac{\mathbf{B}\times\nabla B}{B^2} + \frac{\mathbf{B}\times\nabla\Phi}{B^2}}_{\text{drifts}}$$

parallel motion

curvature drift

ExB drift

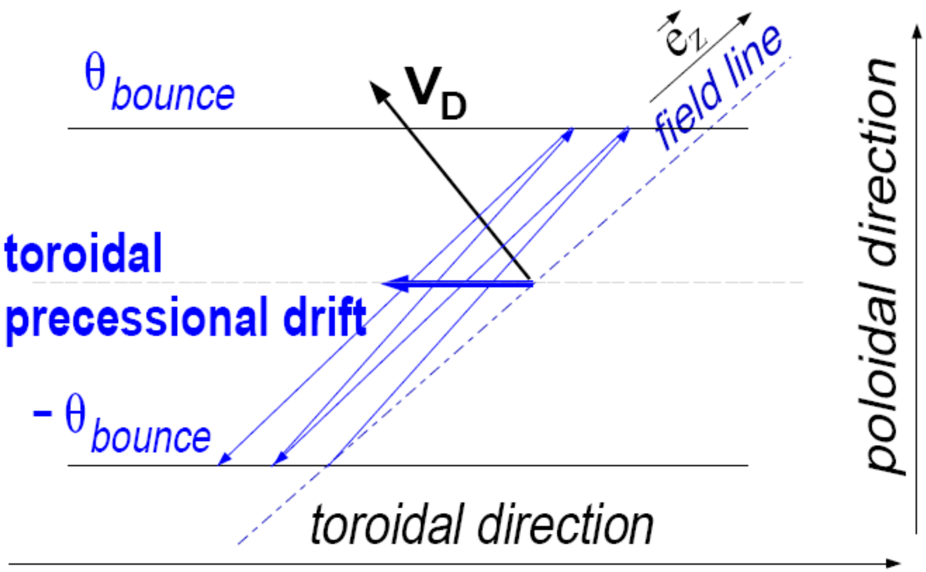
grad B drift

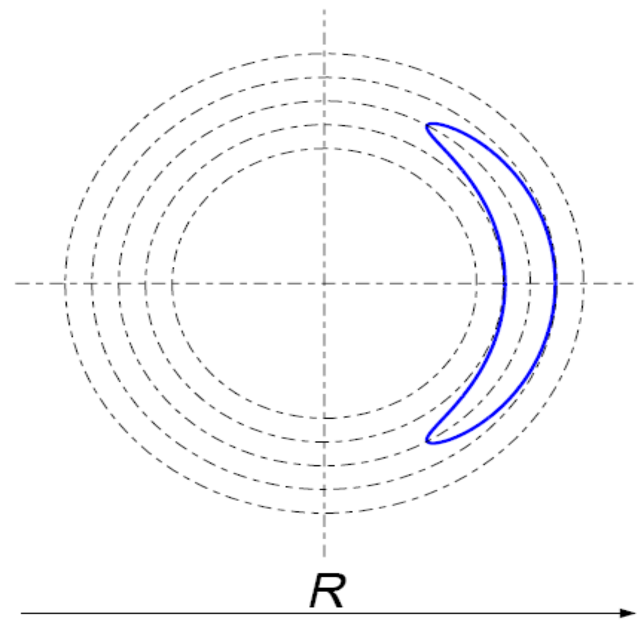


➤ **Mirror force also leads to particle trapping (banana orbits) since the magnetic field increases towards the axis of the torus**

Tokamak geometry, trapping and drifts

➤ **Mirror Force (B ∝ 1/R) ⇒ particle trapping (Banana orbits)**



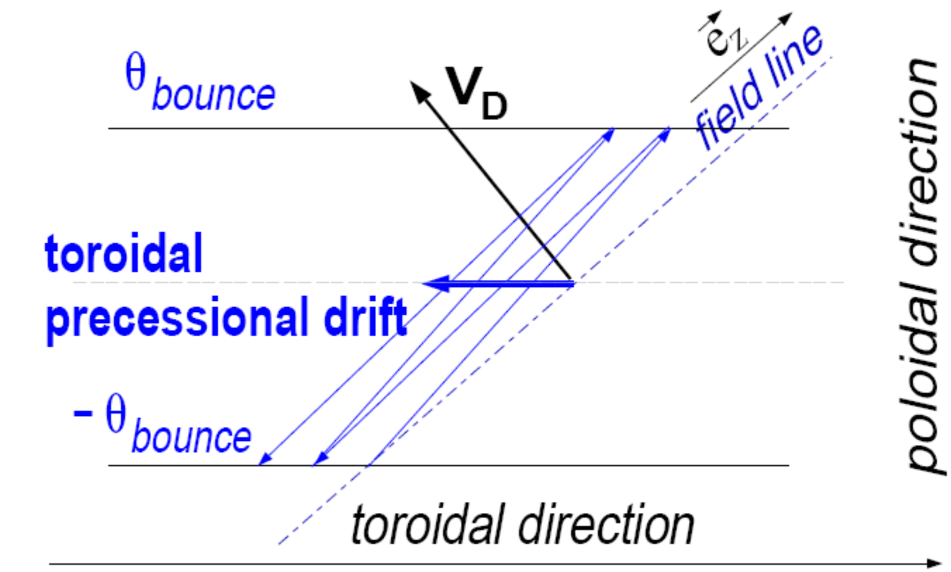


➢ **Curvature and grad B drift**

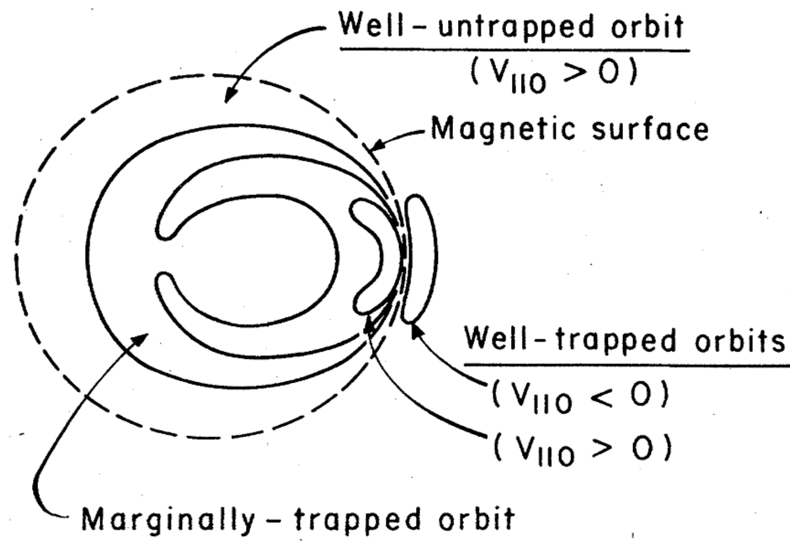
$$\mathbf{v}_C + \mathbf{v}_{\nabla B} = \frac{v_\parallel^2}{\Omega_c}\mathbf{B} \times \mathcal{K}_B + \frac{v_\perp^2}{2\Omega_c}\mathbf{B} \times \frac{\nabla B}{B}$$

Collisional / Neoclassical transport

- **Mirror Force (B ∝ 1/R) ⇒ particle trapping (Banana orbits)**



[Hinton Hazeltine RMP 1976]



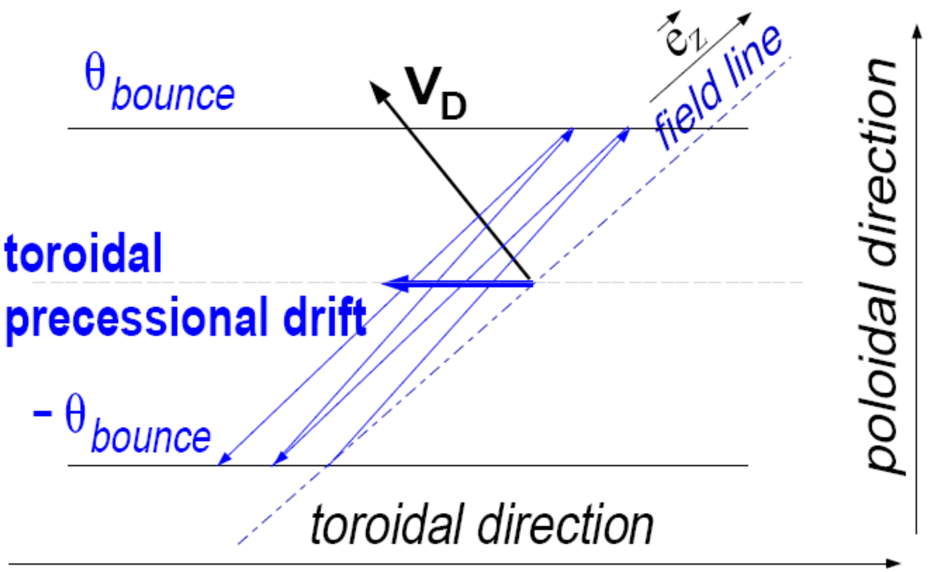
- **Curvature and grad B drift**

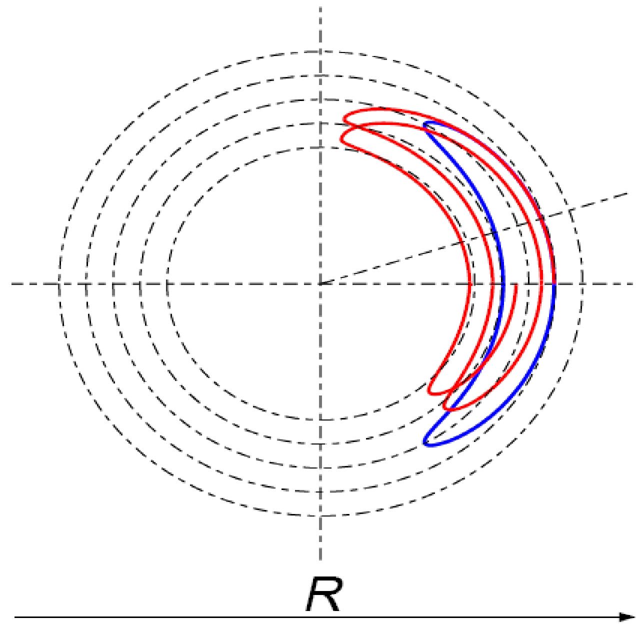
$$\mathbf{v}_C + \mathbf{v}_{\nabla B} = \frac{v_\parallel^2}{\Omega_c}\mathbf{B}\times\mathcal{K}_B + \frac{v_\perp^2}{2\Omega_c}\mathbf{B}\times\frac{\nabla B}{B}$$

- **Coulomb collisions, diffusion in velocity space ⇨ diffusion in real space ⇨**
- **Random displacement by a banana width ⇨ neoclassical (banana) transport**

Tokamak geometry, trapping and drifts

➤ **Mirror Force (B $\propto$ 1/R) $\Rightarrow$ particle trapping (Banana orbits)**





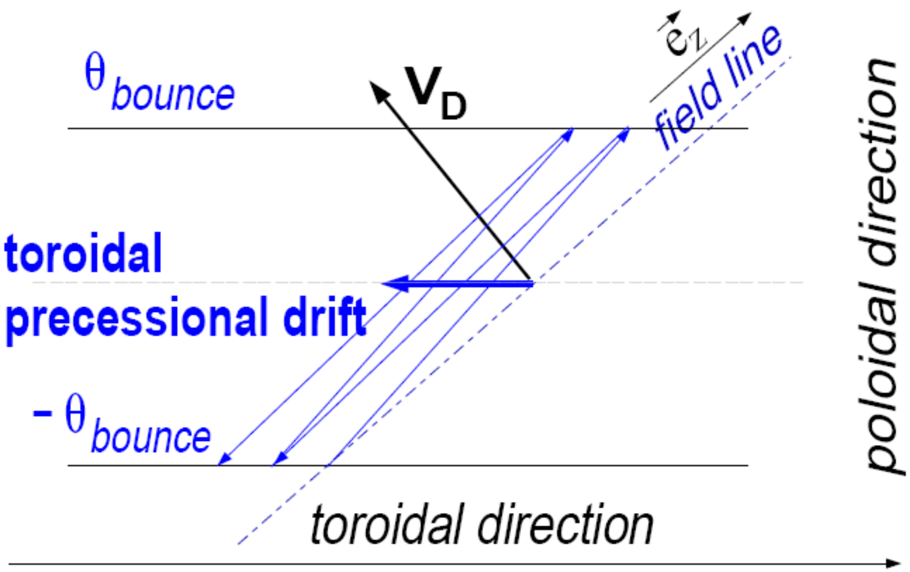
➢ **Curvature and grad B drift**

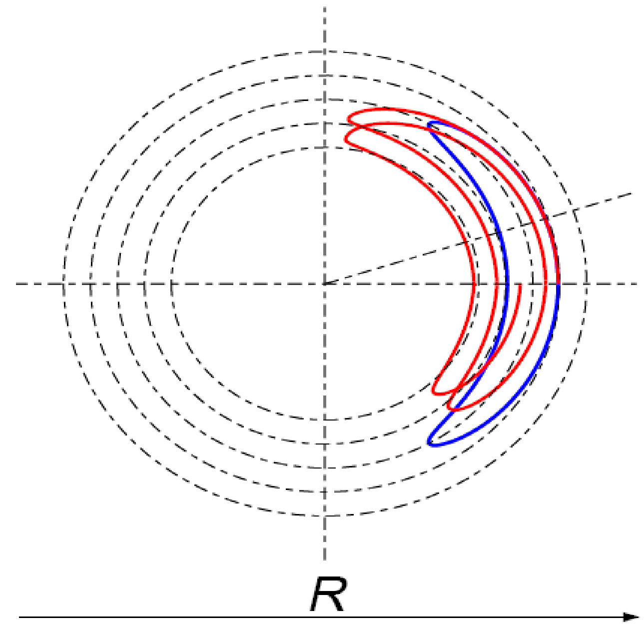
$$\mathbf{v}_C + \mathbf{v}_{\nabla B} = \frac{v_\parallel^2}{\Omega_c}\mathbf{B} \times \mathcal{K}_B + \frac{v_\perp^2}{2\Omega_c}\mathbf{B} \times \frac{\nabla B}{B}$$

➢ **Parallel electric field $\Rightarrow$ orbit displacement $\Rightarrow$ Ware pinch $\Rightarrow$** $\propto \dfrac{1}{q\,T^{5/2}}$

Tokamak geometry, trapping and drifts

➤ **Mirror Force (B ∝ 1/R) ⇒ particle trapping (Banana orbits)**



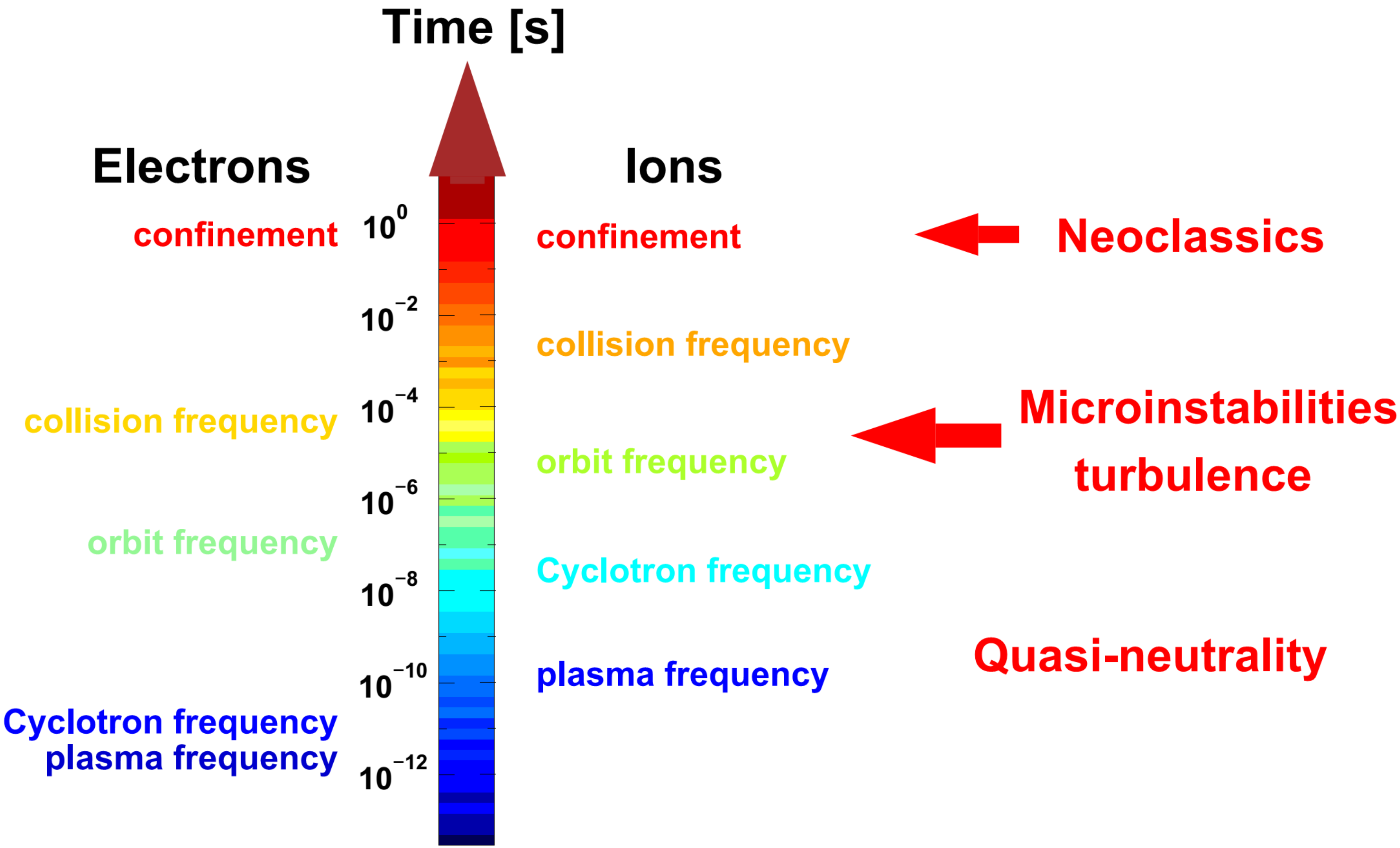


➢**Curvature and grad B drift**

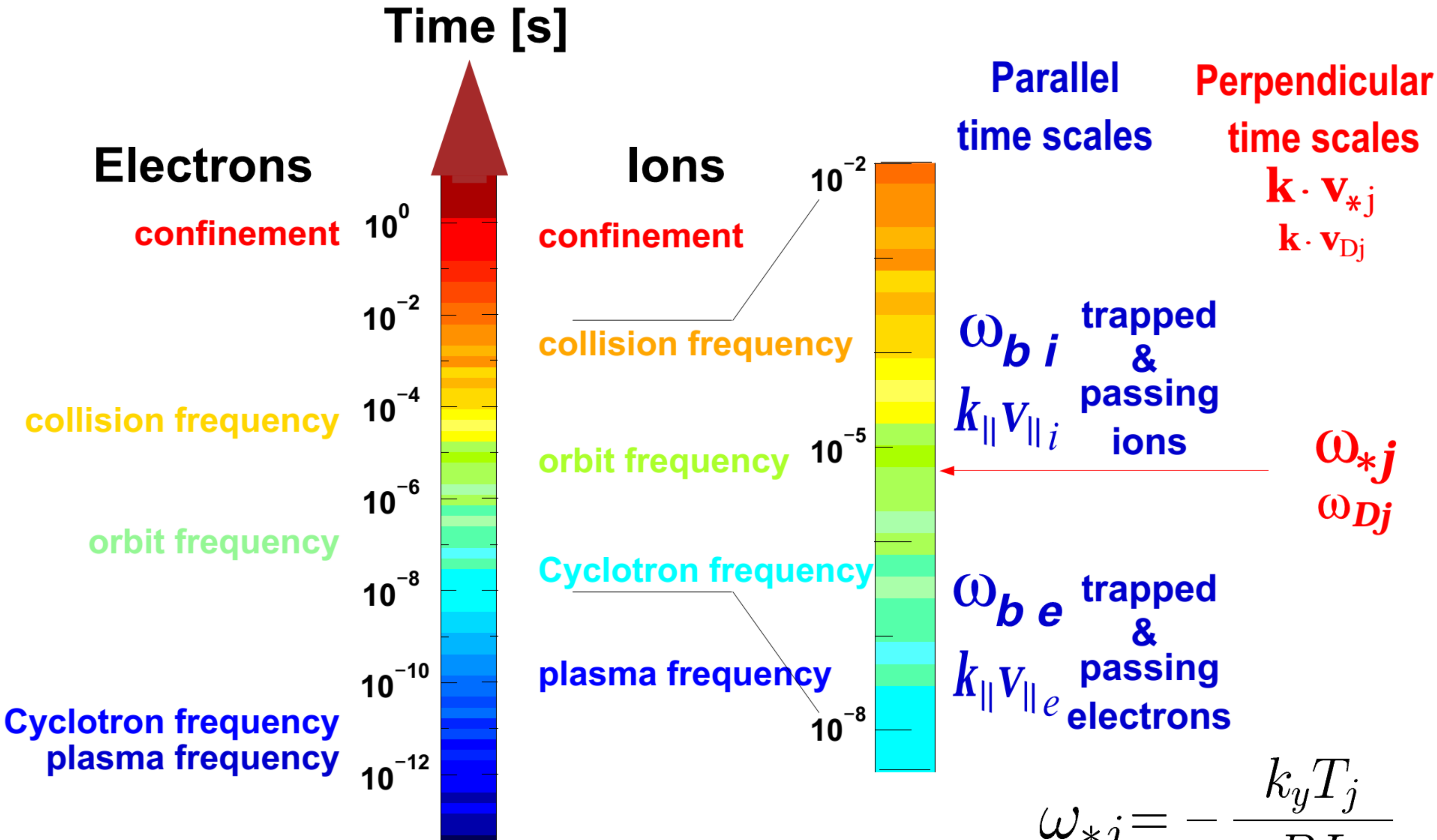
$$\mathbf{v}_C + \mathbf{v}_{\nabla B} = \frac{v_\parallel^2}{\Omega_c}\mathbf{B} \times \mathcal{K}_B + \frac{v_\perp^2}{2\Omega_c}\mathbf{B} \times \frac{\nabla B}{B}$$

$$\mathbf{k} \cdot \mathbf{v}_\mathrm{D} \quad \Rightarrow \quad \omega_D = -\frac{k_y T}{eBR}$$

Time scales relevant to transport



Time scales relevant to transport



$$\omega_{*j} = -\frac{k_y T_j}{q_j B L_{nj}}$$

Turbulent transport, basic principles

- Charged particles $\Rightarrow$ different behaviour along and across a magnetic field
- Transport in fusion plasmas is strongly anisotropic: very fast in the direction parallel to the magnetic field, slow in the direction perpendicular to the magnetic field
- Parallel transport is fast enough to limit the variation of density and pressure along the field lines (on the flux surfaces)
- How large is the perpendicular (radial) transport determines the confinement properties of the plasma.
- Collisional (neoclassical) transport is lower than measured values
  - By about one order of magnitude for the ion heat transport
  - By at least two order to magnitude for the electron heat transport
  - It remains significant for impurities (particularly heavy, highly charged impurities, W is of particular relevance for ITER)

Gyrokinetic equation

➤ **General kinetic equation**

$$\frac{\mathrm{d}F}{\mathrm{d}t} = \frac{\partial F}{\partial t} + \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t}\cdot\frac{\partial F}{\partial \mathbf{x}} + \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}\cdot\frac{\partial F}{\partial \mathbf{v}} = C(F)$$

➤ **Gyrokinetic equation : gyroaverage,** $f = F - F_M$

$$\frac{\partial f}{\partial t} + \mathbf{v}_{gc}\cdot\nabla f + \tilde{\mathbf{v}}_E\cdot\nabla f - \frac{\mu}{m}\nabla_{||}B\frac{\partial f}{\partial v_{||}} = C(f) - \tilde{\mathbf{v}}_E\cdot\nabla F_M - \frac{Ze}{T}F_M\,\mathbf{v}_{gc}\cdot\nabla\phi$$

➤ **Linear gyrokinetic equation in simplified geometry,** $\mathbf{B} = B_0\,R_0\,/\,R\;\mathbf{b}$

$$\hat{\omega}\,f + \frac{m\left(v_{||}^2 + v_\perp^2/2\right)}{ZT}f - \frac{v_{||}k_{||}}{\omega_D}f = \left[\frac{R}{L_n} + \left(\frac{mv_{||}^2 + mv_\perp^2}{2T} - \frac{3}{2}\right)\frac{R}{L_T} - \frac{m\left(v_{||}^2 + v_\perp^2/2\right)}{T} + Z\,\frac{v_{||}k_{||}}{\omega_D}\right]F\,\phi$$

Normalized logarithmic gradients, thermodynamic forces

➤ **Gyrokinetic equation : gyroaverage,** $f = F - F_M$

$$\frac{\partial f}{\partial t} + \mathbf{v}_{gc} \cdot \nabla f + \tilde{\mathbf{v}}_E \cdot \nabla f - \frac{\mu}{m} \nabla_{\|} B \frac{\partial f}{\partial v_{\|}} = C(f) - \boxed{\tilde{\mathbf{v}}_E \cdot \nabla F_M} - \frac{Ze}{T} F_M \mathbf{v}_{gc} \cdot \nabla \phi$$

➤ **Linear gyrokinetic equation in simplified geometry,** $\mathbf{B} = B_0 R_0 / R\, \mathbf{b}$

$$\hat{\omega} f + \frac{m\left(v_{\|}^2 + v_{\perp}^2/2\right)}{ZT} f - \frac{v_{\|} k_{\|}}{\omega_D} f = \left[\boxed{\frac{R}{L_n} + \left(\frac{m v_{\|}^2 + m v_{\perp}^2}{2T} - \frac{3}{2} \right) \frac{R}{L_T}} - \frac{m\left(v_{\|}^2 + v_{\perp}^2/2\right)}{T} + Z \frac{v_{\|} k_{\|}}{\omega_D} \right] F \phi$$

➤ **The (radial) gradient of the Maxwellian**

$$\nabla F_M = \frac{F_M}{R} \left[\frac{R}{L_n} + \left(\frac{E}{T} - \frac{3}{2} \right) \frac{R}{L_T} \right]$$

where $\dfrac{1}{L_X} = -\dfrac{\nabla X}{X}$ **and** $L_B = R$

Fluid model in simplified geometry

- **Continuity (zero moment)**

$$\hat{\omega}\tilde{n} \;+\; 2\,(\tilde{n}+\tilde{T}) \;+\; 4\,\hat{k}_{||}\,\tilde{u} \;=\; \left[\frac{R}{L_n} - 2\right]\hat{\phi}$$

- **Parallel velocity moment**

$$\hat{\omega}\tilde{u} \;+\; 4\tilde{u} \;+\; 2\hat{k}_{||}(\tilde{n}+\tilde{T}) \;=\; -2\,\hat{k}_{||}\,\hat{\phi}$$

- **Energy balance (second order moment)**

$$\hat{\omega}\tilde{T} \;+\; \frac{4}{3}\tilde{n} \;+\; \frac{14}{3}\tilde{T} \;+\; \frac{8}{3}\hat{k}_{||}\tilde{u} \;=\; \left[\frac{R}{L_T} - \frac{4}{3}\right]\hat{\phi}$$

- **Linear gyrokinetic equation in simplified geometry,** $\boldsymbol{B} = B_0\,R_0\,/\,R\;\mathbf{e}_y$

$$\hat{\omega}\,f + \frac{m\,(v_{||}^2 + v_{\perp}^2/2)}{ZT}f - \frac{v_{||}k_{||}}{\omega_D}f \;=\; \left[\frac{R}{L_n} + \left(\frac{mv_{||}^2 + mv_{\perp}^2}{2T} - \frac{3}{2}\right)\frac{R}{L_T} - \frac{m\,(v_{||}^2 + v_{\perp}^2/2)}{T} + Z\frac{v_{||}k_{||}}{\omega_D}\right]F\,\phi$$

Fluid model in simplified geometry

- **Continuity (zero moment)**

$$\boxed{\hat{\omega}\tilde{n}} + \boxed{2\left(\tilde{n}+\tilde{T}\right)} + \boxed{4\,\hat{k}_{\|}\,\tilde{u}} = \boxed{\left[\frac{R}{L_n}-2\right]\hat{\phi}}$$

- **Parallel velocity moment**

$$\boxed{\hat{\omega}\tilde{u}} + \boxed{4\tilde{u}} + \boxed{2\hat{k}_{\|}(\tilde{n}+\tilde{T})} = \boxed{-2\,\hat{k}_{\|}\,\hat{\phi}}$$

- **Energy balance (second order moment)**

$$\boxed{\hat{\omega}\tilde{T}} + \boxed{\frac{4}{3}\tilde{n} + \frac{14}{3}\tilde{T}} + \boxed{\frac{8}{3}\hat{k}_{\|}\tilde{u}} = \boxed{\left[\frac{R}{L_T}-\frac{4}{3}\right]\hat{\phi}}$$

- **Linear gyrokinetic equation in simplified geometry,** $\boldsymbol{B} = B_0\, R_0\, /\, R\; \mathbf{e}_y$

$$\boxed{\hat{\omega}\, f} + \boxed{\frac{m\left(v_{\|}^2+v_{\perp}^2/2\right)}{ZT}\, f} - \boxed{\frac{v_{\|}k_{\|}}{\omega_D}\, f} = \boxed{\left[\frac{R}{L_n}+\left(\frac{mv_{\|}^2+mv_{\perp}^2}{2T}-\frac{3}{2}\right)\frac{R}{L_T}-\frac{m\left(v_{\|}^2+v_{\perp}^2/2\right)}{T}+Z\frac{v_{\|}k_{\|}}{\omega_D}\right] F\,\phi}$$

Normalizations, parallel and perpendicolar time scales

- We are considering harmonic fluctuations $\propto \; exp(\, i\mathbf{k} \cdot \mathbf{x} - i\omega\, t\,)$
- Perpendicular motion characterisitc frequency is the drift frequency

$$\omega_D = -\frac{k_y T}{eBR} \quad \Rightarrow \quad \hat{\omega} = \omega \,/\, \omega_D$$

- Parallel motion : $k_{||} v_{\rm th} \; = \; k_{||}\sqrt{2T/m} \;\Rightarrow\; \hat{k}_{||} \; = \; k_{||} v_{\rm th} \,/ 4\, \omega_D$
- Ions and electrons:
  same mobility perpendicular to the field line,
  very different mobility along the field line
- Parallel velocity moment (parallel force balance)

$$\hat{\omega}\tilde{u} \; + \; 4\tilde{u} \; + \; 2\hat{k}_{||}(\tilde{n} + \tilde{T}) \; = \; -2\,\hat{k}_{||}\,\hat{\phi}$$

- Ions : inertia is dominant $\Rightarrow$ zero order, negligible parallel motion
- Electrons : parallel streaming is dominant $\Rightarrow$ adiabatic response

Parallel and perpendicular time scales, passing and trapped particles

- **Motion $\perp$ to B : Drift frequency** $\omega_D = -\dfrac{k_y T}{eBR}$

- **Motion $\parallel$ to B :**

  Passing particles $k_\parallel v_{\parallel i} << \omega \sim \omega_D << k_\parallel v_{\parallel e}$

  Trapped particles (Mirror force due to B $\propto$ 1/R)

$$\omega_{bi} \simeq \sqrt{\epsilon}\,\frac{v_{Ti}}{qR} << \omega_D \quad \& \quad \omega_{be} \simeq \sqrt{\epsilon}\,\frac{v_{Te}}{qR} >> \omega_D$$

- **Ions: bounce time longer than the characteristic time zero order $\Rightarrow$ no difference between passing and trapped**

  Electrons : many bounces in a characteristic time different behaviour between passing and trapped electrons

Passing electrons: adiabatic response

- Fast unconstrained motion parallel to B

- Parallel force balance for passing electrons

$$\hat{\omega}\tilde{u} \; + \; 4\tilde{u} \; + \; 2\hat{k}_{||}(\tilde{n} + \tilde{T}) \; = \; 2\,\hat{k}_{||}\,\hat{\phi}$$

- Balance of dominant terms $\quad \tilde{n} \; = \; \hat{\phi}$

or in non-normalized form

$$\frac{\delta n_{ep}}{n_{ep}} = \frac{e\,\phi}{T_e}$$

Passing and trapped ions: continuity and energy balance

- Neglect parallel motion (strong inertia) $\tilde{u} = 0$
- Continuity and energy balance

$$\hat{\omega}\tilde{n} \;+\; 2\left(\tilde{n}+\tilde{T}\right) \;=\; \left[\frac{R}{L_n} - 2\right]\hat{\phi}$$

$$\hat{\omega}\tilde{T} \;+\; \frac{4}{3}\tilde{n} \;+\; \frac{14}{3}\tilde{T} \;=\; \left[\frac{R}{L_T} - \frac{4}{3}\right]\hat{\phi}$$

- Quasineutrality condition $\tilde{n}_e = \tilde{n}_i$
- Assuming (for the moment) that all electrons are passing (more precisely adiabatic), quasi-neutrality becomes $\tilde{n} = \hat{\phi}$
  we obtain a homogeneous system of three equations in three unknowns

The mechanism leading to an instability

➤ **We look for the eigenvalues of this homogeneous system**

$$\hat{\omega}\tilde{n} \;+\; 2\,(\tilde{n}+\tilde{T}) \;=\; \left[\frac{R}{L_n} - 2\right]\hat{\phi}$$

$$\hat{\omega}\tilde{T} \;+\; \frac{4}{3}\tilde{n} \;+\; \frac{14}{3}\tilde{T} \;=\; \left[\frac{R}{L_T} - \frac{4}{3}\right]\hat{\phi}$$

$$\tilde{n} \;=\; \hat{\phi}$$

➤ **Can be solved analytically (trivial) here we focus on the basic coupling mechanism leading to an instability**

The mechanism leading to an instability

➤ **We look for the eigenvalues of this homogeneous system**

$$\boxed{\hat{\omega}\tilde{n}} + 2\,(\tilde{n} + \boxed{\tilde{T}}) = \left[\frac{R}{L_n} - 2\right]\hat{\phi}$$

$$\boxed{\hat{\omega}\tilde{T}} + \frac{4}{3}\tilde{n} + \frac{14}{3}\tilde{T} = \left[\boxed{\frac{R}{L_T}} - \frac{4}{3}\right]\hat{\phi}$$

$$\boxed{\tilde{n} = \hat{\phi}}$$

➤ $\hat{\omega}^2\,\hat{\phi} = -2\,\dfrac{R}{L_T}\,\hat{\phi}$ $\Rightarrow$ **imaginary roots**

$\Rightarrow$ **pure growing mode** $\hat{\gamma} = \sqrt{2\,\dfrac{R}{L_T}}$

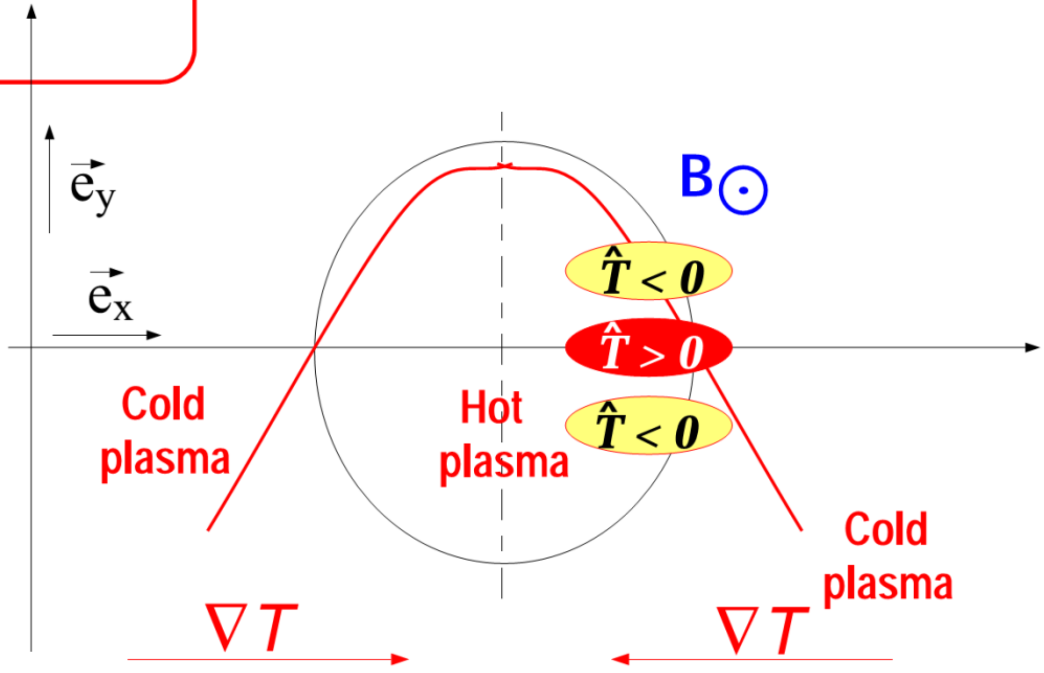
The Ion Temperature Gradient (ITG) mode

$$\hat{\omega}\tilde{n} + 2\tilde{T} = 0$$

$$\hat{\omega}\tilde{T} + \frac{R}{L_T}\hat{\phi} = 0$$

$$\tilde{n} = \hat{\phi}$$

Initial Temperature perturbation



Turbulent transport, basic principles

curvature and ∇B drift

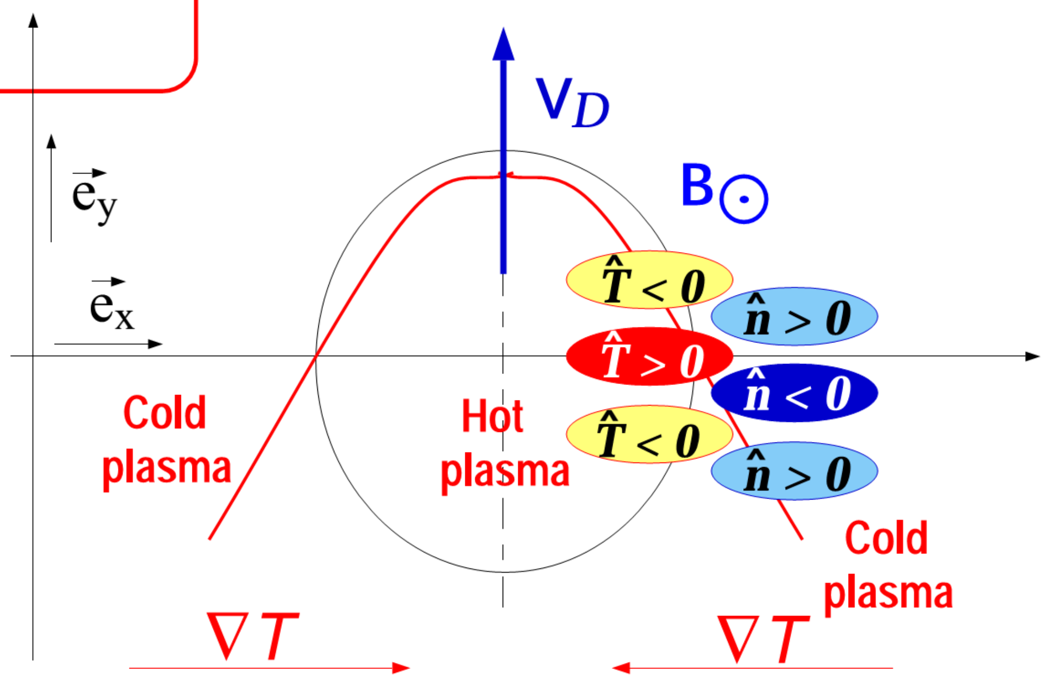
$$\hat{\omega}\tilde{n} + 2\tilde{T} = 0$$

$$\hat{\omega}\tilde{T} + \frac{R}{L_T}\hat{\phi} = 0$$

$$\tilde{n} = \hat{\phi}$$

Initial Temperature perturbation

Generates a density perturbation



The Ion Temperature Gradient (ITG) mode

curvature and ∇B drift

$$\hat{\omega}\tilde{n} + 2\tilde{T} = 0$$

ExB flow advection

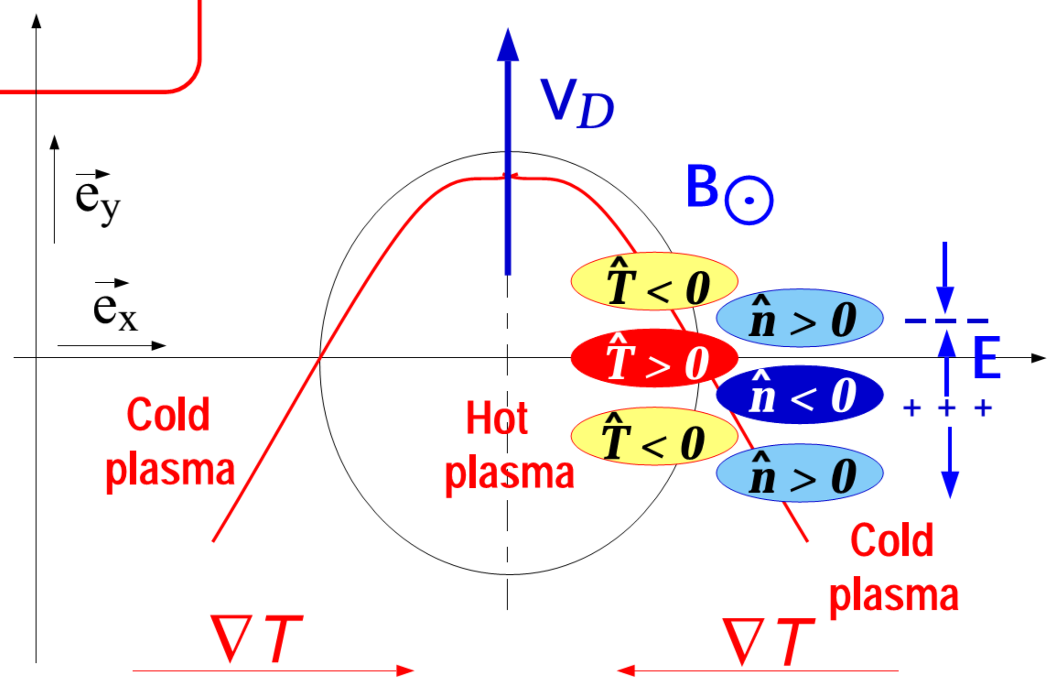
$$\hat{\omega}\tilde{T} + \frac{R}{L_T}\hat{\phi} = 0$$

$$\tilde{n} = \hat{\phi}$$

Initial Temperature perturbation

Generates a density perturbation

Passing electrons neutralise the charge separation



The Ion Temperature Gradient (ITG) mode

curvature and ∇B drift

$$\hat{\omega}\tilde{n} + 2\tilde{T} = 0$$

ExB flow advection

$$\hat{\omega}\tilde{T} + \frac{R}{L_T}\hat{\phi} = 0$$

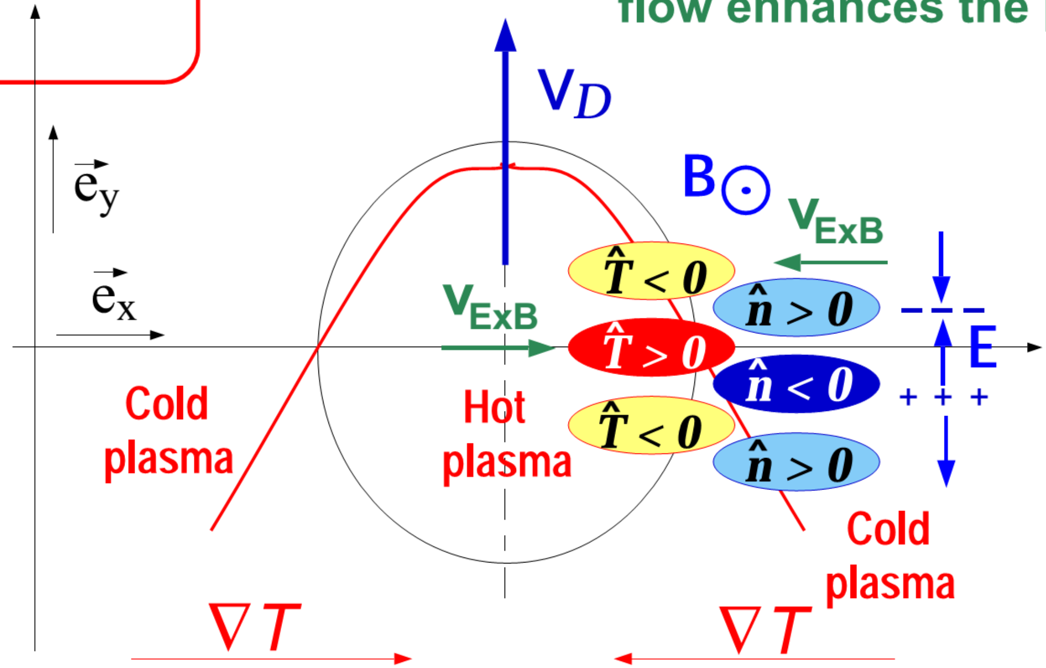
$$\tilde{n} = \hat{\phi}$$

Initial Temperature perturbation

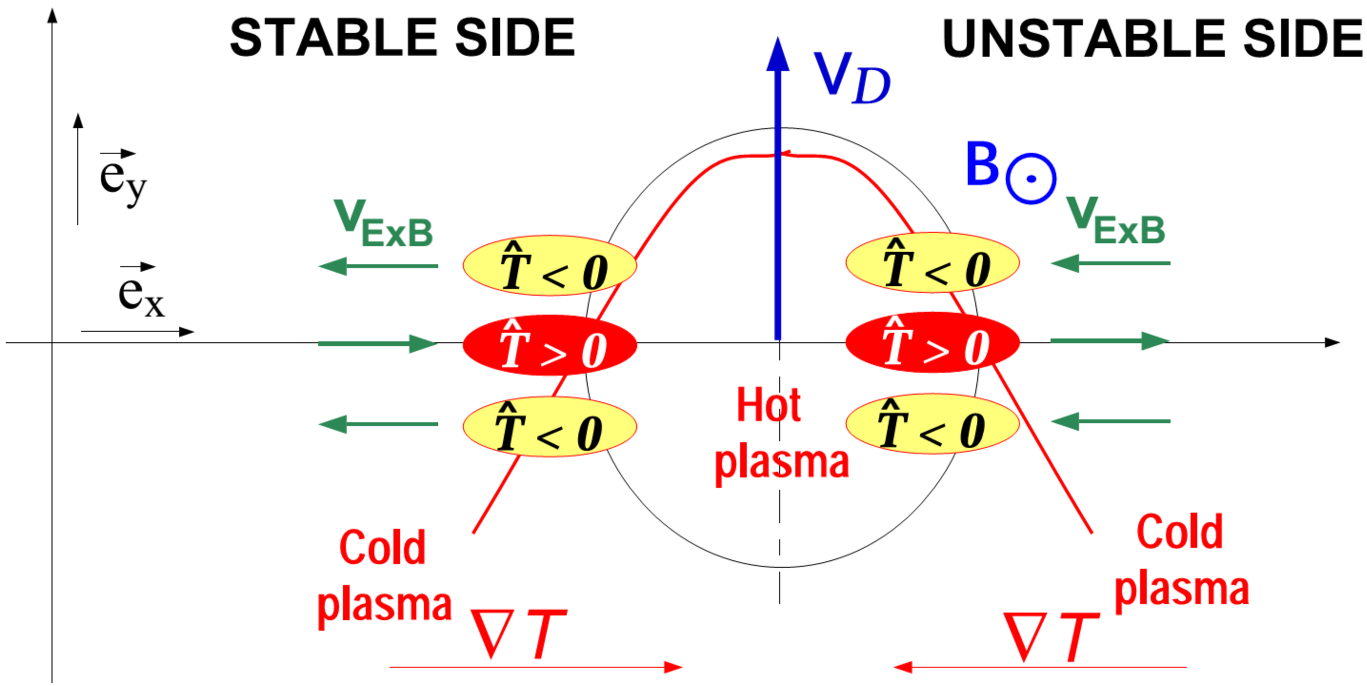
Generates a density perturbation

Passing electrons neutralise the charge separation

Parallel force balance implies an electrostatic potential, the ExB flow enhances the perturbation



ITG mode, the "bad curvature" region



- ➤ **Low field side ⇒ unstable (bad curvature) region :**
  warm plasma moves in the warm regions
- ➤ **High field side ⇒ stable region (reversed temperature gradient)**
  warm plasma moves in the cold regions

ITG mode, what did we neglect so far ?

➤ **Back to our simple fluid model**

$$\boxed{\hat{\omega}\tilde{n}} + 2\left(\tilde{n} + \boxed{\tilde{T}}\right) = \left[\frac{R}{L_n} - 2\right]\hat{\phi}$$

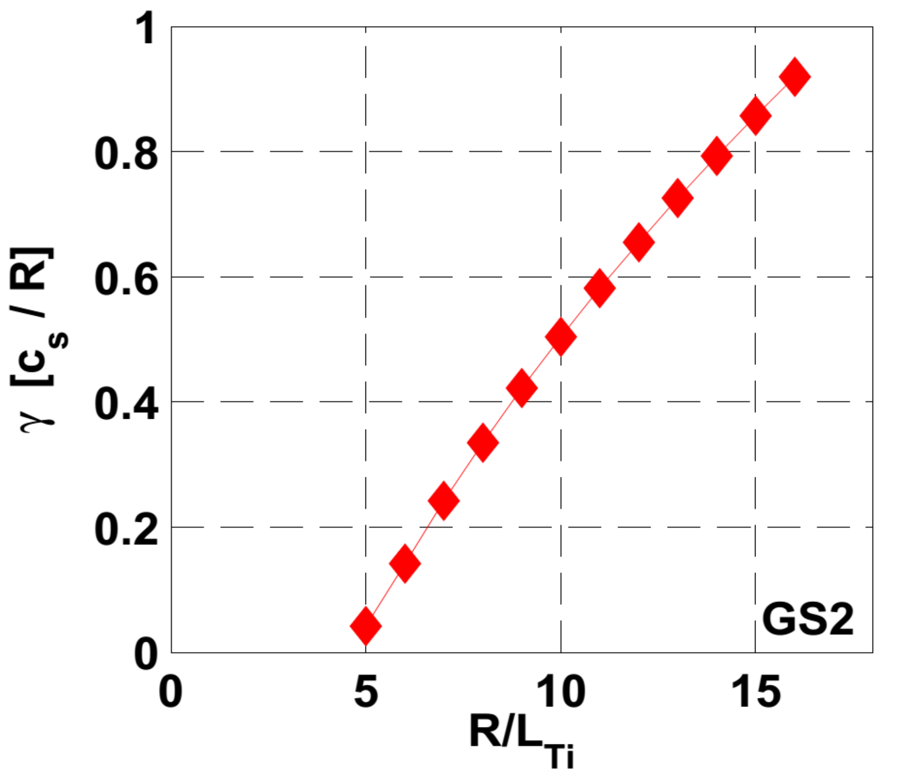
$$\boxed{\hat{\omega}\tilde{T}} + \frac{4}{3}\tilde{n} + \frac{14}{3}\tilde{T} = \left[\boxed{\frac{R}{L_T}} - \frac{4}{3}\right]\hat{\phi}$$

$$\boxed{\tilde{n} = \hat{\phi}}$$

➤ **By keeping all the terms, we would have found that the eigenvalues are not just imaginary, but complex numbers in which an imaginary part (an unstable mode) occurs provided that the normalized logarithmic temperature gradient R/LT exceeds a certain value (threshold) (otherwise only real roots)**

ITG mode : the threshold

- The mode is stable for values of R/L_T smaller than the threshold value (R/L_T critical)



- The threshold value is not a universal number, but depends itself on plasma parameters

- In particular it increases with increasing T_i / T_e and for adiabatic electrons increases with increasing R/L_n (η_i mode , $\eta_i = L_n / L_T$)

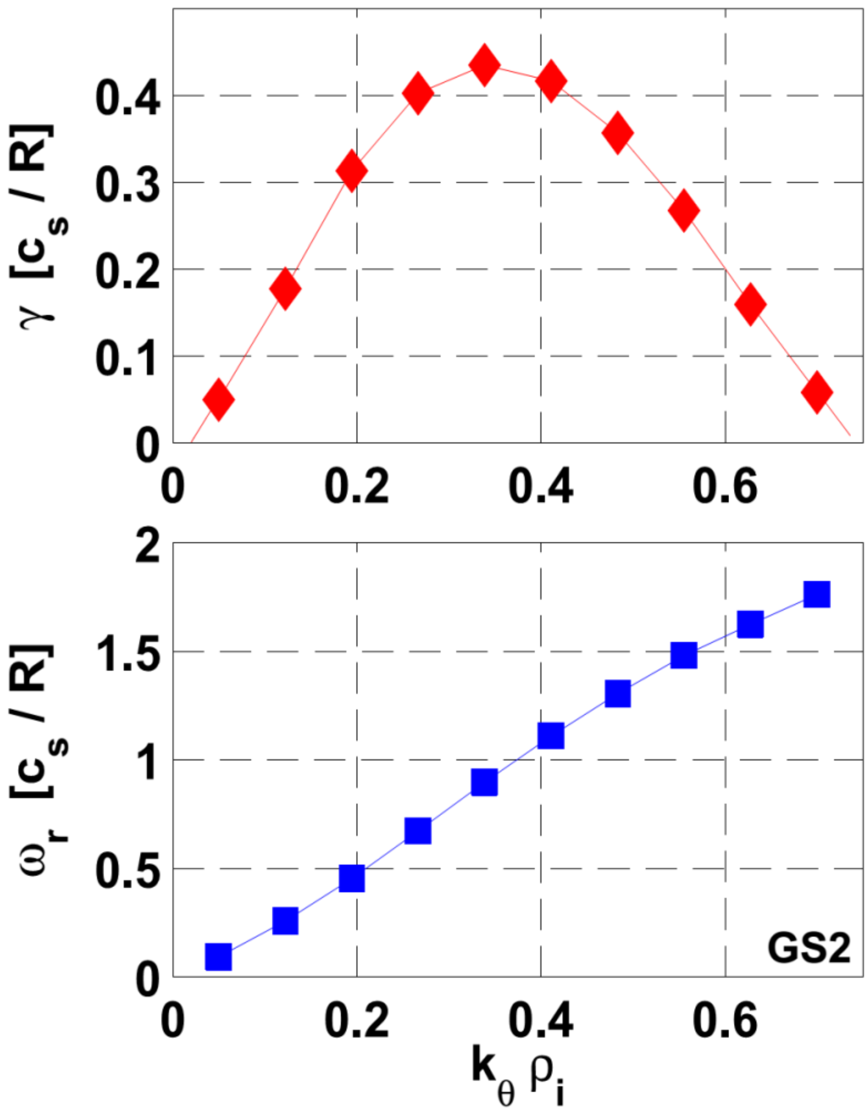
ITG mode : the spectrum

➤ **Simple (reduced) fluid model**

$$\gamma = \omega_D \sqrt{2\frac{R}{L_T}}$$

$$\gamma = k_\theta \rho_s \frac{c_s}{R} \sqrt{2\frac{R}{L_T}}$$

γ is an increasing linear function of the wave number



ITG mode : the spectrum

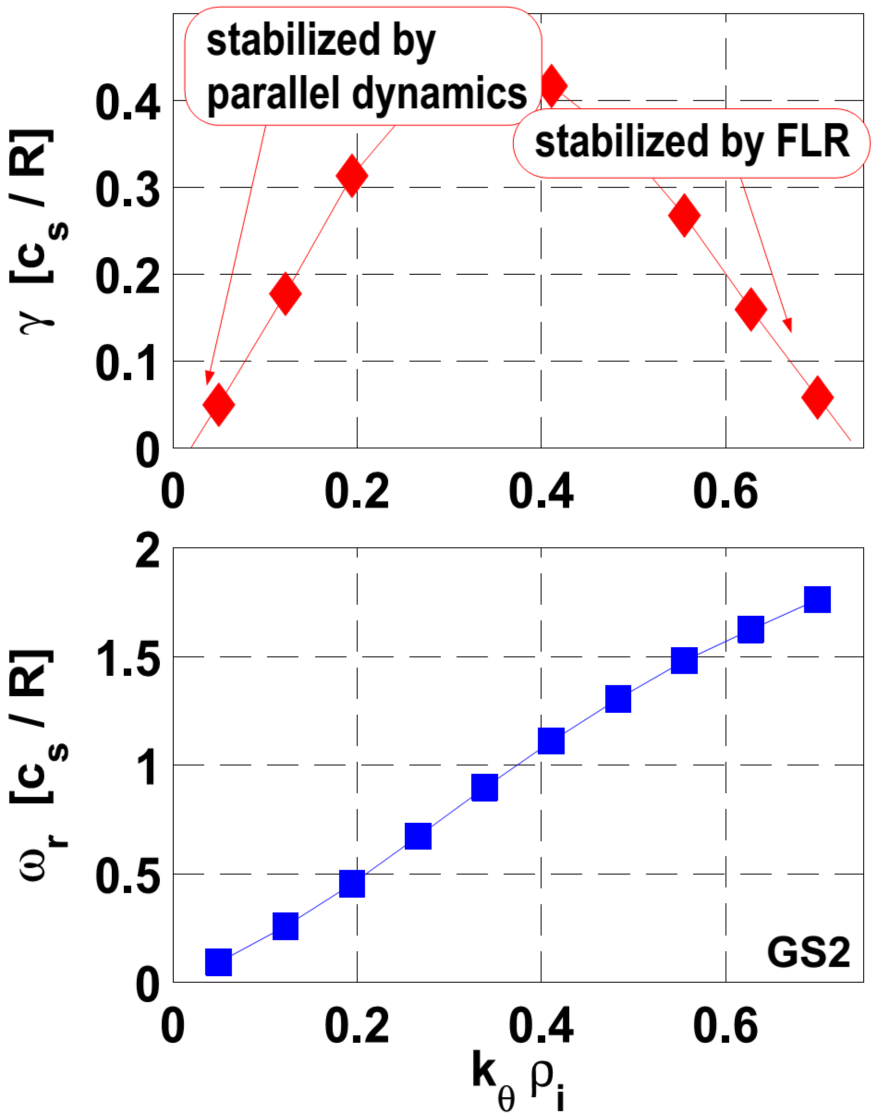
➤ **Simple (reduced) fluid model**

$$\gamma = \omega_D \sqrt{2 \frac{R}{L_T}}$$

$$\gamma = k_\theta \, \rho_s \frac{c_s}{R} \sqrt{2 \frac{R}{L_T}}$$

γ is an increasing linear function of the wave number

➤ **Parallel dynamics and finite Larmor radius effects, which were neglected in the simple fluid model, modify significantly the spectrum**



Kinetic electrons

- So far we have explored the (electrostatic) ITG mode with adiabatic electrons
- The inclusion of the electron dynamics leads to several other modes, in particular the trapped electron mode (TEM) and the electron temperature gradient (ETG) mode
- These are mainly electrostatic modes. Simple pictures can be built for these modes, analogous to that we have presented for the ITG, but this time assuming an adiabatic response of the ions
- In addition, at sufficiently high (electron) plasma pressure, fluctuating currents produce also fluctuations of the magnetic field (and magnetic potential), opening the possibility to instabilities of electromagnetic type, in particular the kinetic ballooning modes (KBM) and the micro-tearing modes (MTM)
- All of these modes can occur concurrently, at different scales, but also at the same scales (e.g. ITG and TEM or ITG, KBM and MTM), and have different parameters dependencies

TEM and ETG

- These are instabilities connected with the electron dynamics, and are **driven by an electron temperature gradient above a critical threshold**

- Next lecture we shall see that **presence of trapped electrons also implies an instability driven by the density gradient**

- The TEM instability can be considered as the analogous of the ITG instability, still at the ion Larmor radius scale, but where the slow (average) motion along the field line of electrons is caused by trapping, as it as caused by inertia for the ions

- The ETG mode is the analogous of the ITG mode at the electron Larmor radius scale (practically where the role of ions and electrons are swapped)

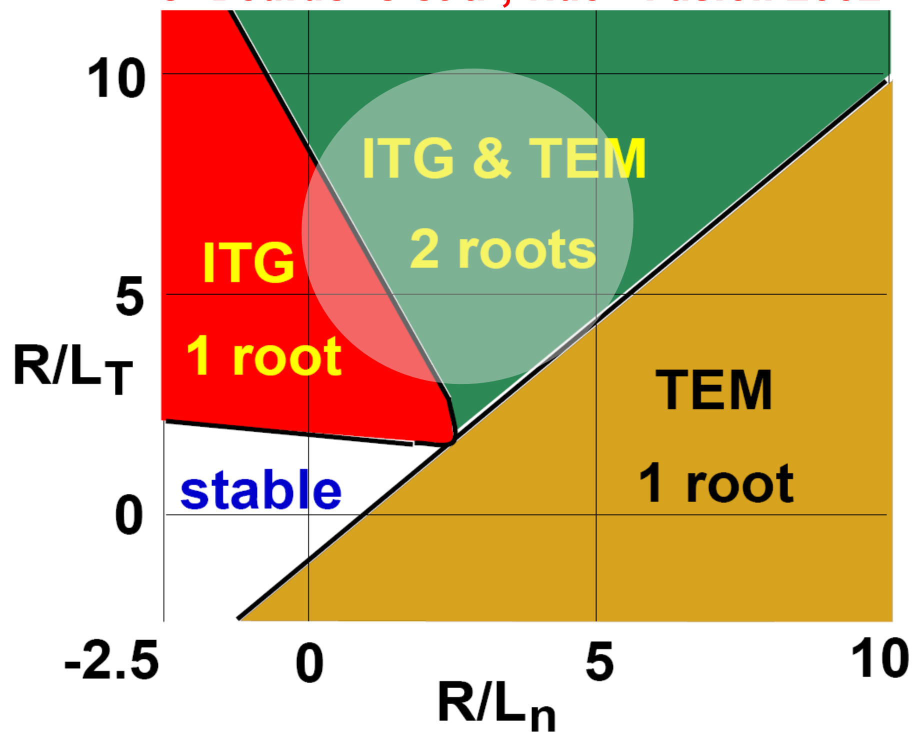
Summary so far:
Example of ITG and TEM instability diagram

- Curvature and grad B drifts depend on particle energy, and electric charge
- This implies coupling between density and temperature fluctuations $\Rightarrow$ leads to instabilities (and turbulence)
- Trapped electrons $\Rightarrow$ slow motion in parallel direction, can lead to instability like ions (slow motion by inertia)
- Presence of trapped electrons implies an additional density gradient driven instability (TEM type, next lecture)
- Instabilities occur when

$$\frac{R}{L_T} = -\frac{R}{T}\frac{dT}{dr} > \frac{R}{L_{T\,crit}}$$

$$\frac{R}{L_n} = -\frac{R}{n}\frac{dn}{dr} > \frac{R}{L_{n\,crit}}$$

C. Bourdelle et al, Nucl. Fusion 2002



Finite plasma pressure, electromagnetic modes

- The consideration of finite plasma pressure introduces magnetic field fluctuations in addition to finite electrostatic potential (electric field) fluctuations

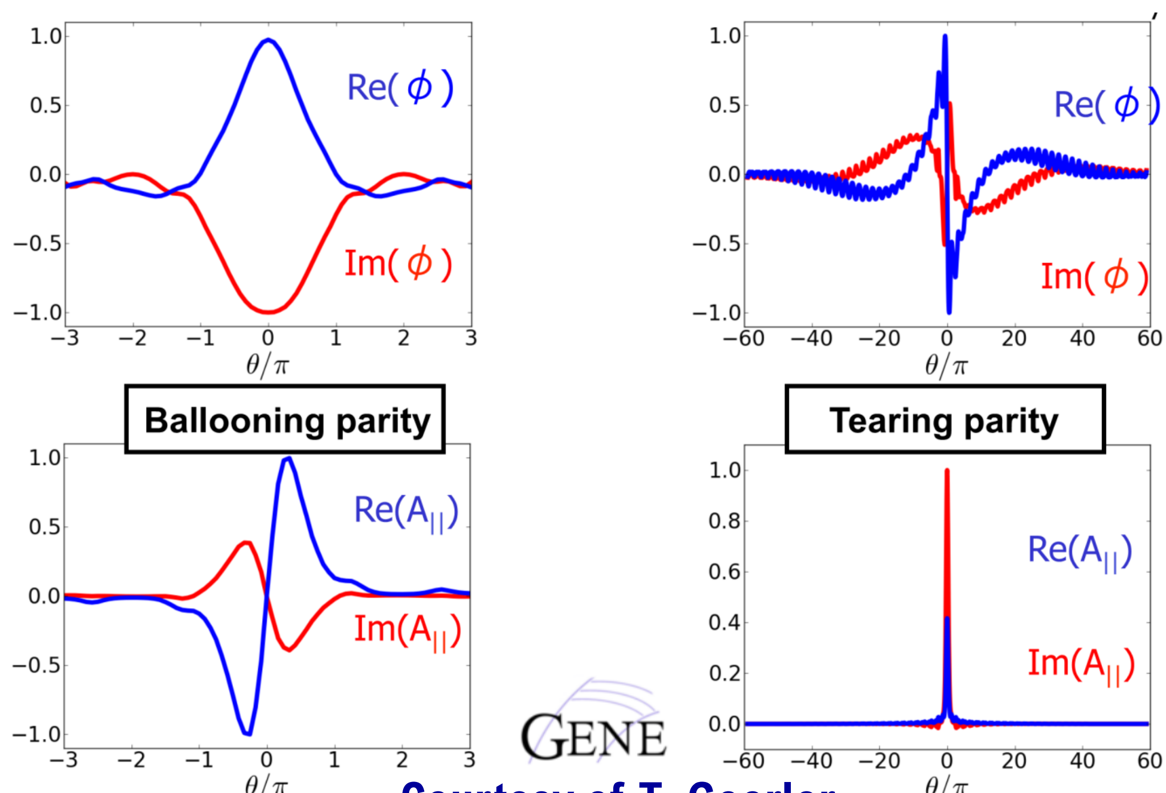
- The system of equations is closed not only by the Poisson (quasi-neutrality) equation but also by Ampère's law

- New instabilities can develop

- Micro-tearing modes have tearing parity

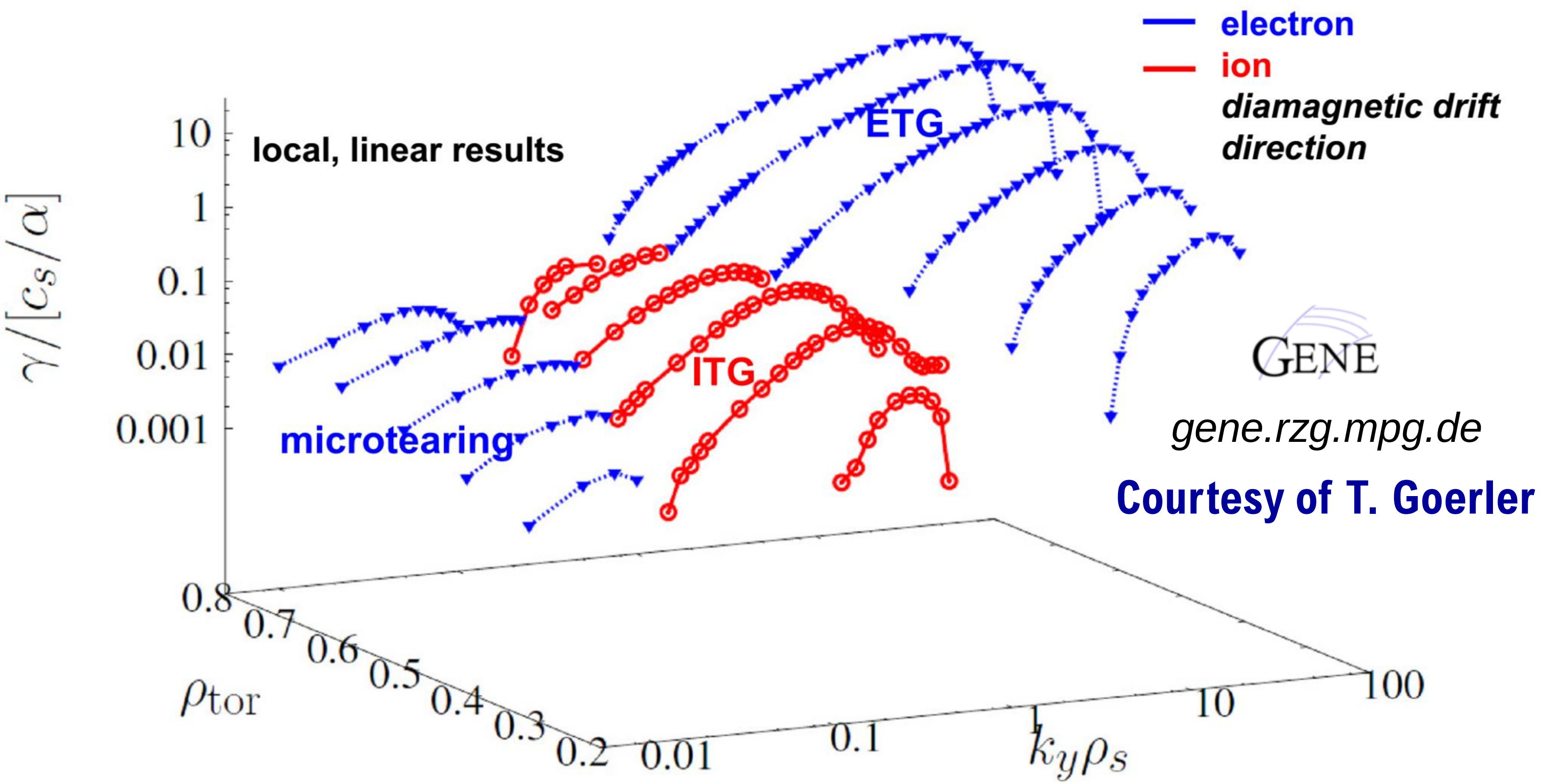
- Kinetic effects also affect the stability of (MHD) ballooning modes: KBM

- Finite beta also affects electrostatic type of instabilities (in particular the ITG, see later)



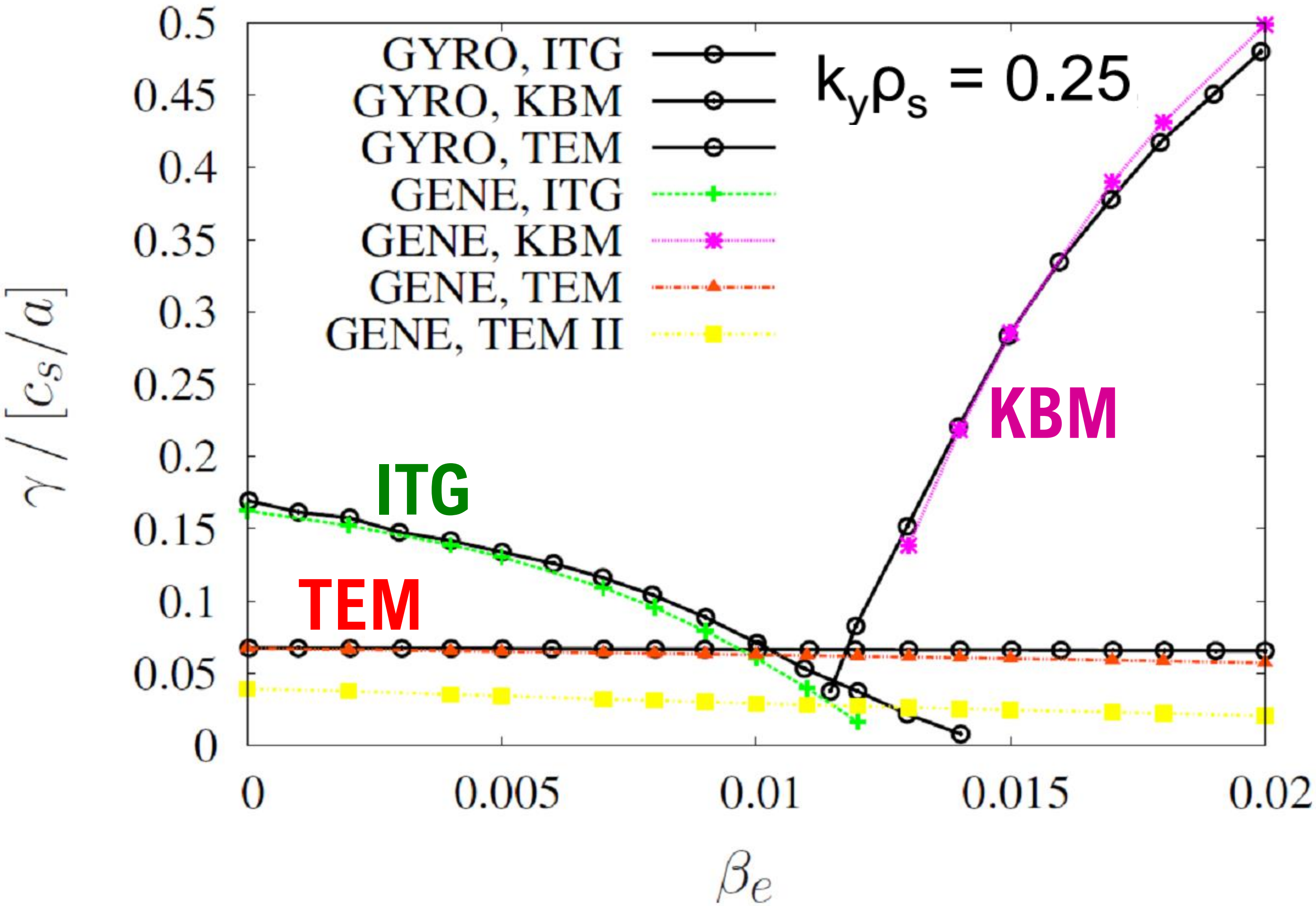
Courtesy of T. Goerler

Modes can coexist at the same radial location



- **Multi-scale problem: Example of binormal spectra at different radial locations for an AUG plasma in L-mode confinement**

Modes can even coexist at the same scale



GENE

gene.rzg.mpg.de

Courtesy of T. Goerler

- **ITG, TEM and KBM can co-exist at the same scale, as combination of dominant and sub-dominant modes (which can impact the turbulent transport)**

Summary of instabilities

$k_\theta \rho_i$: 0.1 — 1 — 10

- **instability** — **ITG** | **TEM** | **ETG**
- **drive** — R/L_{Ti} | R/L_{Te} & R/L_n | R/L_{Te}

pure ITG stabilised by R/L_n

stabilised by e-i collisions

stabilised by R/L_n

~ unaffected by collisions

destabilised by T_e / T_i

stabilised by T_e / T_i

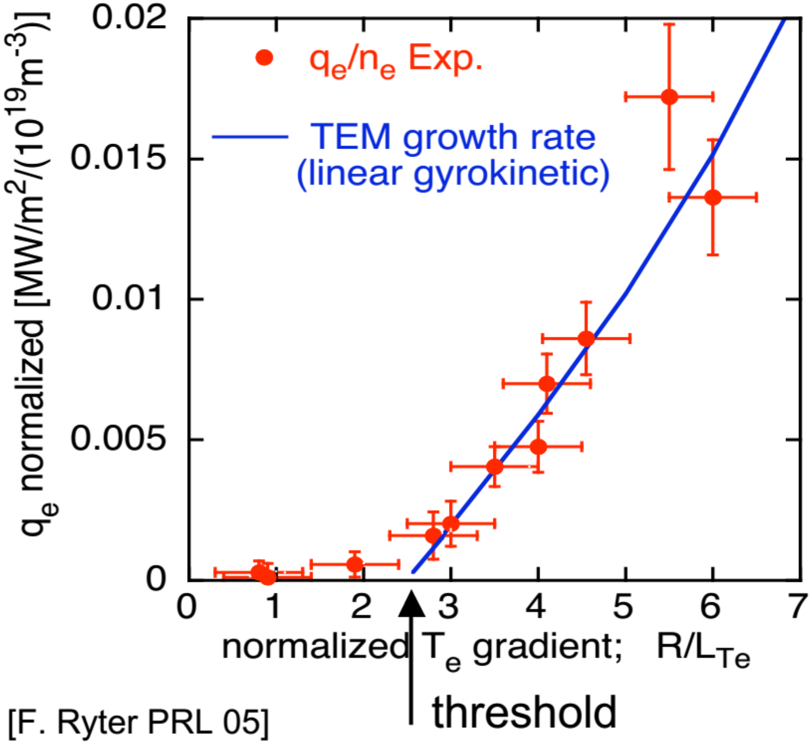
$k_\theta \rho_i$: 0.1 — 1 — 10

- **e.m. instability** — **MTM , KBM**
- **drive** — R/L_{Te}, β_e | $q^2\, \beta_e$, β' (R/L_T, R/L_n)

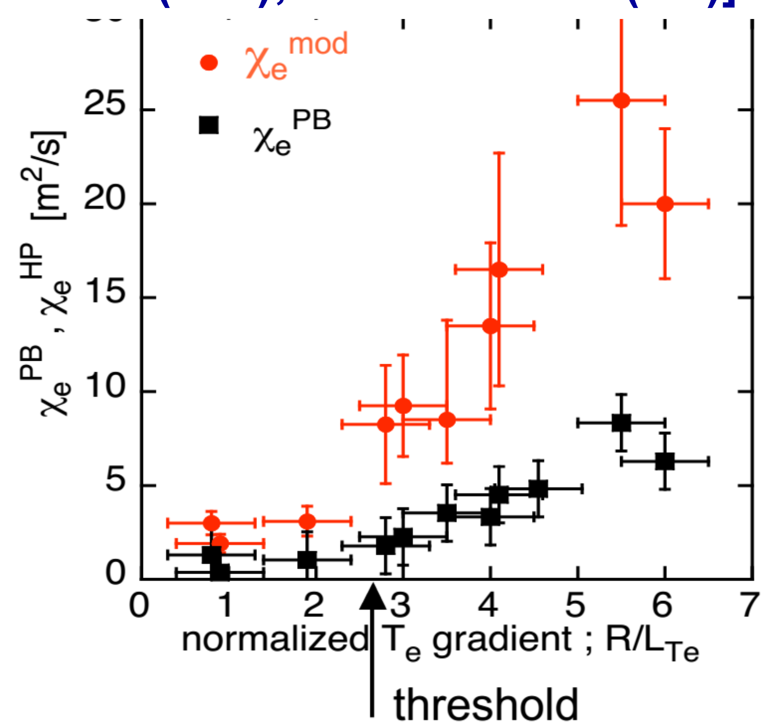
Experimental evidence: existence of threshold

- Existence of threshold can be experimentally investigated by combined stationary and transient (modulation of heating power) experiments
- Experimental evidences have been obtained, which have been identified with thresholds predicted for TEM and ITG turbulence

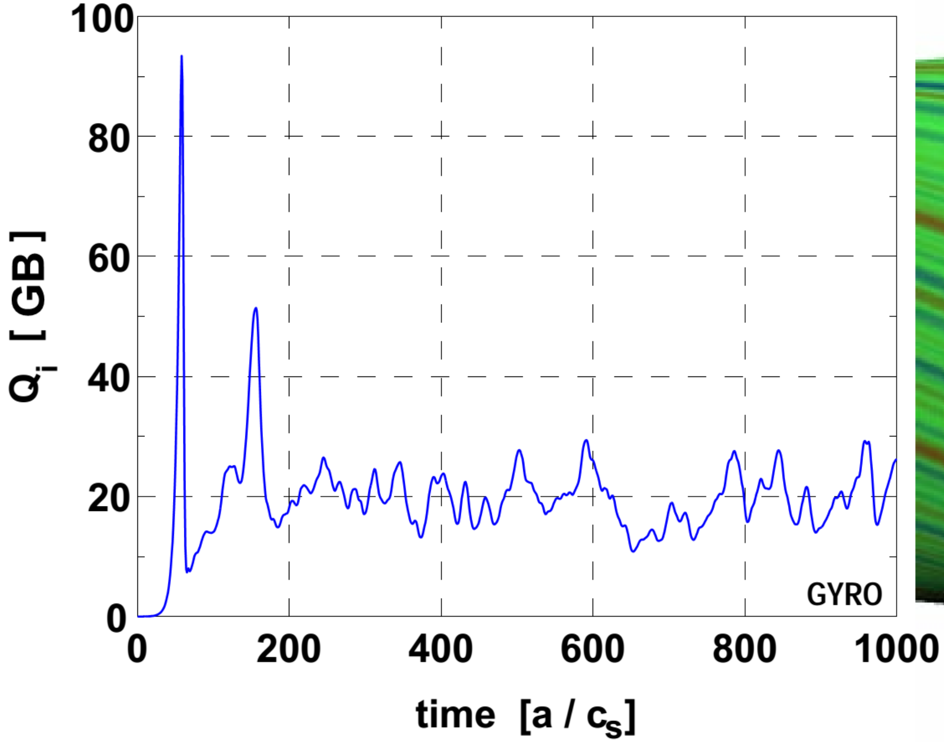
[e.g. Ryter PRL 05 (TEM), Mantica PRL 09 (ITG)]

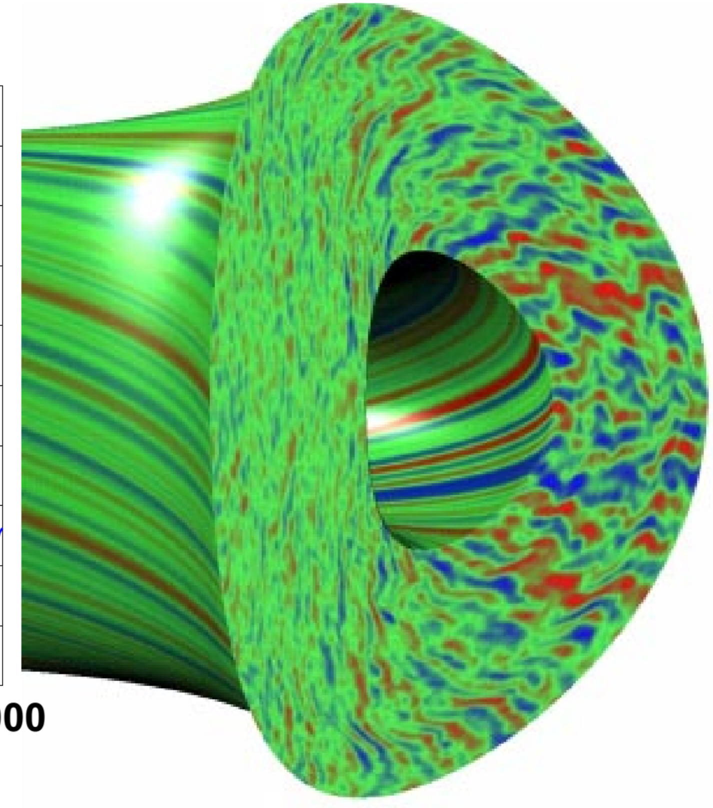


[F. Ryter PRL 05]



From instabilities to turbulence





- Time evolution of the ion heat flux in a gradient driven nonlinear gyrokinetic simulation (code GYRO [Candy and Waltz, General Atomics])

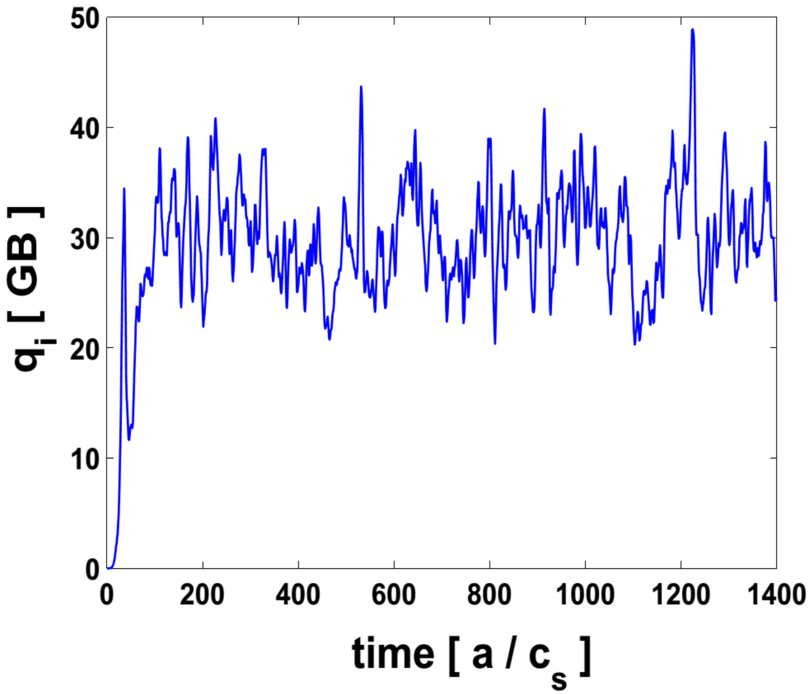
From instabilities to turbulence

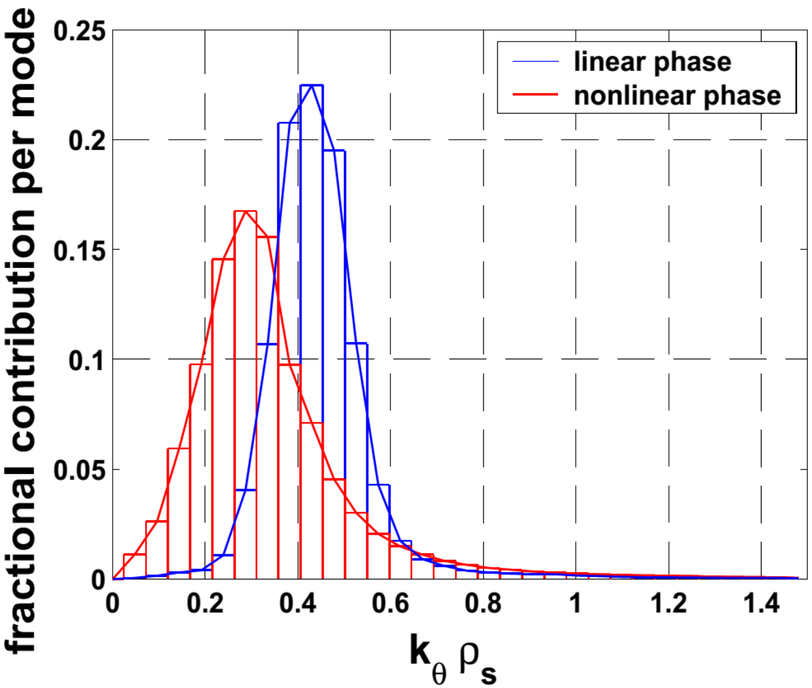
$$\frac{\partial f}{\partial t} + \mathbf{v}_{gc} \cdot \nabla f + \boxed{\tilde{\mathbf{v}}_E \cdot \nabla f} - \frac{\mu}{m} \nabla_{\parallel} B \frac{\partial f}{\partial v_{\parallel}} = C(f) - \tilde{\mathbf{v}}_E \cdot \nabla F_M - \frac{Ze}{T} F_M \, \mathbf{v}_{gc} \cdot \nabla \phi$$

- **So far we considered a linear model, and neglected this term, which describes the effect of the fluctuating potential on the perturbed distribution function**
- **This term introduces a non-linearity, since the fluctuation potential depends on the perturbed distribution function through the Poisson equation**
- **In addition, we introduce toroidal mode coupling, since all toroidal modes are involved in this term (these are de-coupled in the linearized equation)**
- **This equation describes the development of a turbulent state in the plasma. Numerical codes (solving nonlinear gyro-fluid and gyro-kinetic systems) have been developed to compute this state and the consequent transport**

Linear and nonlinear spectra

- Nonlinear transfer through mode coupling implies that the nonlinear spectrum is different from the linear one
- Scales which provide the largest contribution to transport are not those which correspond to the most unstable linear modes

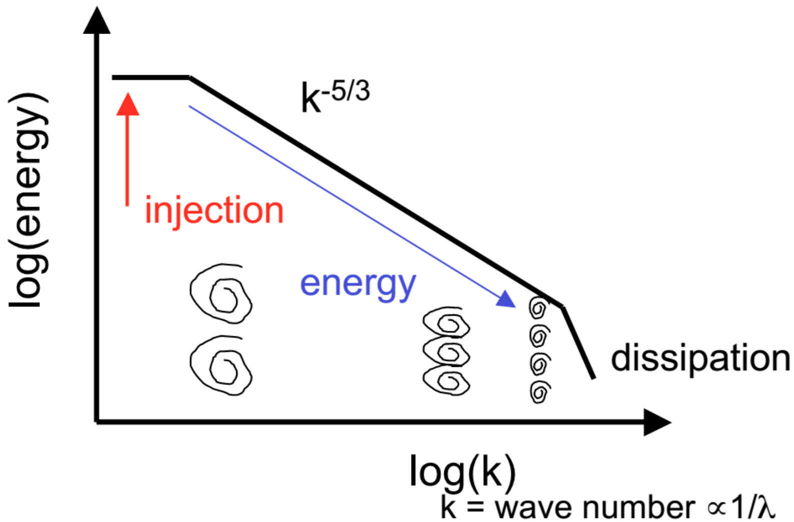




General properties of turbulence spectra

Fluids in general 3D

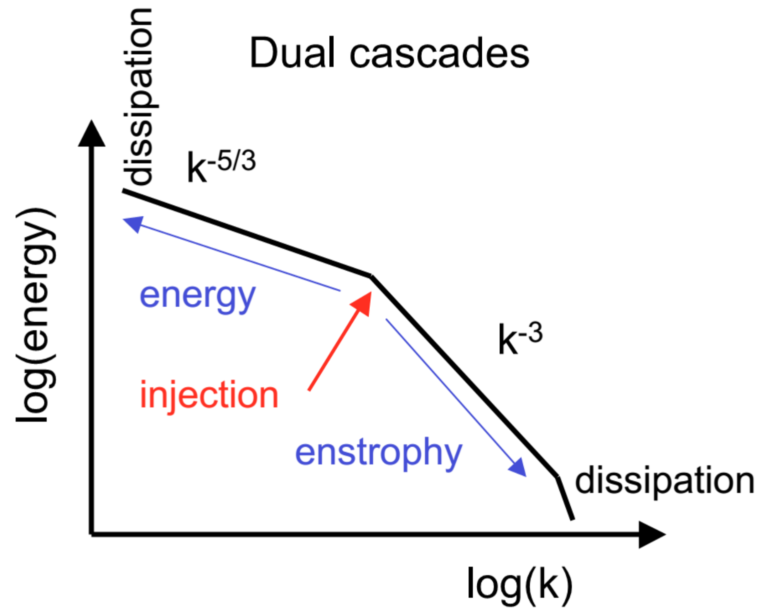
Kolmogorov spectrum



- injection low k values
- energy transfer to smaller structures
- dissipation in small structures by viscosity

Plasmas 2D

Dual cascades

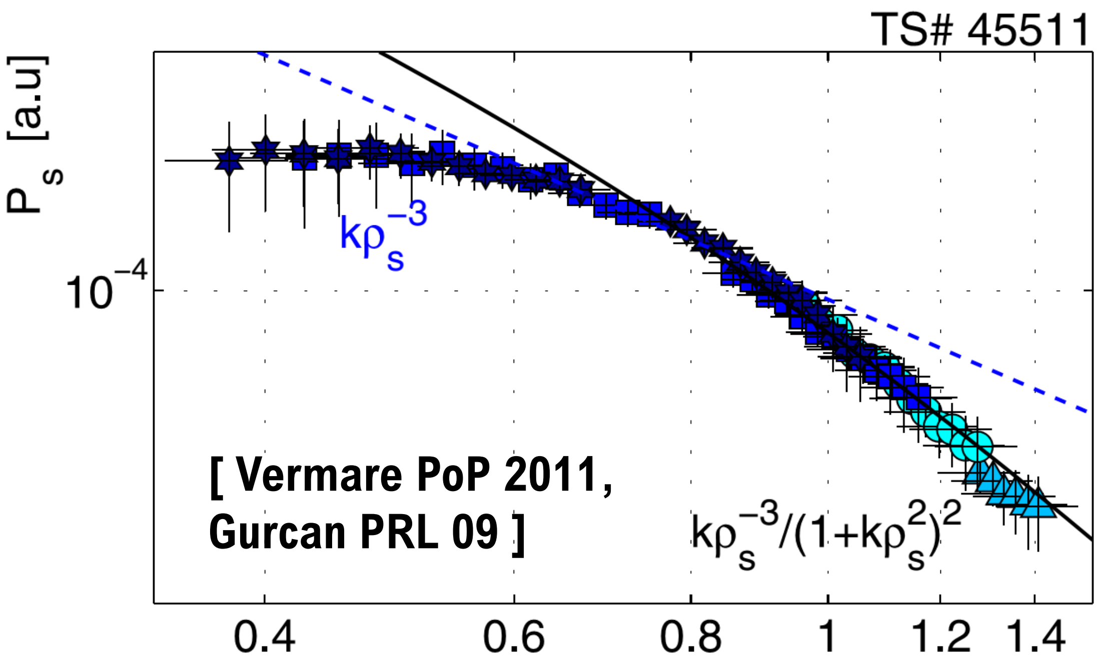


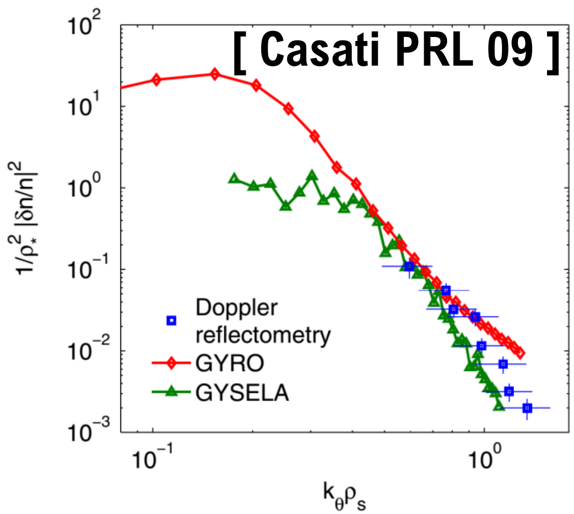
- injection at different k values and with broad spectra (ITG, TEM, ETG)
- main energy transfer through larger structures
- main dissipation in very large structures and to the device

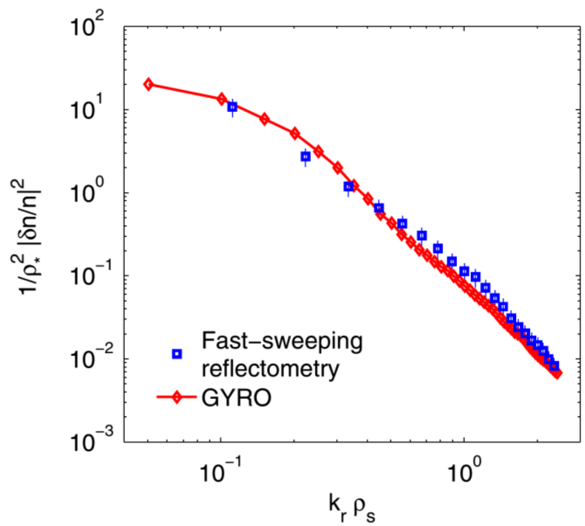
[after U. Stroth, Transport in Toroidal Plasmas, Lect. Notes Phys. 670 (2005) Springer-Verlag]

Measured spectra exhibit features which are theoretically expected

- **Present efforts in understanding plasma turbulence and in validation of theoretical predictions also include comparisons with measurements of the turbulence characteristics (wave number and frequency spectra, amplitudes and cross-phases between fluctuating quantities)**

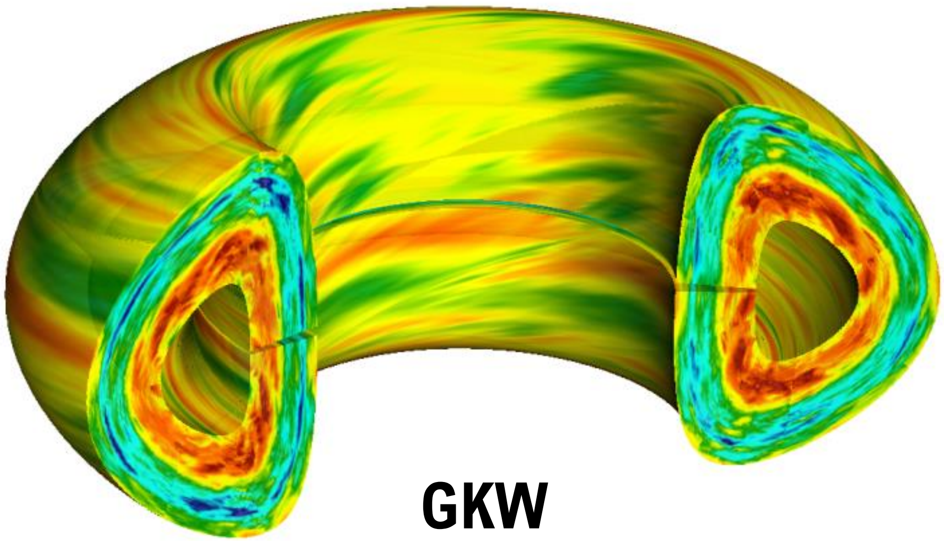






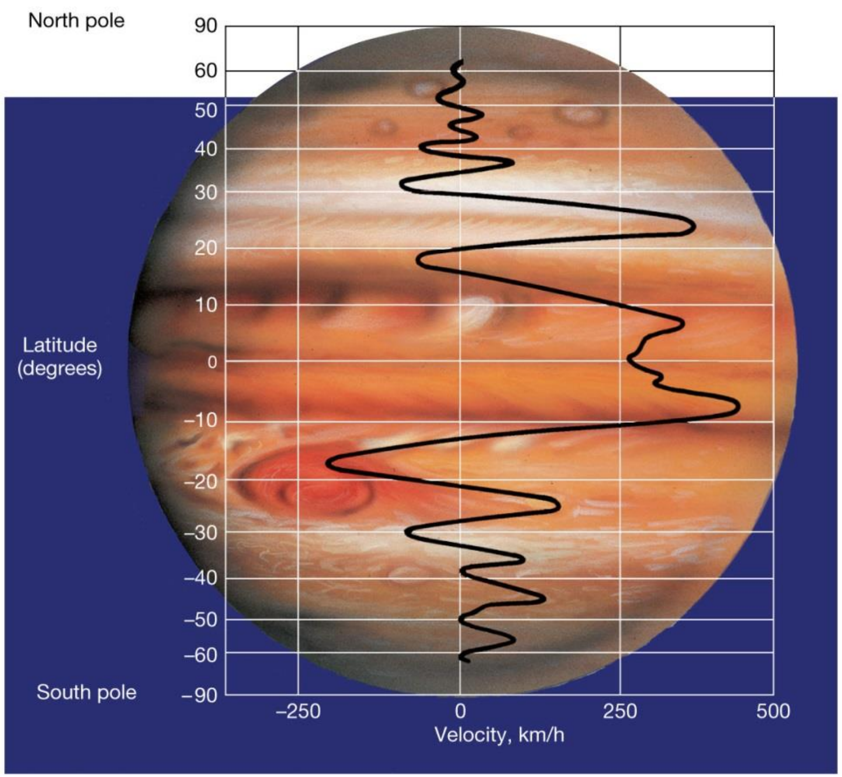
Turbulence saturation and zonal flows

- **Zonal flows (stationary, poloidally and toroidally homogeneous (n=0,m= 0), radially varying ExB flows) are produced by turbulence (self-organization) and act as saturation and self-regulation mechanism of the turbulence**



GKW

https://code.google.com/p/gkw/



Diamond et al, "Zonal flows in plasma-a review", *Plasma Phys. Control. Fusion* 47, R35 (2005)

Turbulence saturation and zonal flows

Code: **GYRO**

Authors: Jeff Candy and Ron Waltz

https://fusion.gat.com/theory/Gyromovies

The local scaling of turbulent transport

- The natural scaling of turbulent transport can be obtained by considering a displacement of one Larmor radius in the time of one inverse growth rate

$$\rho_s = \frac{c_s}{\Omega_c} \quad \text{and} \quad \gamma \sim \frac{c_s}{R} \quad \Rightarrow D \propto \rho_s^2 \, \gamma \quad \Rightarrow D \propto \frac{c_s^3}{\Omega_c R}$$

- It is called gyroBohm, and it is the natural scaling of local transport, derived from the dimensionless form of the equations

- The name gyroBohm shows that it is given by Bohm diffusion reduced by one normalized gyroradius factor

- Bohm diffusion is given by a displacement of one Larmor radius in a time of the order of one gyroperiod

$$D_B = \rho_s^2 \, \Omega_c = \frac{T}{e\,B} \Rightarrow D_{GB} = D_B \frac{\rho_s}{a} = \frac{m^{1/2}\, T^{3/2}}{e^2 \, B^2} \frac{1}{a}$$

Profile stiffness

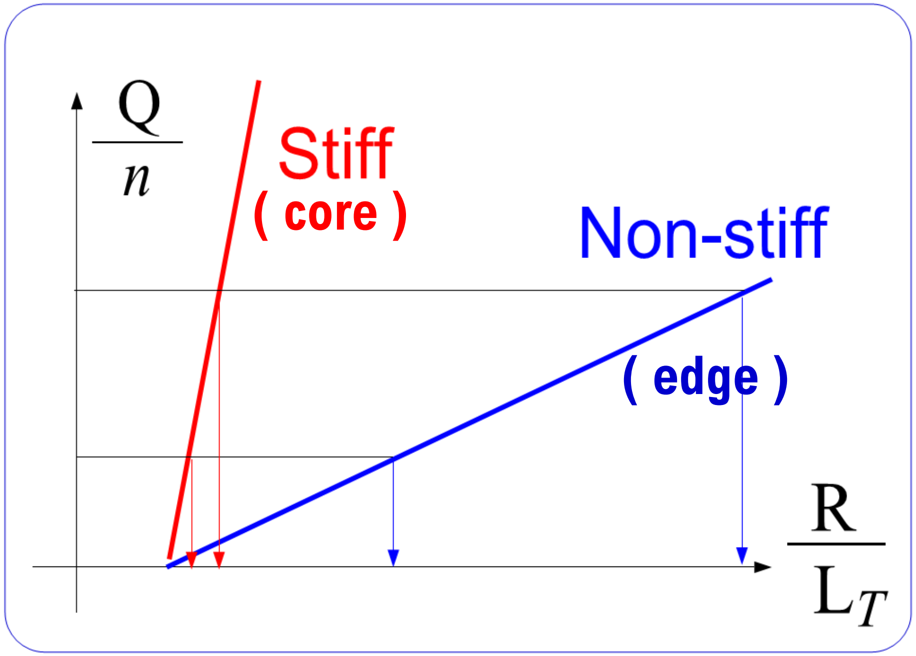
- **We look now at the scaling of the heat flux**

$$\frac{Q}{n} = -T\chi\frac{\nabla T}{T}$$

$$\chi_{GB} = D_B\frac{\rho_s}{a} = \frac{m^{1/2}T^{3/2}}{e^2B^2}\frac{1}{a}$$

$$\frac{Q}{n} = \hat{\chi}_s T\frac{\chi_{GB}}{R}\left(\frac{R}{L_T} - \frac{R}{L_{Tcrit}}\right) = \hat{\chi}_s\frac{m^{1/2}T^{5/2}}{e^2B^2R^2}\left(\frac{R}{L_T} - \frac{R}{L_{Tcrit}}\right)$$

- **Strong scaling of the heat flux with temperature**
- **Core, high temperature, is stiff (little change of gradient in response to large change of heat flux)**
- **Edge, low temperature, is non stiff**
- **In ITER high temperatures, core profiles will be close to the threshold**



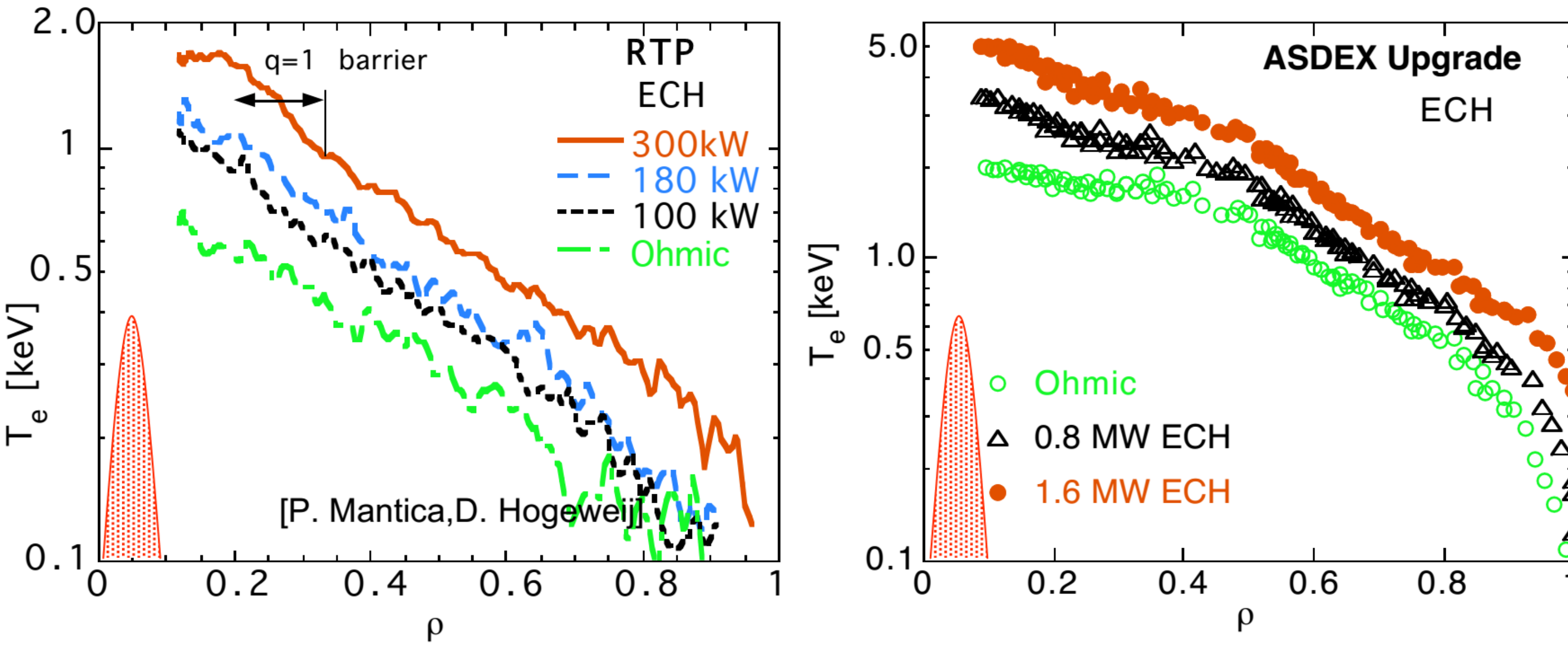
Profile stiffness, experimental evidences

➢ **With central heating, temperature increases maintaining self-similar profile shape**

Comparable B_T and n_e, central ECH, different size [F. Ryter ppcf 2001]

R = 0.72 m R = 1.65 m

Log scale

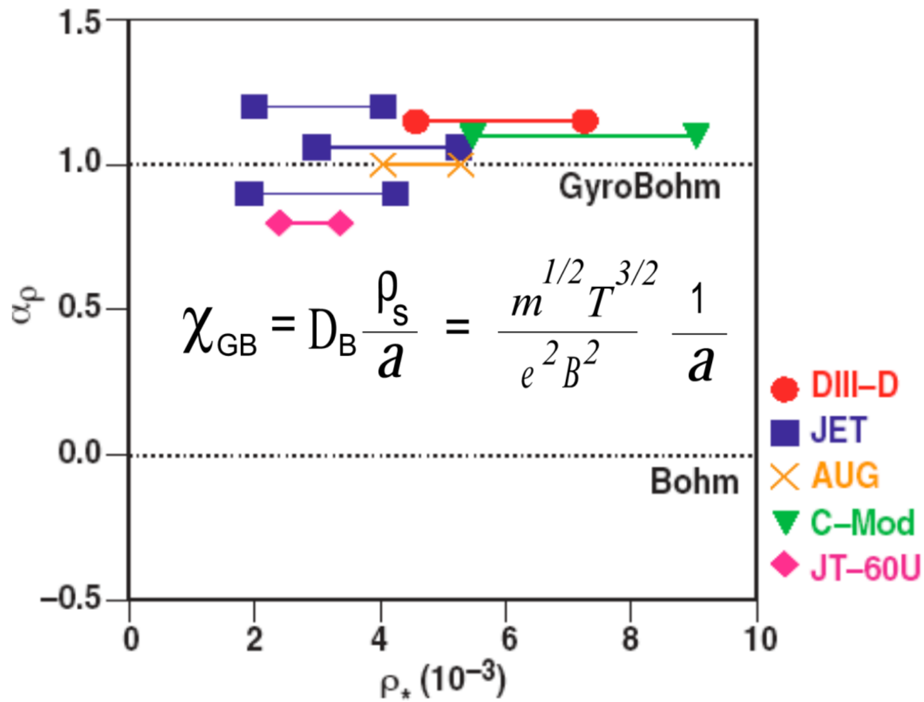


T_e profiles: const $\nabla T_e/T_e$ shifted according to edge T_e

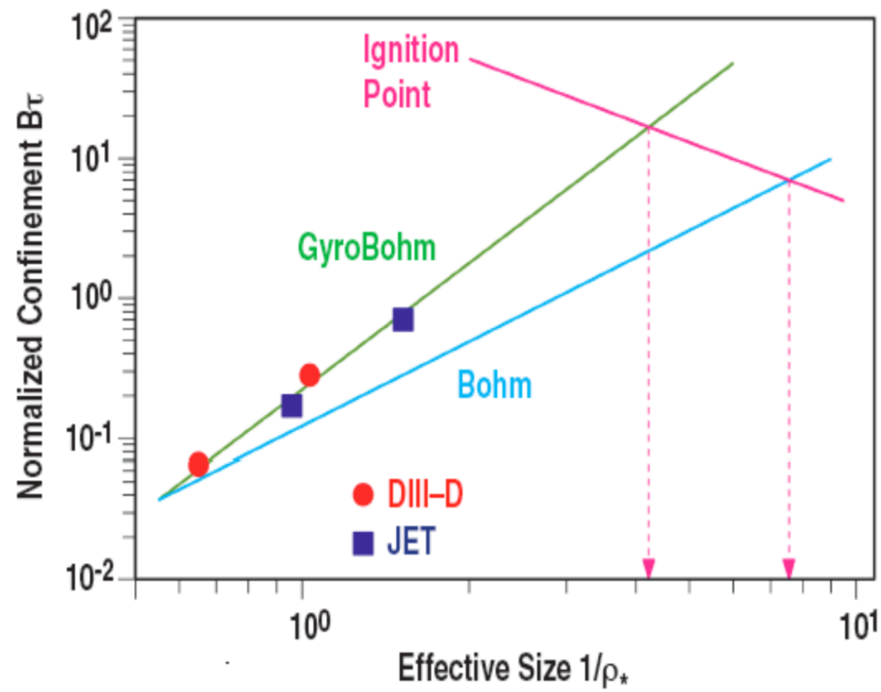
"Profile resilience" observed since 1980 in tokamaks
(TFR, TFTR, T10, ASDEX, DIII-D, ASDEX Upgrade, Tore Supra, ... JET)

Scaling of confinement time vs effective size of a device

Dependence of global confinement time gyroBohm in several tokamaks



GyroBohm favourable for ITER



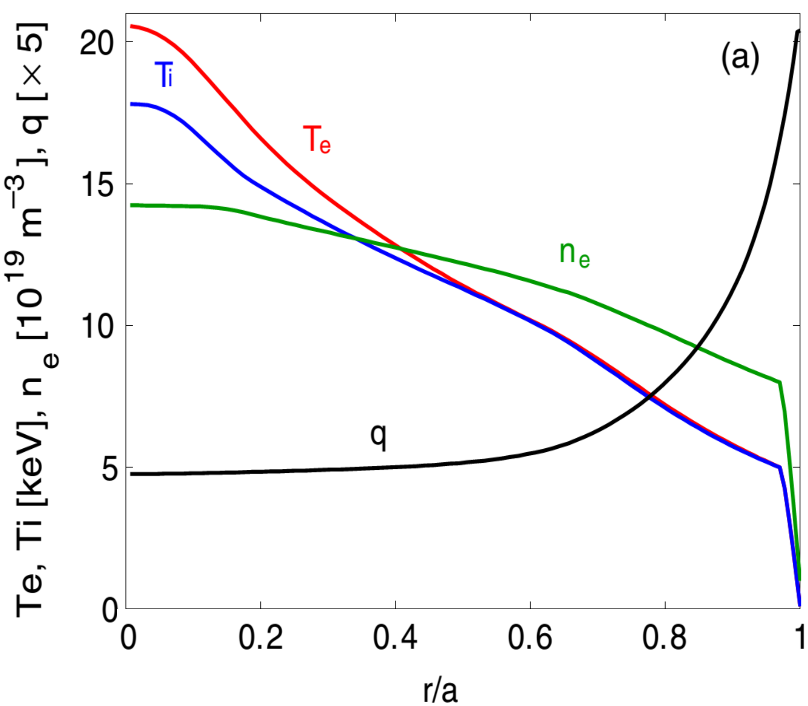
[C.C. Petty APS 2006, Phys. Plasmas 2006, web.gat.com/pubs-ext/APS06/Pettyrevvgs.pdf]

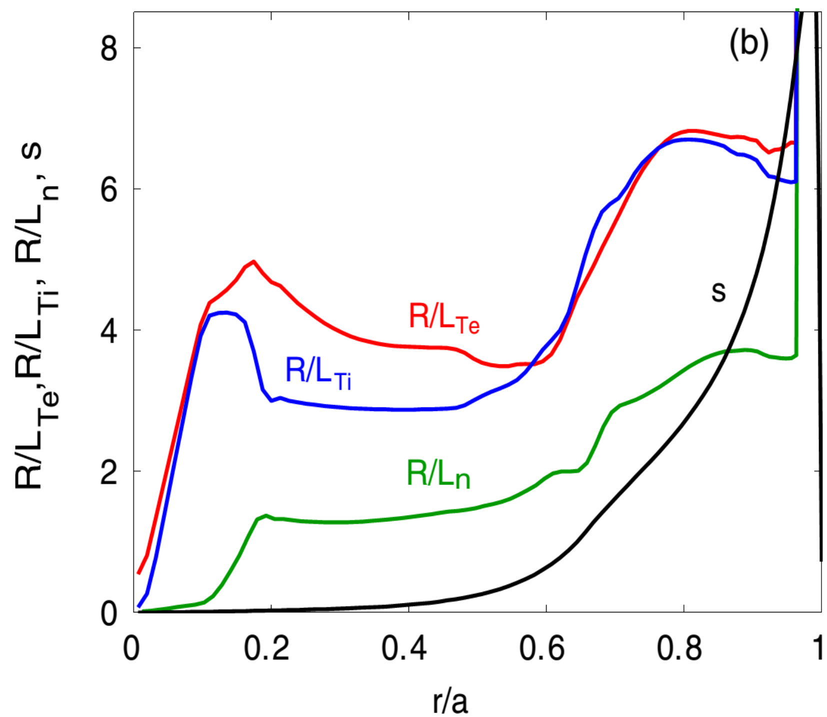
Note: ρ^* in ITER smaller than in present devices by about factor 5 - 10

$$\rho_* = \rho_s / a = (c_s / \Omega_i)/a, c_s = \sqrt{T_e / m_i}, \Omega_i = eB/m_i c$$

ITER profiles close to marginal stability

- **The height of the pedestal at the edge plays the most critical role**





[ASTRA with GLF23, Pereverzev NF 05]

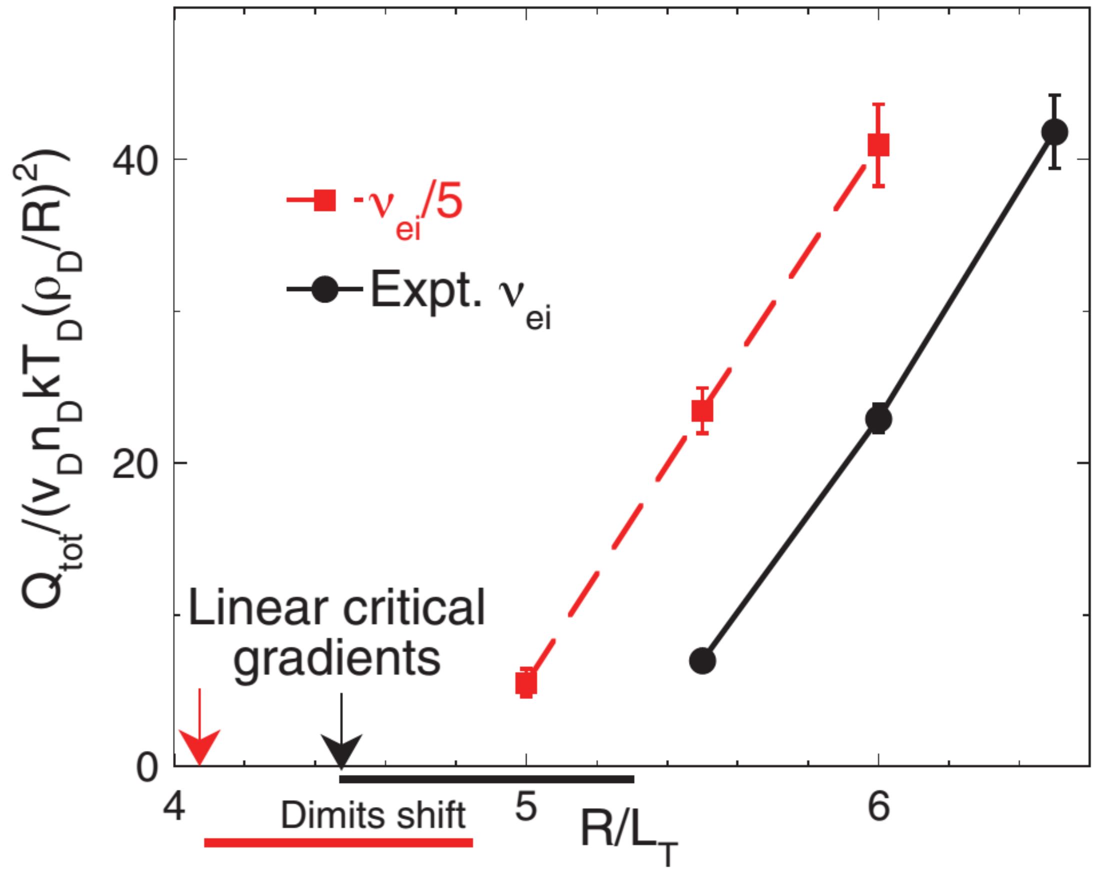
- **But good core confinement plays a stronger role in ITER than in present devices, since fusion reaction rate increases with increasing temperature**

Stabilizing mechanisms, nonlinear upshift of the threshold (Dimits shift)

- **Nonlinear simulations of ITG turbulence are found to have a threshold in R/LT which is up-shifted with respect to corresponding linear simulations**

- **Due to various exp. uncertainties and theoretical sensitivity on parameters, it is difficult to get a clear experimental evidence of the (non-) existence of this effect**

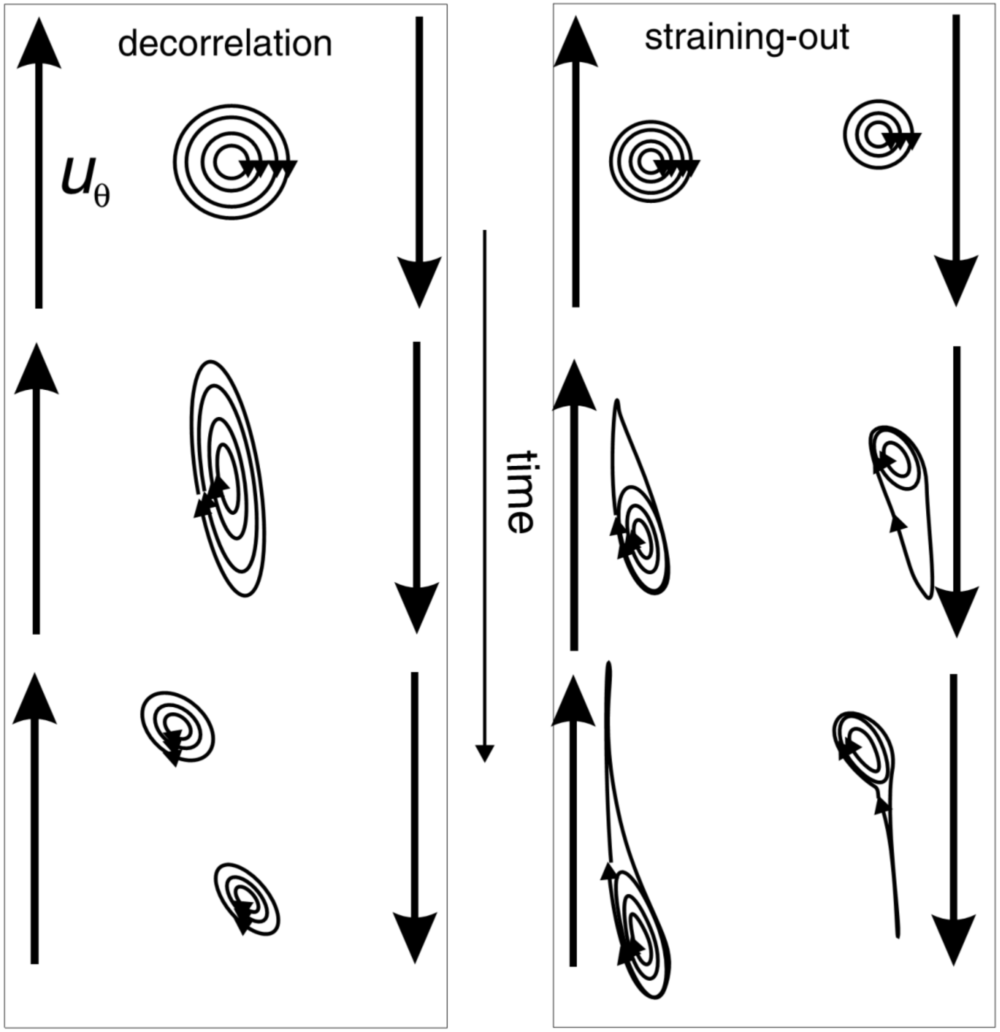
- **This upshift is produced by zonal flows close to marginal stability (high intermittency)**



[Dimits PoP 2000, Mikkelsen PRL 08]

Stabilizing mechanisms, sheared flows

- More in general, any sheared flow can produce a reduction of the turbulence and of the associated transport
- Strong rotational shear (gradient in rotation velocity) can even quench the turbulence
- This mechanism is identified as the key player producing the transport barrier at the edge (H-mode pedestal)



[P. Manz PRL 09, U. Stroth PPCF 11]

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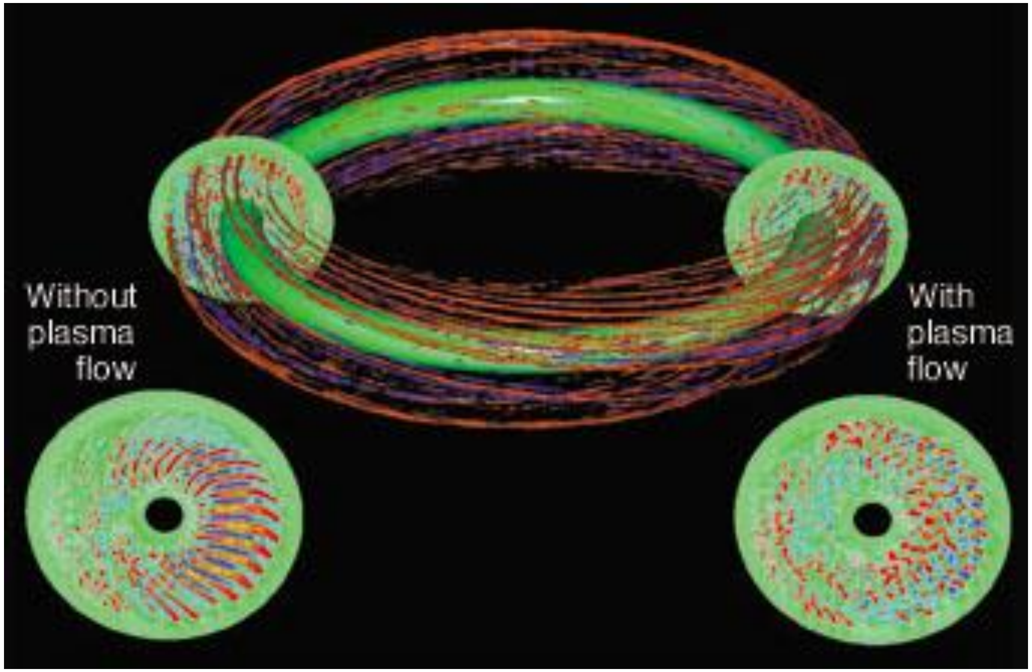
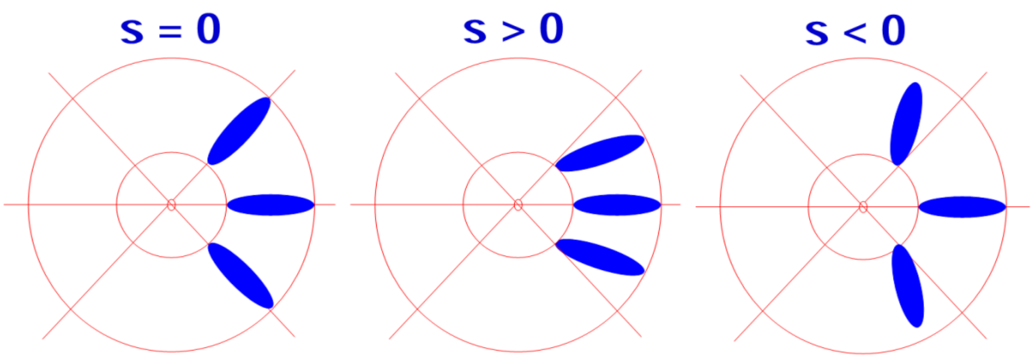


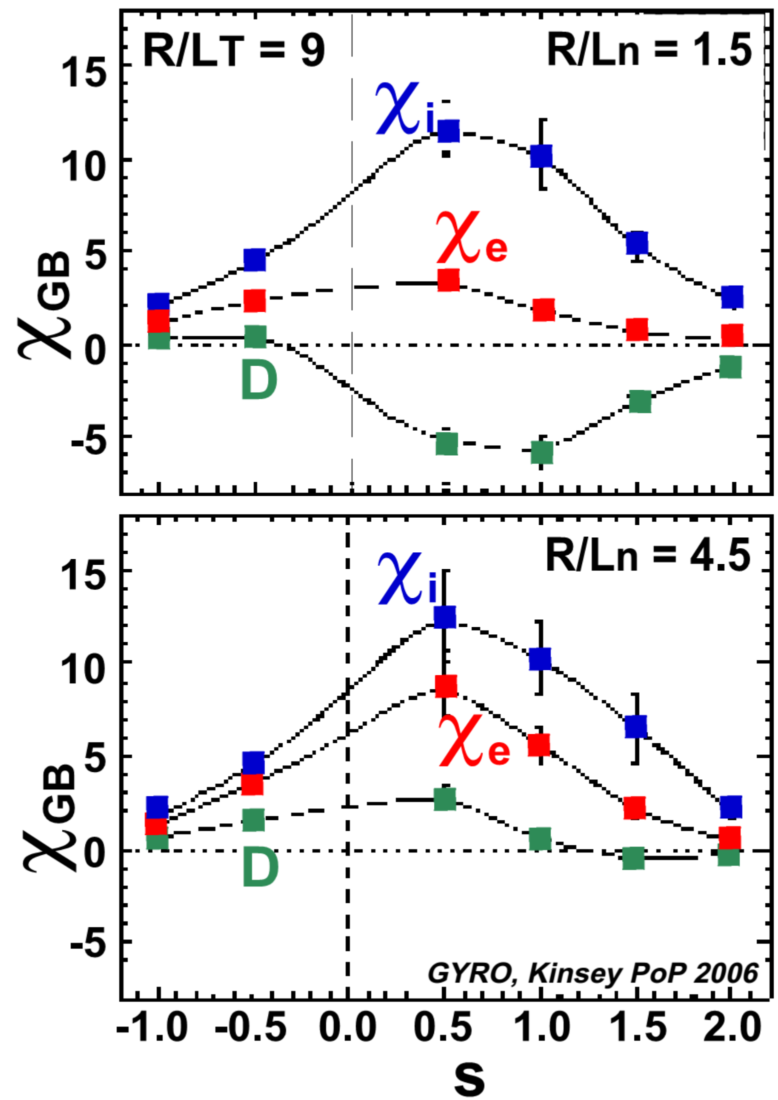
[Z. Lin Science 281, 1835 (1998)]

- It can also be important in the core, however ITER core plasmas are not expected to benefit from this mechanism, since ratio of external torque to plasma inertia is small in ITER compared to present devices

Stabilizing mechanisms, magnetic shear

- Also a radial variation of the pitch of the magnetic field lines has beneficial effects





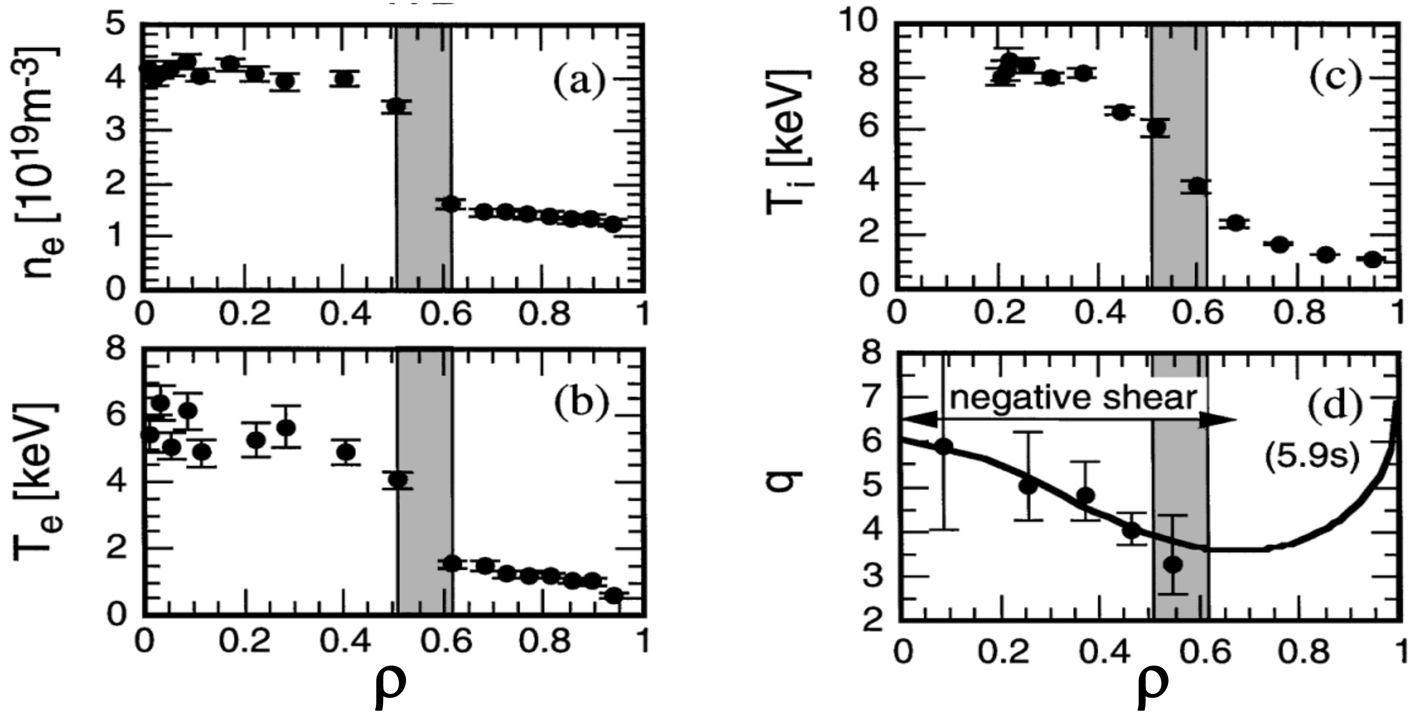
- Perturbations are field aligned, magnetic shear tilts the eddies and reduces the drive

- Maximum transport around s = 0.5

Stabilizing mechanisms, magnetic shear

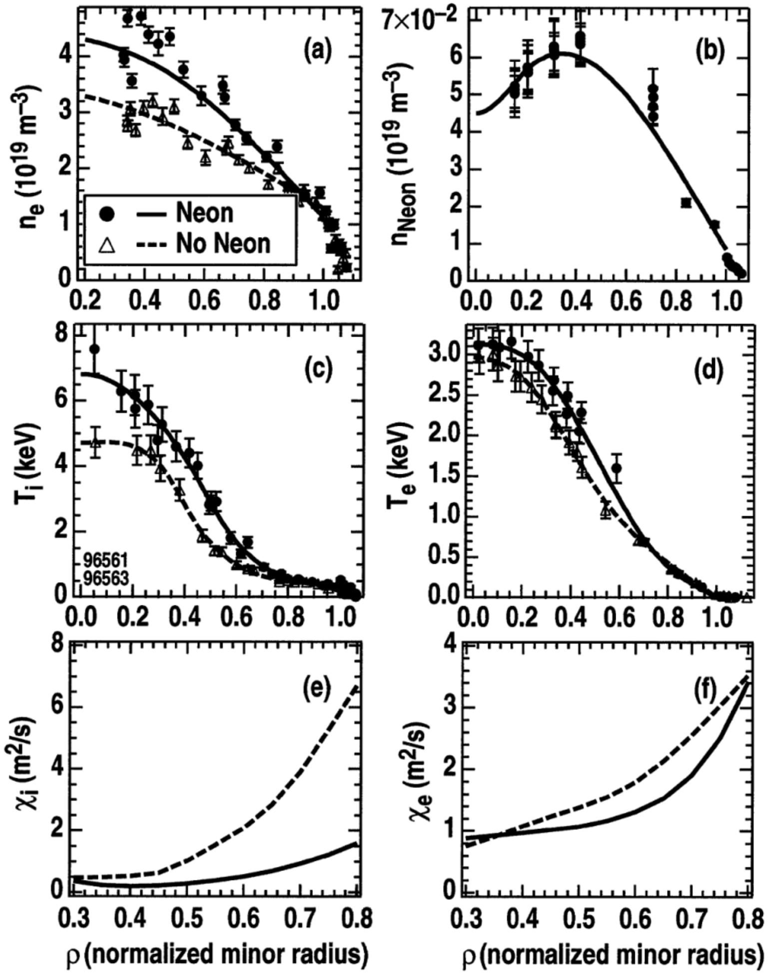
- Internal transport barriers are obtained in the presence of centrally reversed safety factor profiles (observations in JT-60U, DIII-D, TCV, JET, AUG)
- Advanced (steady-state) tokamak scenarios strongly rely on this effect
- Reversal of current density profile is generated by combination of bootstrap and auxiliary driven currents

JT-60U [Fujita PRL 78, 2377 (1997)]



Stabilizing mechanisms, impurity dilution

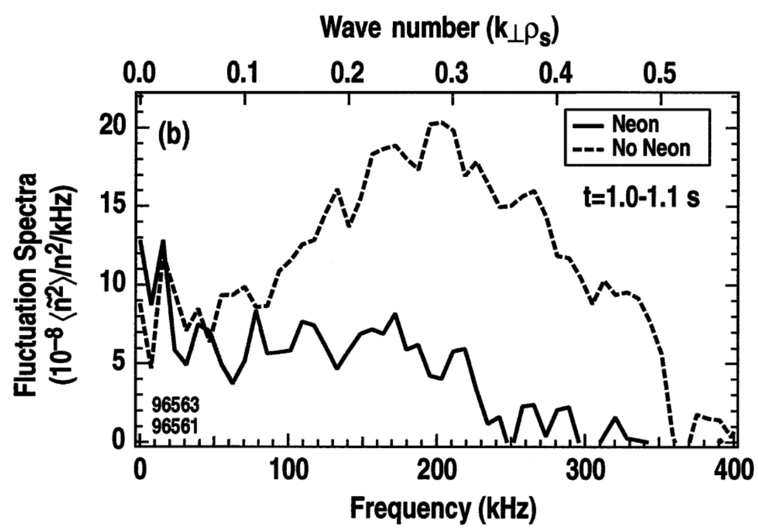
- Presence of impurities reduces the number of resonant (main ion) particles $\Rightarrow$ this reduces the ITG drive and linear growth rate, as well as the turbulence level



- Theoretical expectations (early nineties) have been verified in many experiments

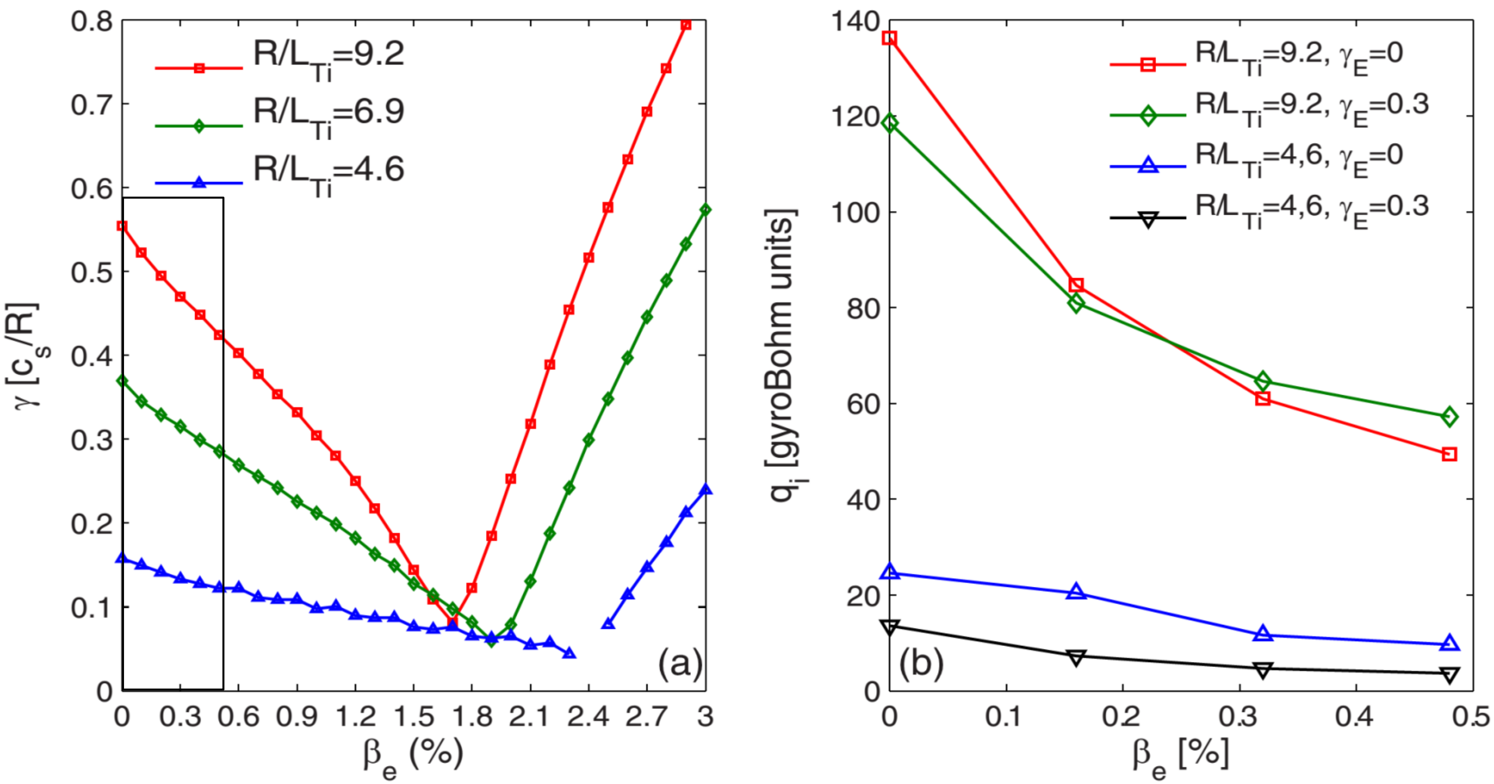
- One of the first examples:

DIII-D [McKee PRL 84, 1922 (2000)]



Stabilizing mechanisms, beta

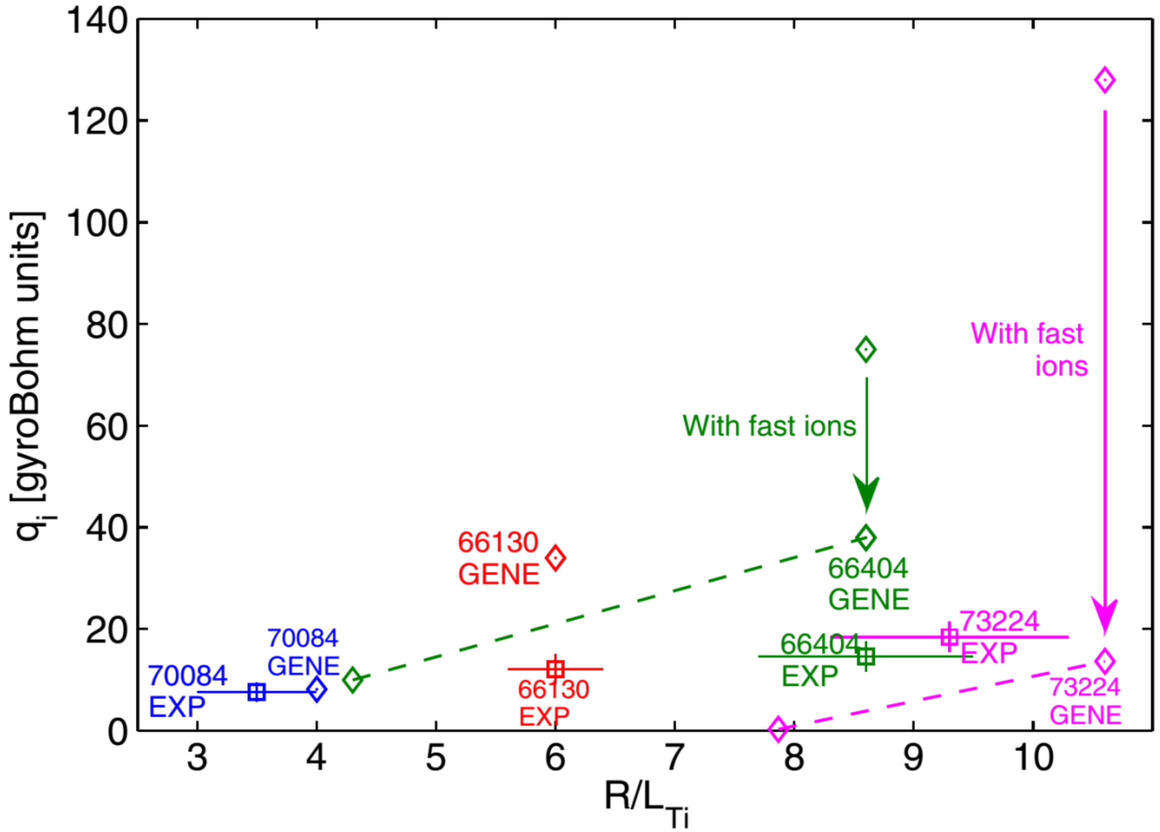
- **Electromagnetic effects reduce the drive of the electrostatic instabilities (ITG in particular): the increase of βe reduces the growth rate**
- **The transport reduction is much stronger in nonlinear simulations**



[J. Citrin Nuclear Fusion 54, 023008 (2014)]

Stabilizing mechanisms, beta

- **The effect is magnified due to the presence of additional pressure (β) brought by fast particles [Citrin PRL 13, NF 14]**



- **Expected beneficial effect due to alpha particle population in ITER and in a reactor**

DPG School ‚The Physics of ITER'
Physikzentrum Bad Honnef, 23.09.2014

Particle Transport,
Theory (and Experiment)

Clemente Angioni

Special acknowledgments for material and discussions

C. Bourdelle, J. Candy, D. Ernst, E. Fable, X. Garbet, G.T. Hoang, M. Maslov, A.G. Peeters, G. Pereverzev, R.E. Waltz, H. Weisen

Relevance to controlled nuclear fusion

- **Fusion power** $P_{\text{fus}} = \int d\text{Vol}\, E_{\text{fus}} \langle \sigma v \rangle n_D n_T \propto n^2$
- **Cost of energy** $\text{coe} \propto \beta_N^{-0.4} (n/n_G)^{-0.3} \propto T^{-0.4} n^{-0.7}$
  [Ward FED 05]
- **Conceptual studies for future power plants foresee operation up to 1.5 above the Greenwald density limit** **[Maisonnier FED 05]**
- **Density peaking can lead to such a target, keeping the edge density below the density limit**
- **It will be shown that a peaked density profile can be expected in ITER (and in a reactor more in general)**

Transport of main plasma (electrons), light and heavy impurities

- Particle transport covers different plasma species: electrons, light (e.g. He, Be, B, C) and heavy (e.g. Mo, W) impurities
- Different transport processes dominate for different species :
- electron transport is usually dominated by turbulence
- light impurity transport can be produced by both turbulent and neoclassical components, with dominance of turbulent component
- Heavy impurity transport can be produced by both turbulent and neoclassical components, with dominance of the neoclassical component

Outline

- Basic introductory concepts
- Analytical expression of the particle flux from the gyrokinetic equation
- Diffusion, thermo-diffusion and convection
  - Simple physical pictures
  - Selection of recent experimental and theory modelling results connected with each process
- Implications of present understanding for a reactor (ITER standard scenario)

Density profiles are peaked in the centre

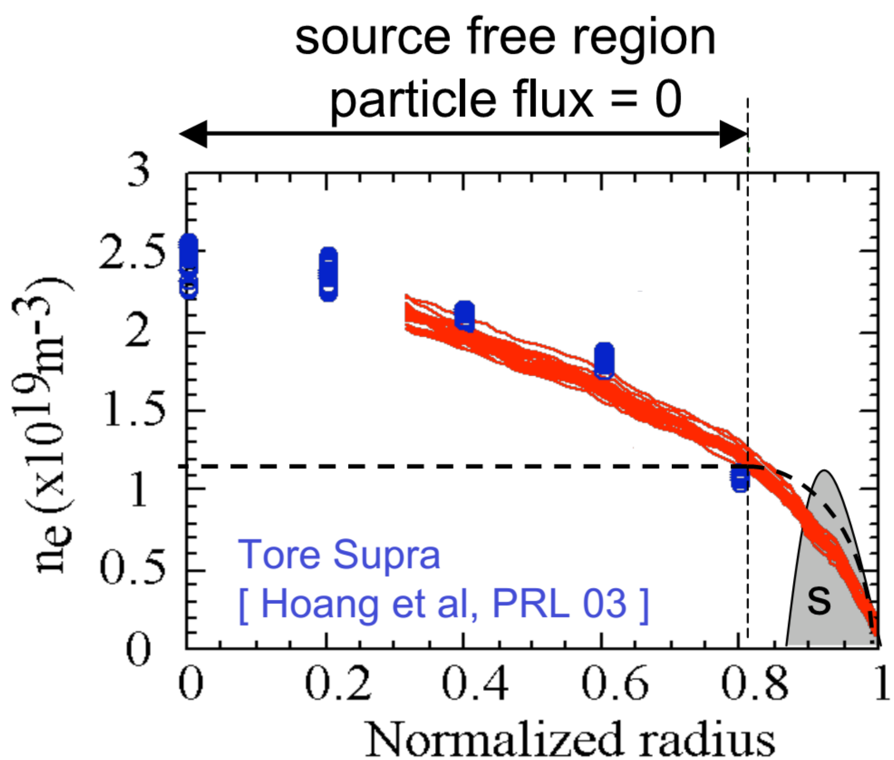
- In most conditions, electron density profiles in tokamaks are centrally peaked, even when the particle source is peripheral

- Existence of inward convection (pinch) balancing outward diffusion

$$\Gamma = -D\nabla n_e + n_e V$$

$$\Downarrow$$

$$-\frac{\nabla n_e}{n_e} = -\frac{V}{D} + \frac{\Gamma}{n_e D}$$



- Physics processes driving inward convection subject of intense research since decades

[Ware PRL 70, Coppi PRL 78, PF 81, Weiland NF 89]

Turbulence produces diffusion & convection

- **Curvature & grad B drift :** $$\mathbf{v}_C + \mathbf{v}_{\nabla B} = \frac{v_\parallel^2}{\Omega_c}\mathbf{B} \times \mathcal{K}_B + \frac{v_\perp^2}{2\Omega_c}\mathbf{B} \times \frac{\nabla B}{B}$$
  - **Depends on the energy of the particles**
  - **Confining magnetic field has gradient and curvature**

$$\Downarrow$$

- **Exists an off-diagonal term related to the electron temperature gradient (thermo-diffusion)**
- **Exists a pure convective term connected to the magnetic field geometry ("curvature" pinch)**

$$\frac{\Gamma}{n} = -D\frac{\nabla n}{n} - D_T\frac{\nabla T}{T} + V_p$$

- **This is an appropriate physical decomposition, but not a linear relationship**
- **It can be derived consistently from the gyrokinetic equation**

Derivation of an expression for the turbulent particle flux starting from a formal solution of the gyrokinetic equation

Short reminder, gyrokinetic equation and normalized logarithmic gradients

➤ **Gyrokinetic equation : gyroaverage,** $f = F - F_M$

$$\frac{\partial f}{\partial t} + \mathbf{v}_{gc}\cdot\nabla f + \tilde{\mathbf{v}}_E\cdot\nabla f - \frac{\mu}{m}\nabla_{\parallel}B\frac{\partial f}{\partial v_{\parallel}} = C(f) - \boxed{\tilde{\mathbf{v}}_E\cdot\nabla F_M} - \frac{Ze}{T}F_M\,\mathbf{v}_{gc}\cdot\nabla\phi$$

➤ **Linear gyrokinetic equation in simplified geometry,** $\mathbf{B} = B_0 R_0 / R\,\mathbf{b}$

$$\hat{\omega}\,f + \frac{m\left(v_{\parallel}^2 + v_{\perp}^2/2\right)}{ZT}f - \frac{v_{\parallel}k_{\parallel}}{\omega_D}f = \left[\boxed{\frac{R}{L_n} + \left(\frac{mv_{\parallel}^2 + mv_{\perp}^2}{2T} - \frac{3}{2}\right)\frac{R}{L_T}} - \frac{m\left(v_{\parallel}^2 + v_{\perp}^2/2\right)}{T} + Z\frac{v_{\parallel}k_{\parallel}}{\omega_D}\right]F\,\phi$$

➤ **The (radial) gradient of the Maxwellian**

$$\nabla F_M = \frac{F_M}{R}\left[\frac{R}{L_n} + \left(\frac{E}{T} - \frac{3}{2}\right)\frac{R}{L_T}\right]$$

where $\displaystyle \frac{1}{L_X} = -\frac{\nabla X}{X}$ **and** $L_B = R$

Derivation from the gyrokinetic equation

- **Linearized gyrokinetic equation for electrons**
  (ballooning representation, simple Krook collision operator)

$$(\omega - k_{||}v_{||} - \omega_{de} + i\nu_{ei})\, g \;=\; F_M \left\{ \omega_{De} \left[\frac{R}{L_n} + \left(\frac{E}{T_e} - \frac{3}{2} \right) \frac{R}{L_{Te}} \right] - \omega \right\} J_0(k_\perp \rho_s)\hat{\phi}$$

- **Compute formally the QL particle flux for trapped electrons ($k_{||}$ = *0*)**

$$\Gamma_k = \frac{k_y c_s^2}{\Omega_{ci}} \left\langle \int d^3v F_M \frac{(\hat{\gamma}_k + \hat{\nu}_k)[R/L_n + (E/T_e - 3/2)R/L_{Te}] - (\hat{\gamma}_k \hat{\omega}_d - \hat{\omega}_{rk}\hat{\nu}_k)}{(\hat{\omega}_{rk} + \hat{\omega}_d)^2 + (\hat{\gamma}_k + \hat{\nu}_k)^2} \; J_0(k_\perp \rho_s)^2 |\hat{\phi}_k^2| \right\rangle$$

- **All frequencies normalized to** $\omega_D = k_y \rho_s c_s / R$ **and** $\hat{\omega}_d = \frac{E}{T_e}\left(1 + \frac{v_{||}^2}{v^2}\right) G(\theta)$

Derivation from the gyrokinetic equation

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ExB advection
(diffusion, always outward)

Thermo-diffusion
(direction mode dependent)

ExB compression
(pinch)

Collisional term
(mode dependent)

- **All frequencies normalized to** $\omega_D = k_y \rho_s c_s / R$ **and** $\hat{\omega}_d = \frac{E}{T_e}\left(1 + \frac{v_{||}^2}{v^2}\right) G(\theta)$

Electron Particle Flux can be decomposed in this form

$$\Gamma_\sigma = \frac{n_\sigma}{R}\left(D_\sigma \frac{R}{L_{n\sigma}} + D_{T\sigma}\frac{R}{L_{T\sigma}} + RV_{p\sigma}\right)$$

$$\frac{R\,\Gamma_\sigma}{n_\sigma} = D_\sigma\left(\frac{R}{L_{n\sigma}} + C_{T\sigma}\frac{R}{L_{T\sigma}} + C_{p\sigma}\right)$$

$$C_{T\sigma} = \frac{D_{T\sigma}}{D_\sigma} \quad ; \quad C_{P\sigma} = \frac{RV_{p\sigma}}{D_\sigma}$$

- **Includes both electrostatic and magnetic fluctuations, E x B and magnetic flutter transport**
- **A physical decomposition, not a linear relationship (linear for trace species)**

General expression for electrons and impurity ions, for a rotating plasma

$$\frac{\Gamma_n}{n} = -D_n \frac{1}{n}\frac{\partial n}{\partial r} - D_T \frac{1}{T}\frac{\partial T}{\partial r} - D_u \frac{1}{v_{th}}\frac{\partial U_\phi}{\partial r} + V_{pn}$$

Diffusion

Thermo-diffusion
[Coppi PRL 78, Weiland NF 89, Angioni NF04, PRL 06]

Roto-diffusion
[Camenen PoP 09, Casson PoP 10, Angioni NF 11]

Pure convection
[Weiland NF 89, Yanko JETP 94, Garbet PRL 03,PoP05 Angioni PRL 03, PRL 06]

- **Off-diagonal terms can reverse direction or change parameter dependences depending on the type of turbulence (ITG / TEM)**

[Angioni et al NF 04 and PoP 05, Garbet PRL 03, Fable PPCF 2010]

[Reviews in Angioni et al PPCF 09, NF 12]

Contributions inward /outward, for electrons and impurities, produce a complex pattern

<table>
<tr><th colspan="2" rowspan="2"></th><th colspan="2">Thermodiffusion</th><th colspan="2">Pure Convection</th><th colspan="2">Collisions (electron)
Roto-diffusion (imp)</th></tr>
<tr><th>ITG</th><th>TEM</th><th>ITG</th><th>TEM</th><th>ITG</th><th>TEM</th></tr>
<tr><td rowspan="2">Curvature & ∇B resonance only</td><td>electrons</td><td>in</td><td>out</td><td colspan="2" rowspan="2">in</td><td>out</td><td>in</td></tr>
<tr><td>impurities</td><td>out</td><td>in</td><td>out</td><td>in</td></tr>
<tr><td rowspan="2">slab resonance limit</td><td>electrons</td><td colspan="2" rowspan="2">in</td><td>out</td><td>in</td><td>out</td><td>in</td></tr>
<tr><td>impurities</td><td>in</td><td>out</td><td>out</td><td>in</td></tr>
</table>

- **Framework for theory validation : do experiments exhibit (qualitatively, quantitatively) the same pattern ?**

Impurity charge provides additional handle to identify different transport processes

[Bourdelle PoP 07]

	Heavy trace impurities	
	Compressibility	Thermodiffusion
Curvature only	No dependences	Scales as $1/Z_s$
Slab limit	Scales as Z_s/A_s	Scales as $1/A_s$

- **Although electrostatic turbulent transport is produced by fluctuating ExB drift, dependences on Z and A arise from the resonances, provided by the perpendicular and parallel gyro-centre motions**
  - **Perpendicular motion, curvature and grad B drift prop. to 1/Z**
  - **Parallel motion, electric force term proportional to Z/A, pressure term proportional to 1/A**

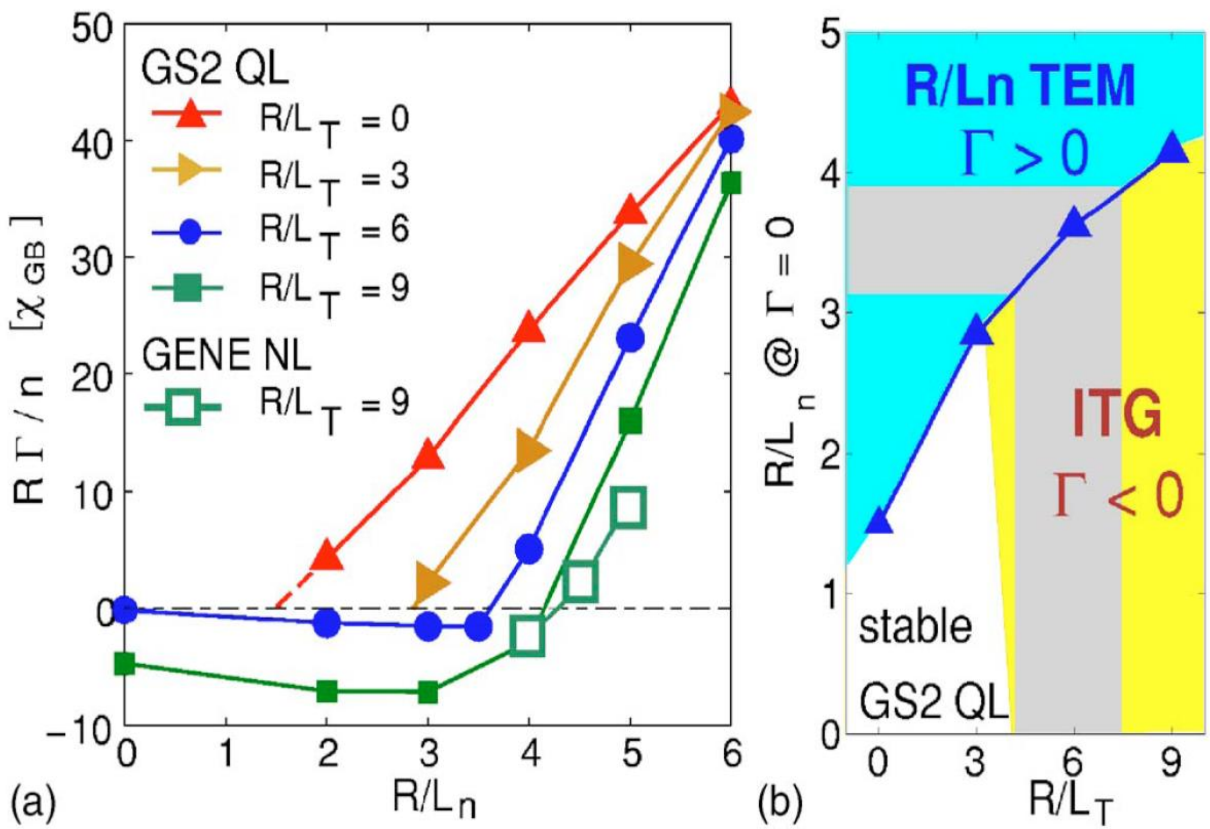
[Angioni PRL 06]

Diffusion and Convection,

A simple physical picture

General relationship between particle flux & density gradient is not linear

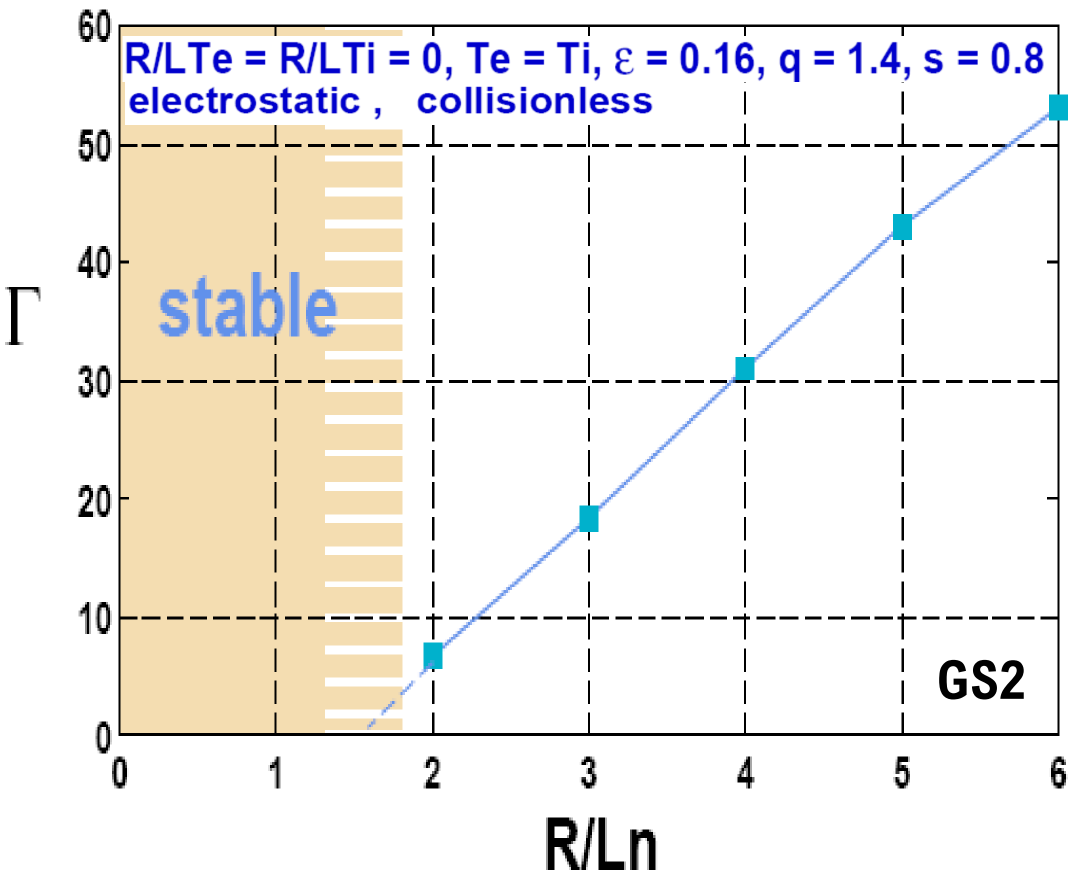
- **Identify local slope of the profile (R/Ln) for given set of plasma parameters by looking for R/Ln at which $\Gamma(R/L_n) = \Gamma_s - \Gamma_{neo} \approx 0$**



[Angioni PoP 07, GENE Jenko PPCF 05]

Linear instability above critical logarithmic gradient

- An instability takes place above a critical log density gradient, in the absence of any temperature gradient
- This is the most efficient instability leading to outward transport of particles
- These are results from a GK code (GS2)
- A simple picture can be given to understand from what this comes



Reminder from previous lecture: Fluid model in simplified geometry

- **Continuity (zero moment)**

$$\boxed{\hat{\omega}\tilde{n}} + \boxed{2\left(\tilde{n}+\tilde{T}\right)} + \boxed{4\,\hat{k}_{||}\,\tilde{u}} = \boxed{\left[\frac{R}{L_n}-2\right]\hat{\phi}}$$

- **Parallel velocity moment**

$$\boxed{\hat{\omega}\tilde{u}} + \boxed{4\tilde{u}} + \boxed{2\hat{k}_{||}(\tilde{n}+\tilde{T})} = \boxed{-2\,\hat{k}_{||}\,\hat{\phi}}$$

- **Energy balance (second order moment)**

$$\boxed{\hat{\omega}\tilde{T}} + \boxed{\frac{4}{3}\tilde{n} + \frac{14}{3}\tilde{T}} + \boxed{\frac{8}{3}\hat{k}_{||}\tilde{u}} = \boxed{\left[\frac{R}{L_T}-\frac{4}{3}\right]\hat{\phi}}$$

- **Linear gyrokinetic equation in simplified geometry,** $\boldsymbol{B} = B_0\, R_0 / R\; \mathbf{e}_y$

$$\boxed{\hat{\omega}\,f} + \boxed{\frac{m\left(v_{||}^2+v_{\perp}^2/2\right)}{ZT}f} - \boxed{\frac{v_{||}k_{||}}{\omega_D}f} = \boxed{\left[\frac{R}{L_n}+\left(\frac{mv_{||}^2+mv_{\perp}^2}{2T}-\frac{3}{2}\right)\frac{R}{L_T}-\frac{m\left(v_{||}^2+v_{\perp}^2/2\right)}{T}+Z\frac{v_{||}k_{||}}{\omega_D}\right]F\,\phi}$$

Physical picture from the simple fluid model

- We consider ions, trapped electrons and adiabatic passing electrons, and we neglect for the time being temperature fluctuations

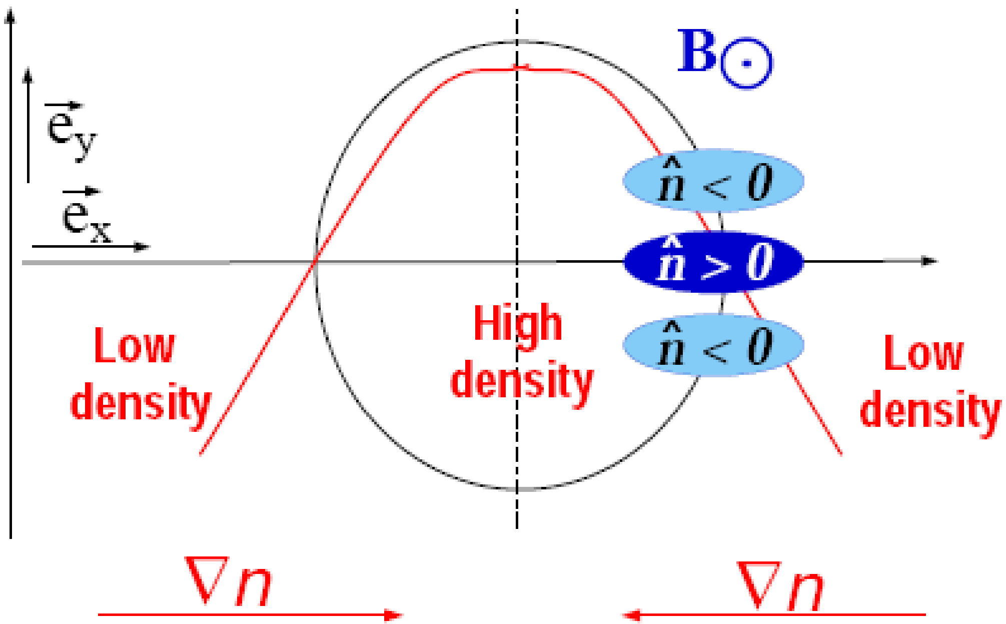
$$\omega \hat{n}_i + \boxed{2\omega_{Di}\hat{n}_i} - \boxed{\omega_{Di}\frac{Z_i T_e}{T_i}\left(\frac{R}{L_n} - 2\right)\hat{\phi}} = 0,$$

$$\omega \hat{n}_{et} + \boxed{2\omega_{De}\hat{n}_{et}} + \boxed{\omega_{De}\left(\frac{R}{L_n} - 2\right)\hat{\phi}} = 0,$$

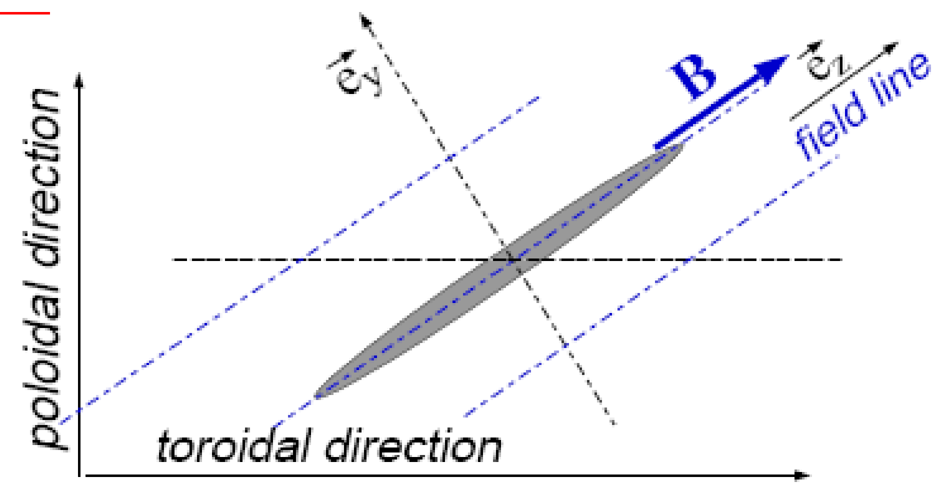
$$\boxed{f_t \hat{n}_{et} + f_p \hat{\phi} = \hat{n}_i.}$$

- Initial density perturbation propagates in opposite directions in the binormal (mainly poloidal) direction
- Charge separation neutralized by parallel motion of passing electrons
- Resulting fluctuating electric field generates an ExB motion in the radial direction which enhances the initial perturbation in the presence of a density gradient

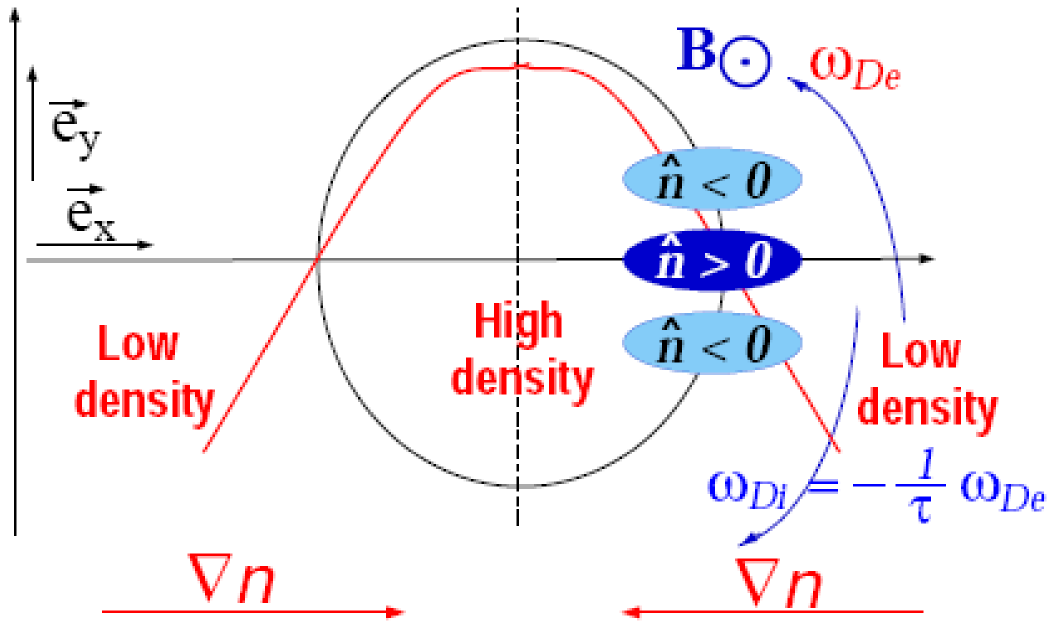
Simple physical picture, main instability transporting particles



Initial density perturbation

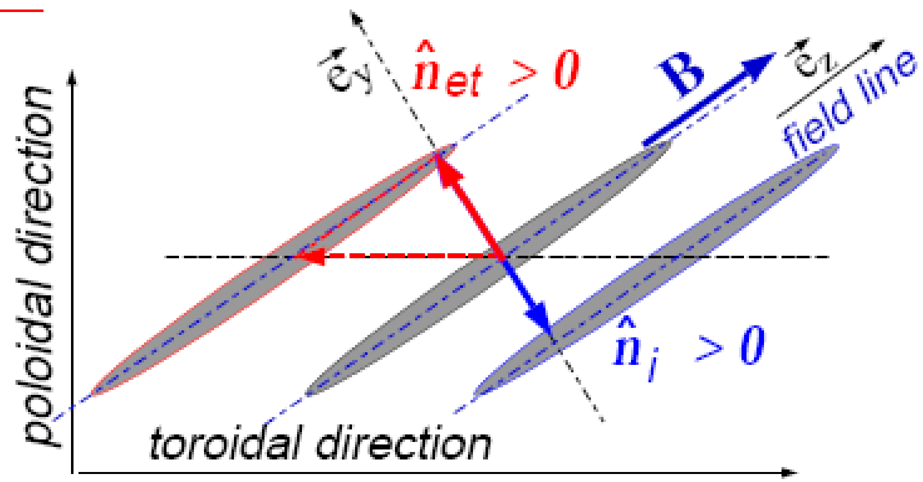


Simple physical picture, main instability transporting particles

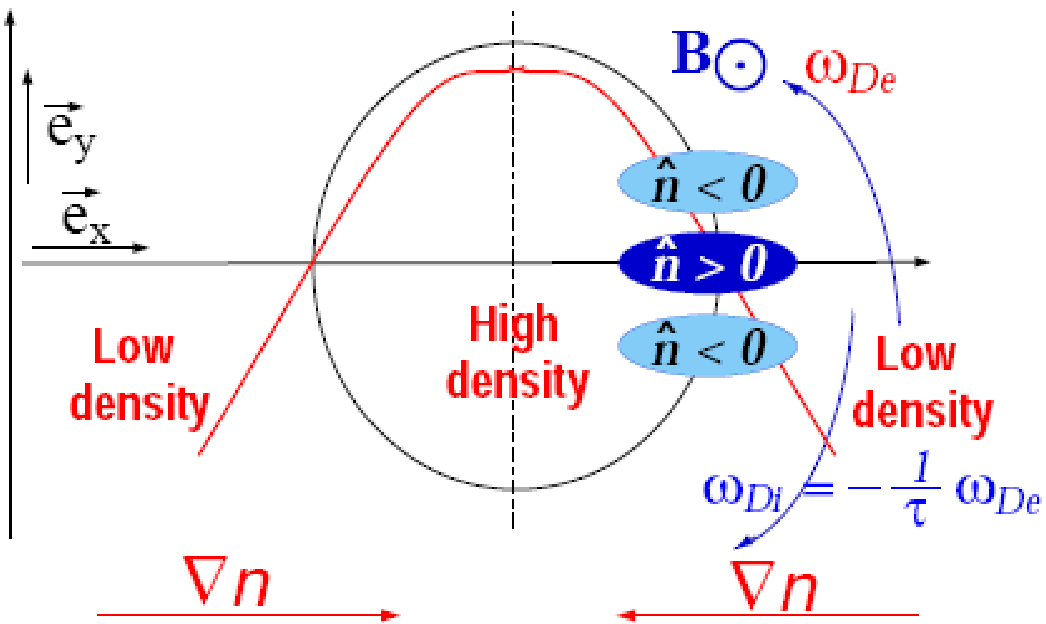


Initial density perturbation

Opposite curvature & ∇B drifts for trapped electrons and ions



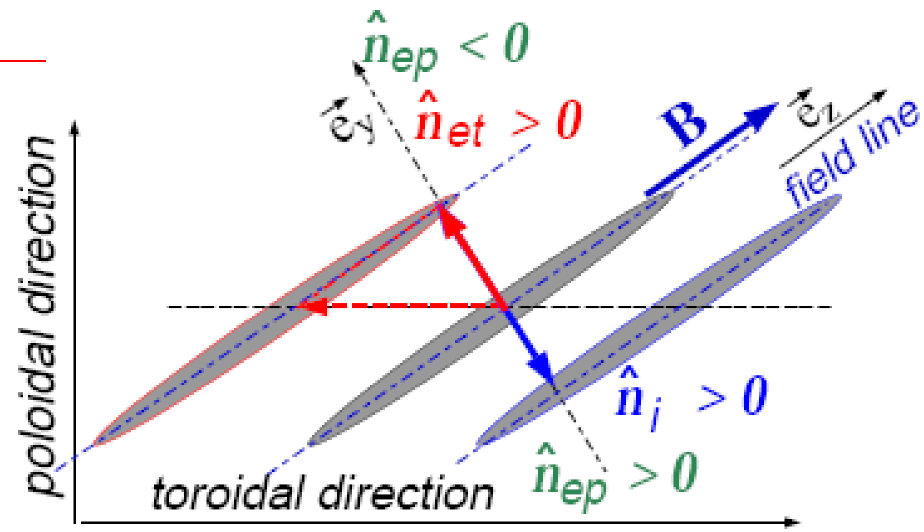
Simple physical picture, main instability transporting particles



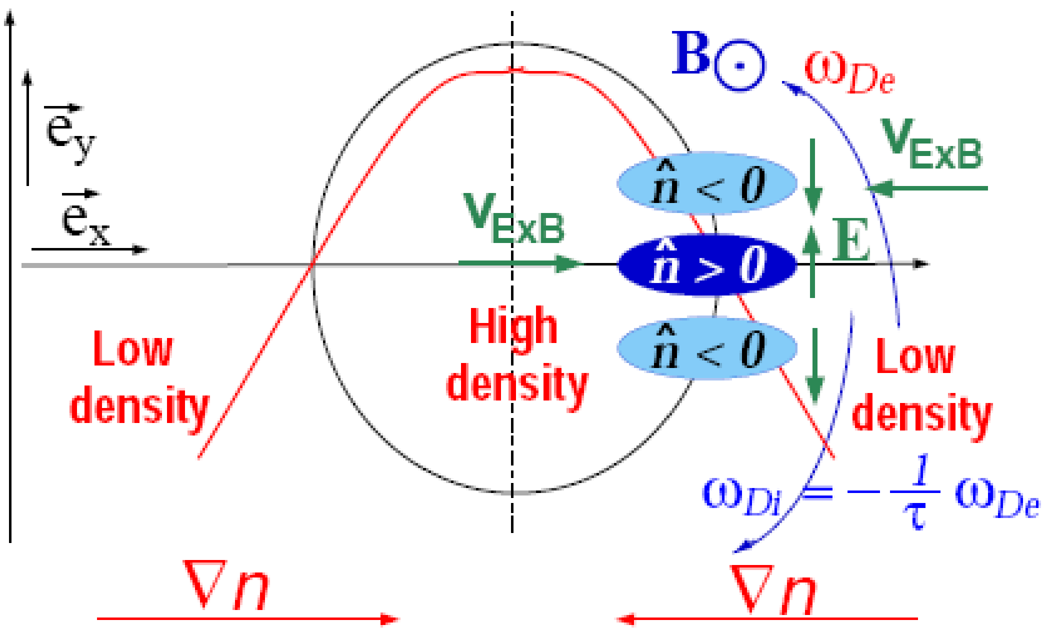
Initial density perturbation

Opposite curvature & ∇B drifts for trapped electrons and ions

Passing electrons neutralise the charge separation



Simple physical picture, main instability transporting particles

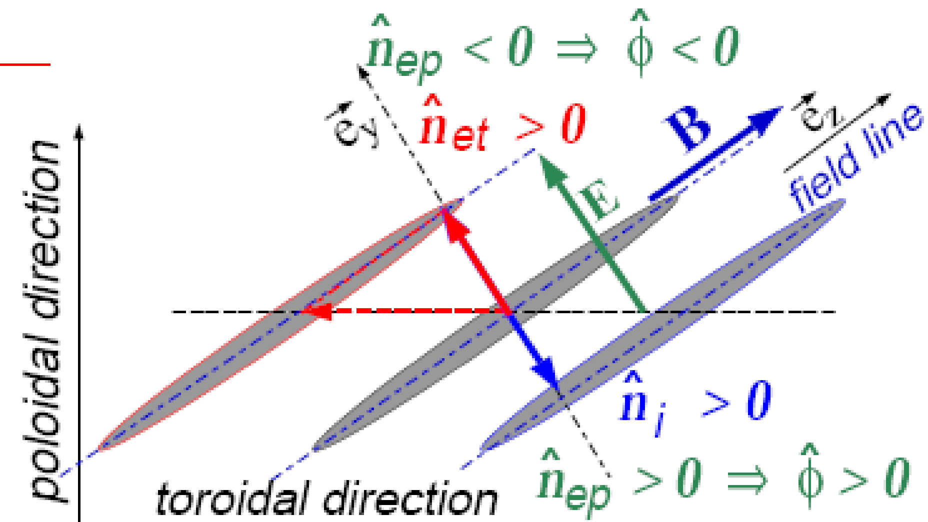


Initial density perturbation

Opposite curvature & ∇B drifts for trapped electrons and ions

Passing electrons neutralise the charge separation

Parallel force balance implies an electrostatic potential, the ExB flow enhances the perturbation



Physical picture from the simple fluid model, the particle flux

- **Dispersion relation of simple fluid model has imaginary roots (unstable modes) when** $$\frac{R}{L_n} > 2\left[2\frac{1+f_t/\tau}{1-f_t} + \frac{1}{\tau} - \sqrt{\left(2\frac{1+f_t/\tau}{1-f_t} + \frac{1}{\tau} - 1\right)^2 - \left(1+\frac{1}{\tau}\right)^2}\,\right]$$
- **An instability takes place, in the absence of temperature gradients, when R/Ln exceeds a certain threshold**
- **QL particle flux** $$\frac{R\Gamma_{ql}}{n} = f_t \sum_k \frac{\gamma_k}{k_x^2} \hat{D}_k \left(\frac{R}{L_n} - 2\right)$$

- **Particle flux is always positive (outward), for R/Ln < 2 the mode is stable**
- **A negative total particle flux can be obtained ONLY in the presence of finite log temperature gradients**
- **This has a profound reason, connected with the requirement of positive entropy production rate [Garbet PoP 2005]**

Physical picture from the simple fluid model, the particle flux

- **Dispersion relation of simple fluid model has imaginary roots (unstable modes) when** $\frac{R}{L_n} > 2\left[2\frac{1+f_t/\tau}{1-f_t} + \frac{1}{\tau} - \sqrt{\left(2\frac{1+f_t/\tau}{1-f_t} + \frac{1}{\tau} - 1\right)^2 - \left(1+\frac{1}{\tau}\right)^2}\right]$

- **An instability takes place, in the absence of temperature gradients, when R/Ln exceeds a certain threshold**

- **QL particle flux** $$\frac{R\Gamma_{ql}}{n} = f_t \sum_k \frac{\gamma_k}{k_x^2} \hat{D}_k \left(\frac{R}{L_n} - 2\right)$$

- **Recalling some relations allows us to understand the origin of the two terms**

$$\nabla \cdot (n\tilde{\mathbf{v}}_{E\times B}) = \nabla n \cdot \tilde{\mathbf{v}}_{E\times B} + n\nabla \cdot \tilde{\mathbf{v}}_{E\times B}$$

$$\nabla \cdot \tilde{\mathbf{v}}_{E\times B} = \nabla\tilde{\phi} \cdot \mathbf{B} \times (\nabla B/B + \mathcal{K}_B)/B^2 \Longrightarrow \nabla \cdot \tilde{\mathbf{v}}_{E\times B} \simeq -i(k_y\tilde{\phi}/B)\,2/R$$

Pure Convection, main physical mechanism from toroidal resonance

- **Basic mechanism of turbulent convection connected with inhomogeneity of the magnetic field**
- **Fluctuations of trapped electron density** $\tilde{n}$ **and electrostatic potential** $\tilde{\phi}$
- **Continuity equation :** $$\frac{\partial \tilde{n}}{\partial t} + \nabla \cdot \left(n \, \tilde{\mathbf{v}}_{E\times B} \right) = 0$$

$$\frac{\partial \tilde{n}}{\partial t} + \nabla n \cdot \tilde{\mathbf{v}}_{E\times B} = 0$$

- **Constant B with straight field lines :** $\nabla \cdot \tilde{\mathbf{v}}_{E\times B} = 0 \Rightarrow$ **Diffusion only**

Pure Convection, main physical mechanism from toroidal resonance

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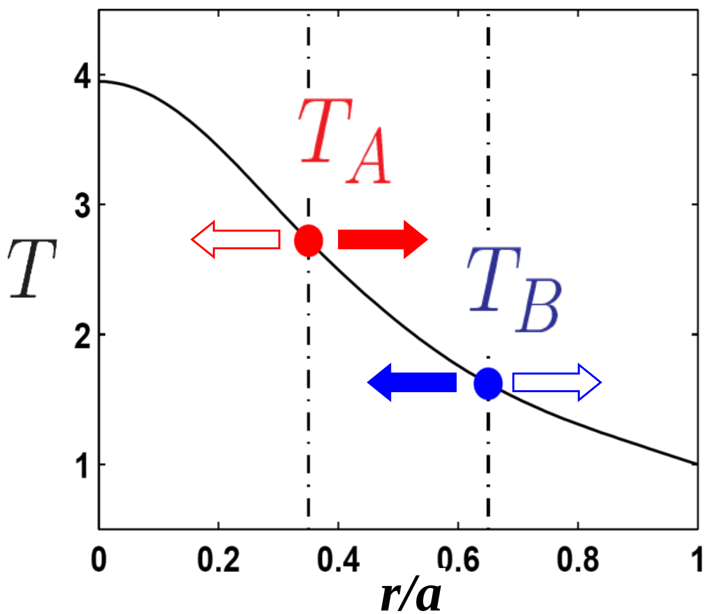
- **Constant B with straight field lines :** $\nabla \cdot \tilde{\mathbf{v}}_{E\times B} = 0$

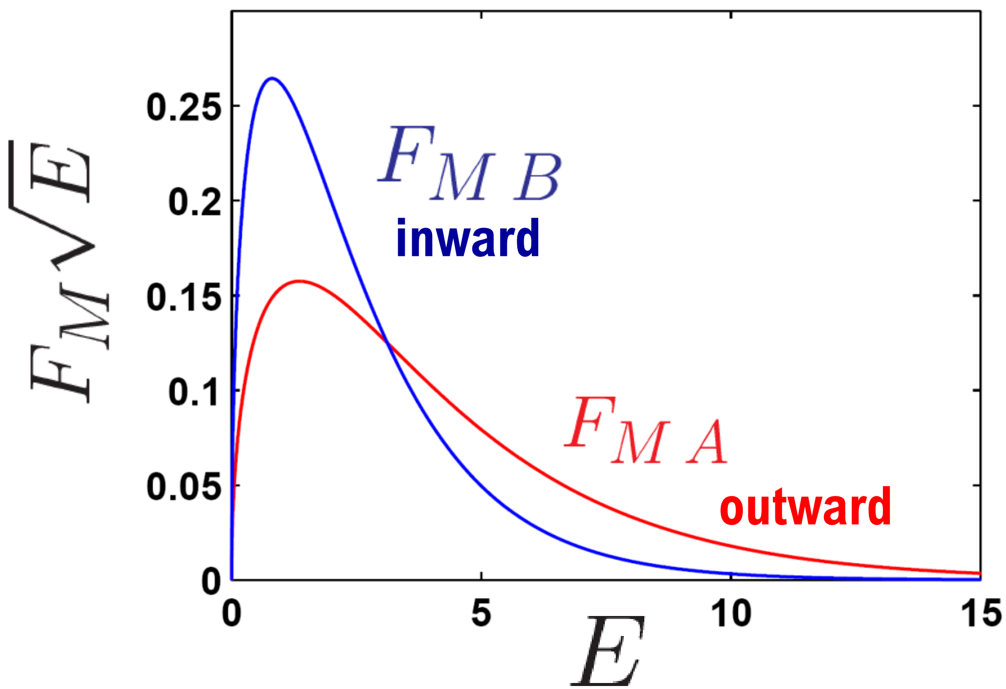
- **Magnetic field in tokamak geometry:** $\nabla \cdot \tilde{\mathbf{v}}_{E\times B} = \nabla \tilde{\phi} \cdot \mathbf{B} \times \left(\frac{\nabla B}{B} + \mathcal{K}_B \right)$

Curvature and grad B yield a compressional term, produces the "curvature" pinch

Off-diagonal terms can reverse direction depending on instability, thermo-diffusion

➢ **Temperature profile, temperature gradient**

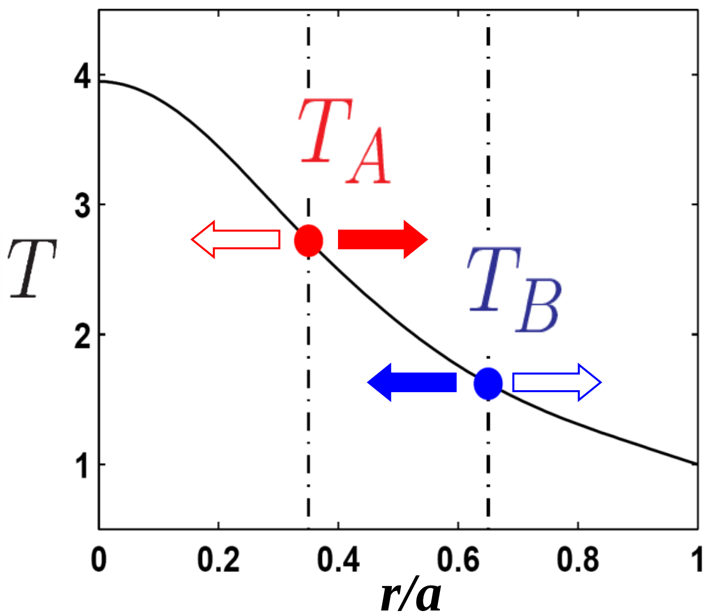


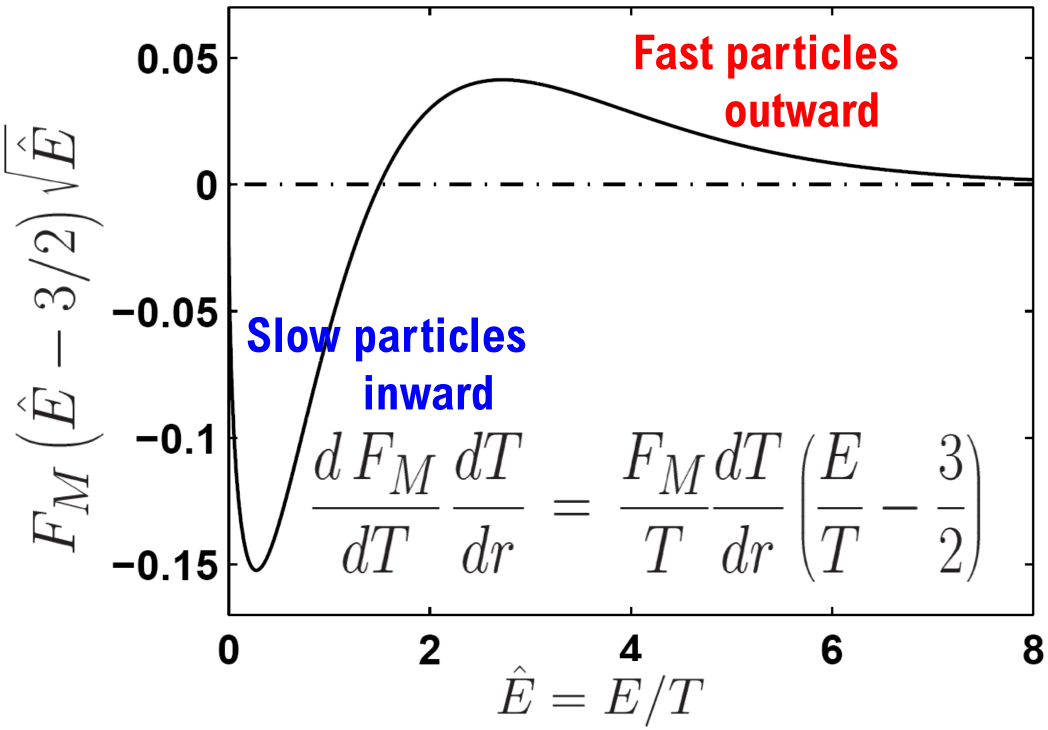


[Angioni et al NF 12]

Off-diagonal terms can reverse direction depending on instability, thermo-diffusion

- Temperature profile, temperature gradient





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Off-diagonal terms can reverse direction depending on instability, thermo-diffusion

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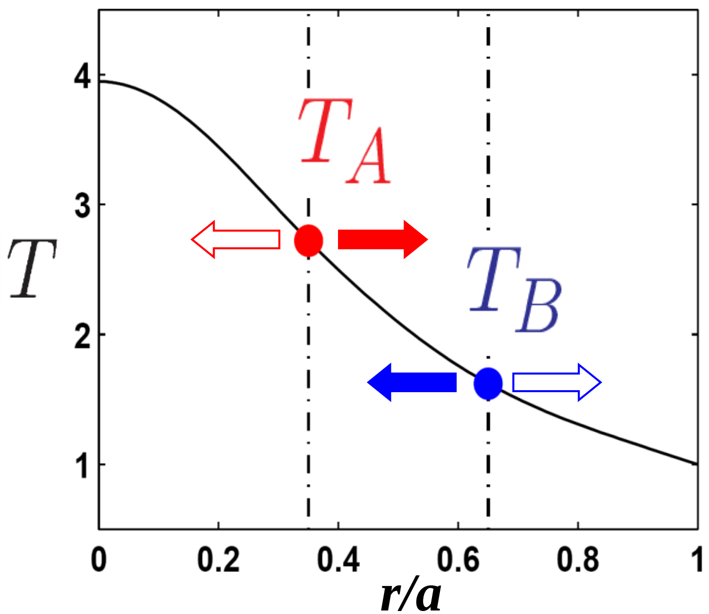
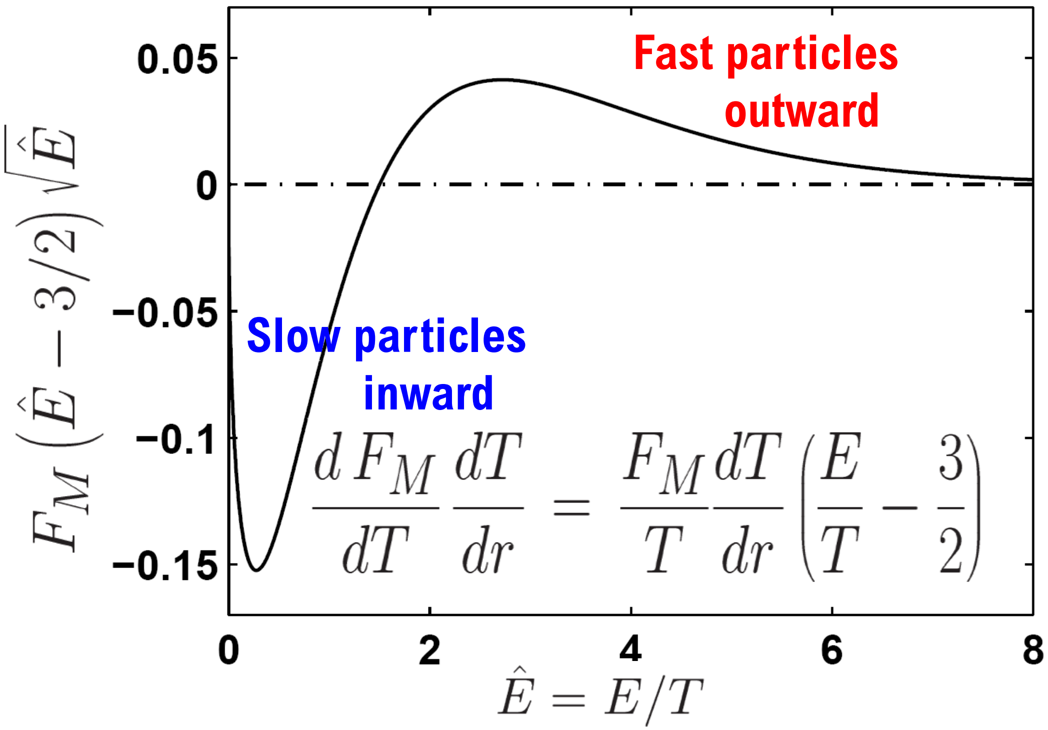


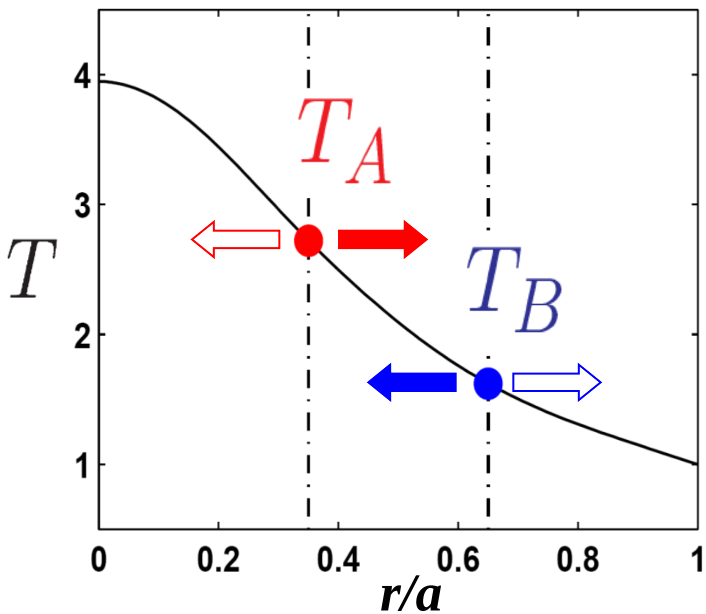


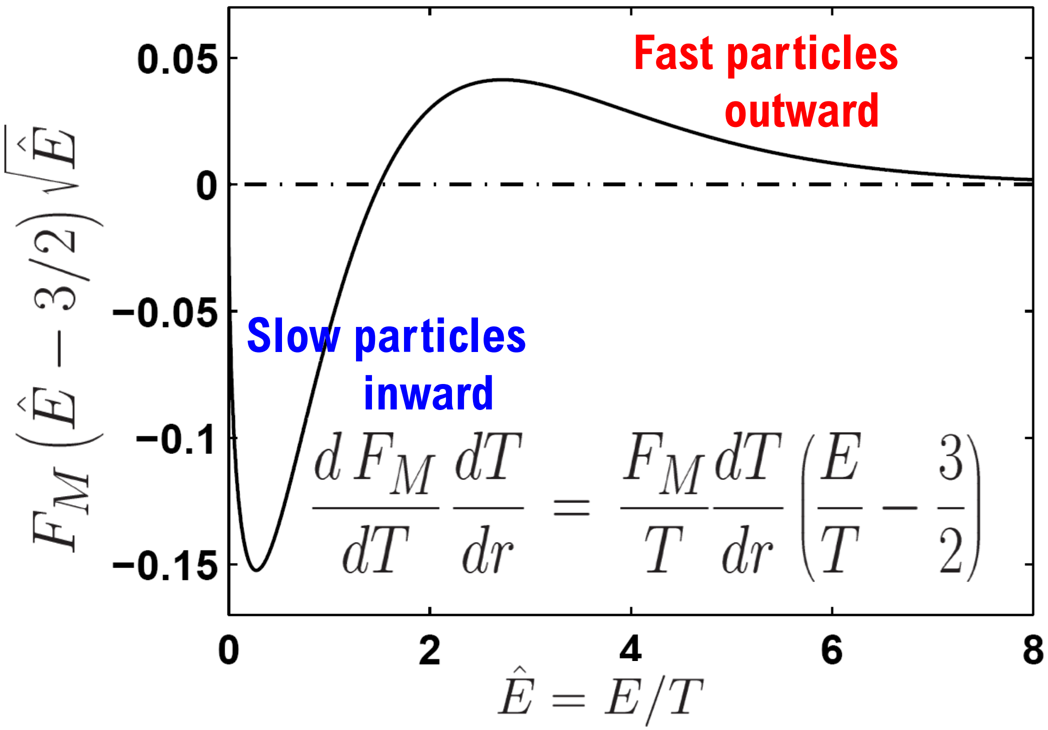
- ➢ No perturbation, no net flux

[Angioni et al NF 12]

Off-diagonal terms can reverse direction depending on instability, thermo-diffusion

- Temperature profile, temperature gradient





$$\frac{d\,F_M}{dT}\frac{dT}{dr}=\frac{F_M}{T}\frac{dT}{dr}\left(\frac{E}{T}-\frac{3}{2}\right)$$

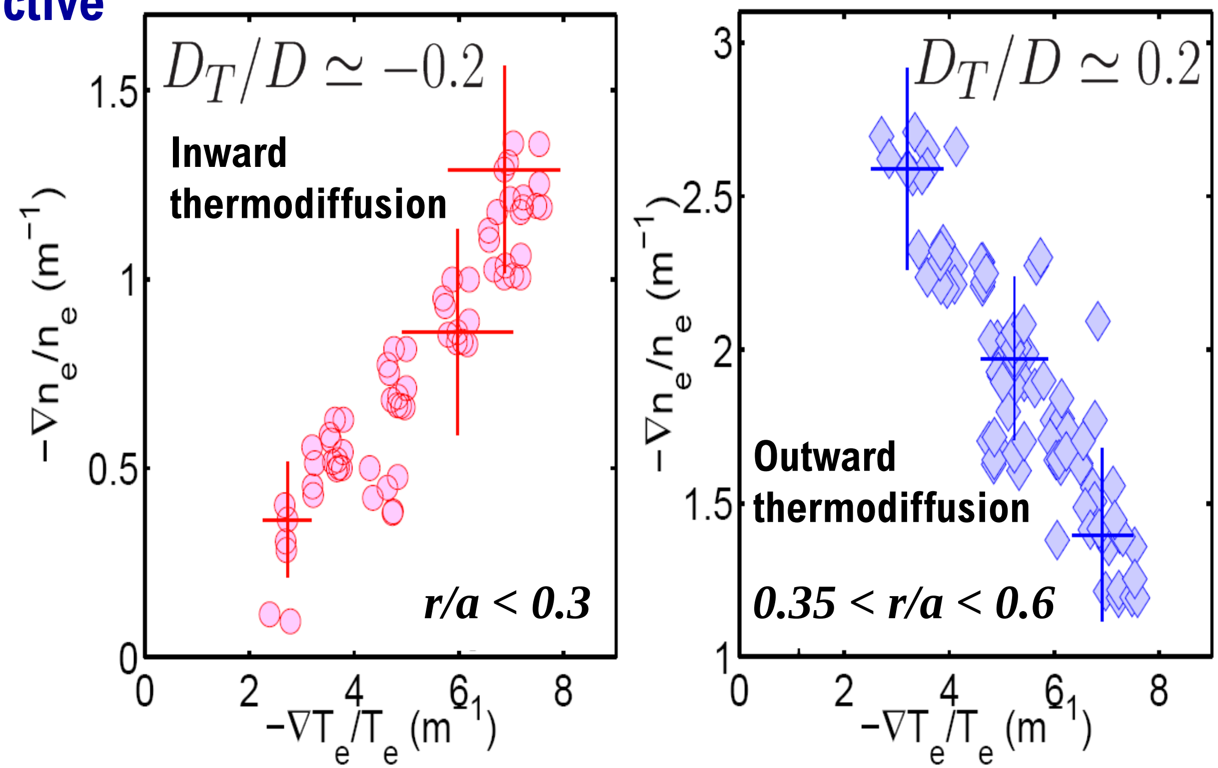
- No perturbation, no net flux
- Energy dependent perturbation: net flux is produced
- Inward if resonance at low energy (ITG)
- Outward if resonance at sufficiently large energy (TEM)

[Angioni et al NF 12]

Reversal of thermodiffusion observed in experiments, consistent with theory

- Tore Supra, fully non-inductive discharges
- Density gradient increases / decreases with increasing temperature gradient in the centre / at midradius
- Calculations indicate ITG dominant in the centre, TEM at mid-radius

Tore Supra, Hoang et al, PRL 04

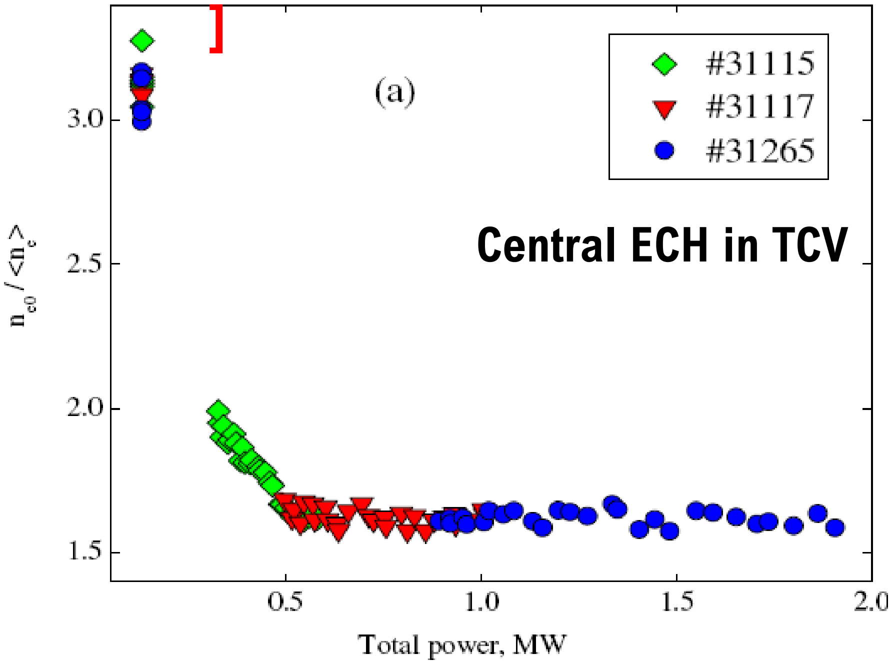


- Observations confirmed in FTU [M. Romanelli et al, PPCF 07]
- Experimental relevance of outward thermodiffusion first pointed out in relation to density pump-out with central electron heating (TEM destabilization with increasing Te/Ti) [Angioni et al, NF 2004]

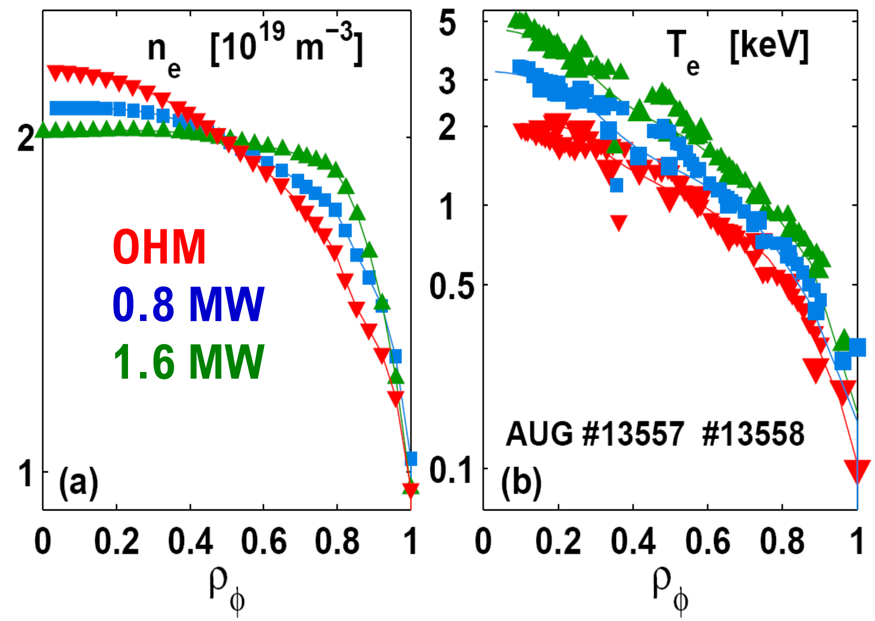
Density profile flattening with central ECH

- Density peaking has been often observed to be reduced by the application of central electron heating, 2 examples here

TCV [Zabolotsky PPCF 06]



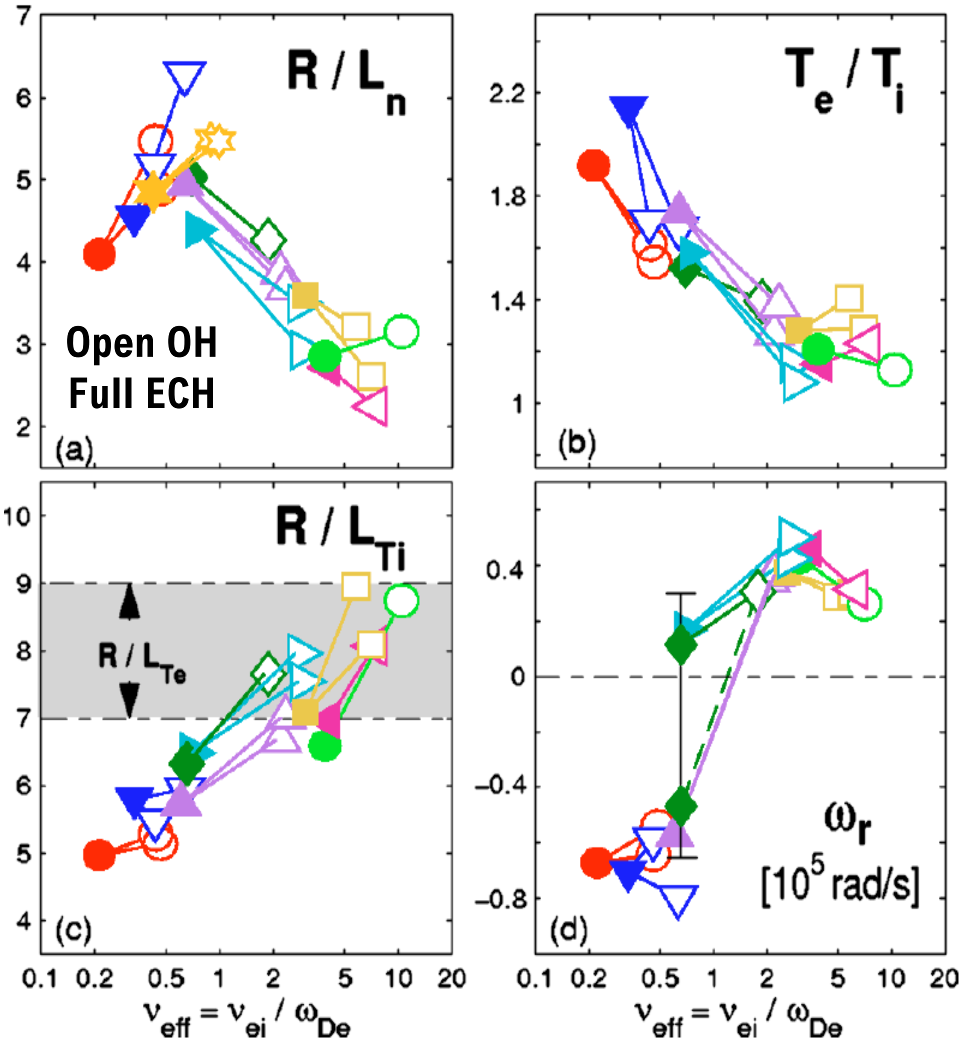
AUG [Angioni NF 04]



- This behaviour however is NOT universal
- Similar observations can be obtained in high density plasmas, due to reduction of Ware pinch effect (different mechanism of flattening)

Central ECH can lead to density flattening or density peaking depending on turbulence type

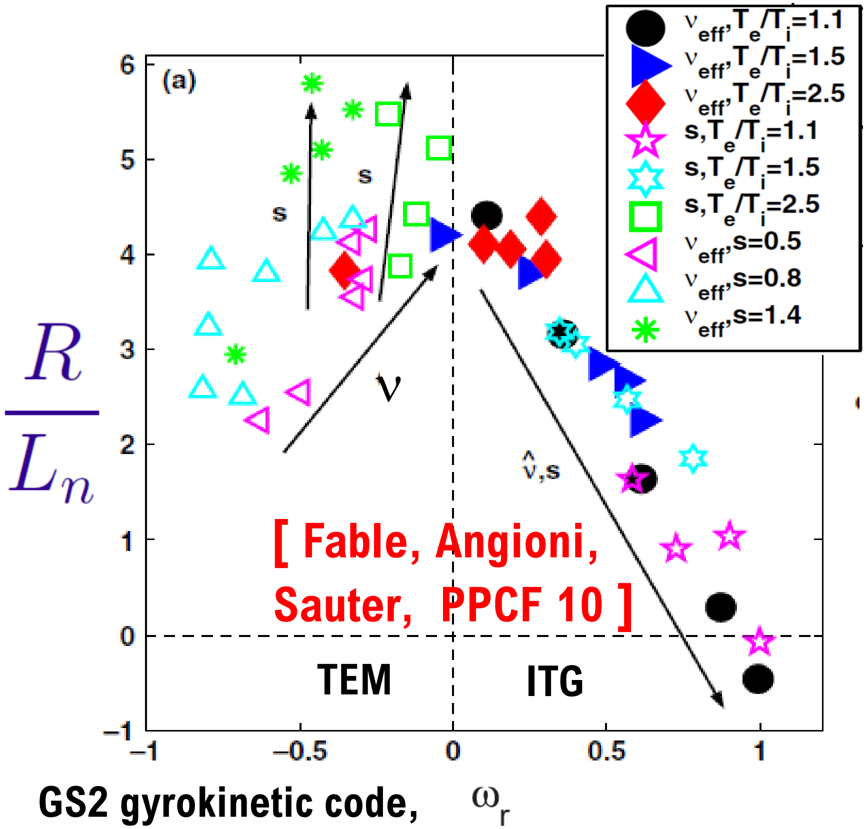
- Different response of density peaking to central ECH can be explained by the reversal of thermodiffusion from inward (ITG) to outward (strong TEM)



- Consistent with results obtained in TORE SUPRA [Hoang PRL 04]
- An increase of peaking is obtained when microinstability calculations indicate ITG most unstable mode
- A flattening when calculations find TEM as most unstable mode

[Angioni PoP 05]

Complicated dependences of off-diagonal electron transport are captured by the mode real frequency



GS2 gyrokinetic code, [Dorland PRL 00]

$$\frac{\Gamma}{n} = -D\frac{\nabla n}{n} - D_T\frac{\nabla T}{T} + V_p$$

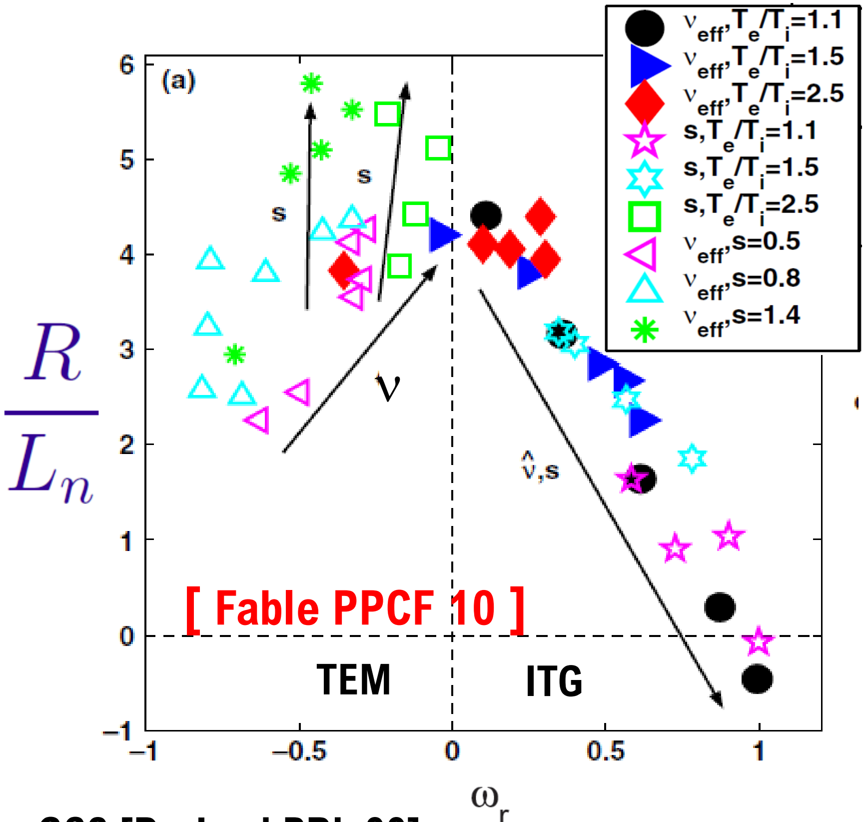
$$\frac{R}{L_n} = -\frac{D_T}{D}\frac{R}{L_T} - \frac{RV_p}{D}$$

- Max peaking in TEM, close to TEM-ITG transition [Angioni et al PoP 2005, Fable et al PPCF 2010]

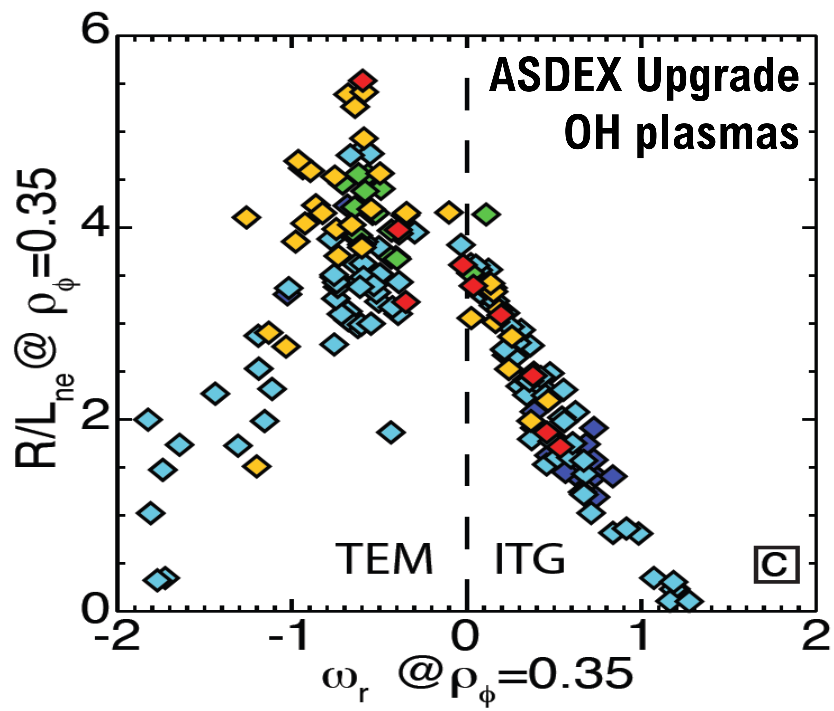
- Density peaking is reduced at large frequencies of the mode

[Review in Angioni et al NF 12]

Experimental data reproduce predicted dependence on mode frequency



GS2 [Dorland PRL 00]

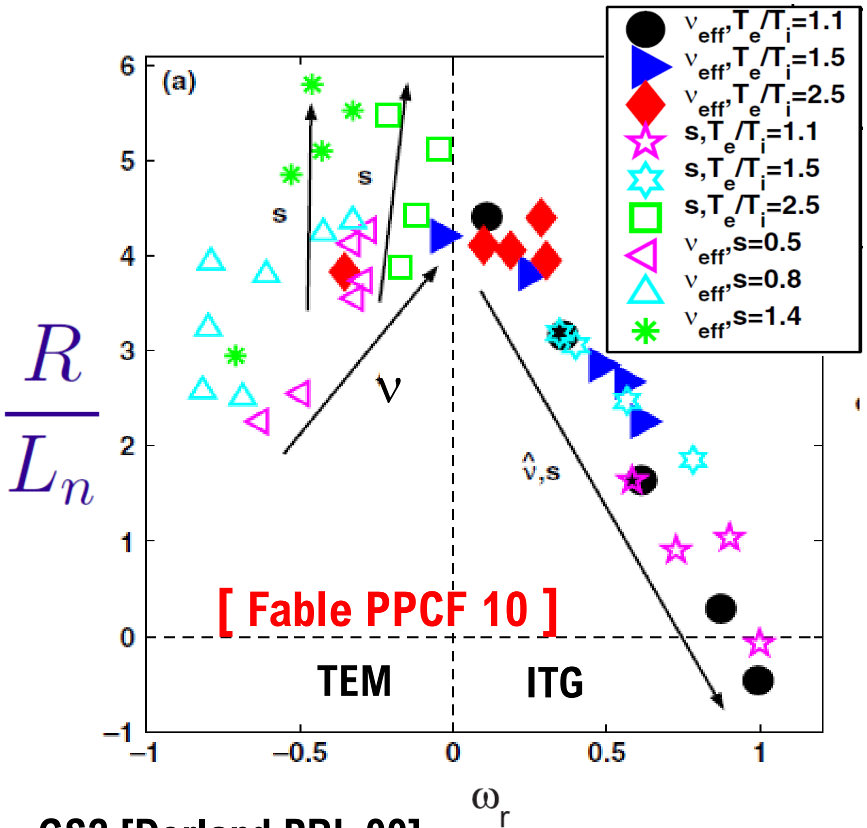


[McDermott et al NF 2014]

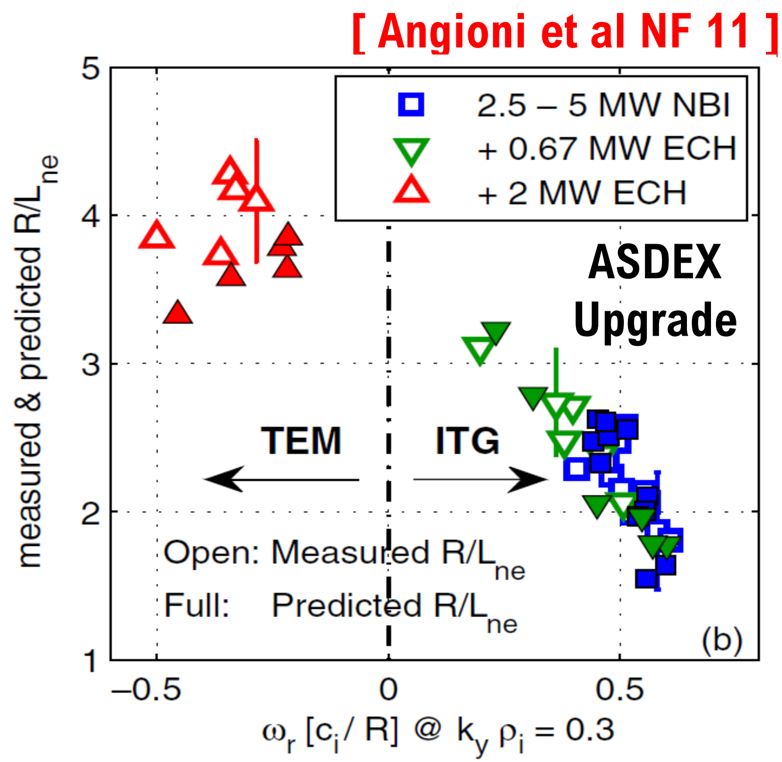
- Predicted curve followed by a large database of observations without external heating

[Review in Angioni et al NF 12]

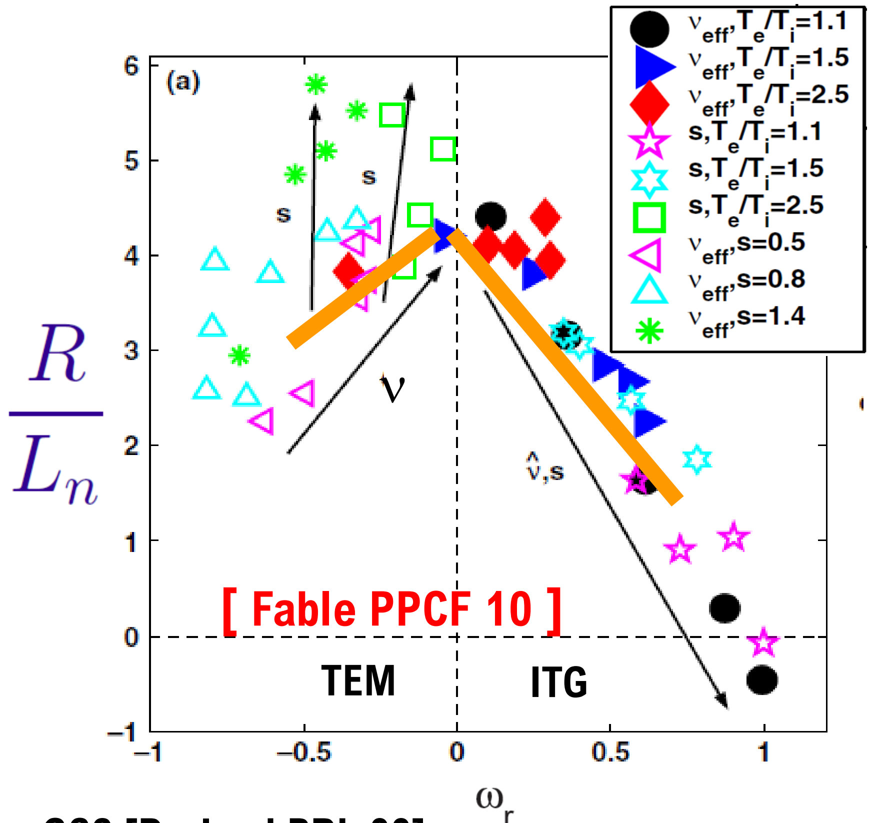
Dedicated modelling of specific data follows the same curve, and demonstrate quantitative match



GS2 [Dorland PRL 00]

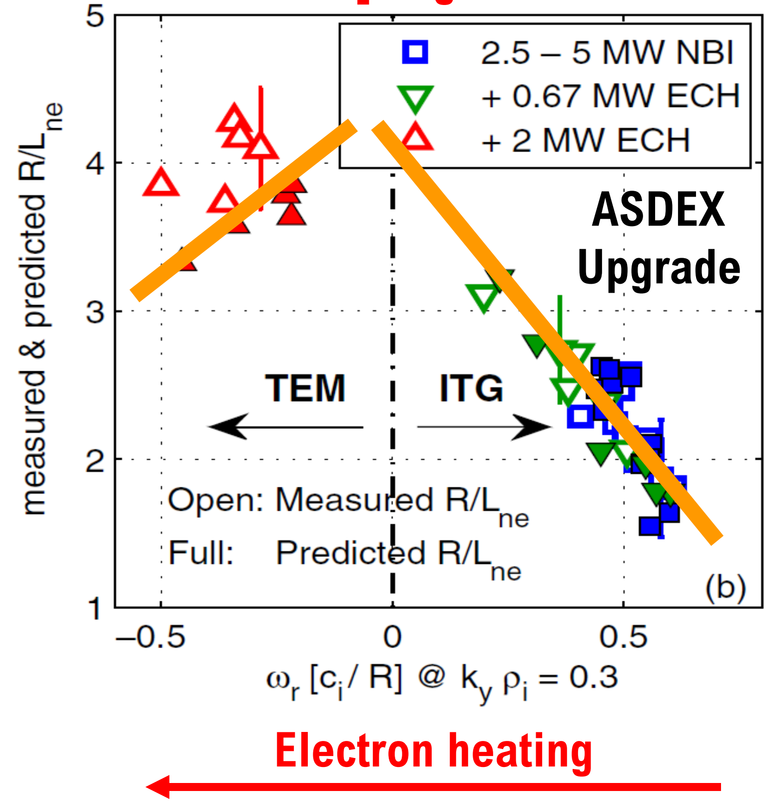


Dedicated modelling of specific data follow the same curve, and demonstrate quantitative match



GS2 [Dorland PRL 00]

[Angioni et al NF 11]

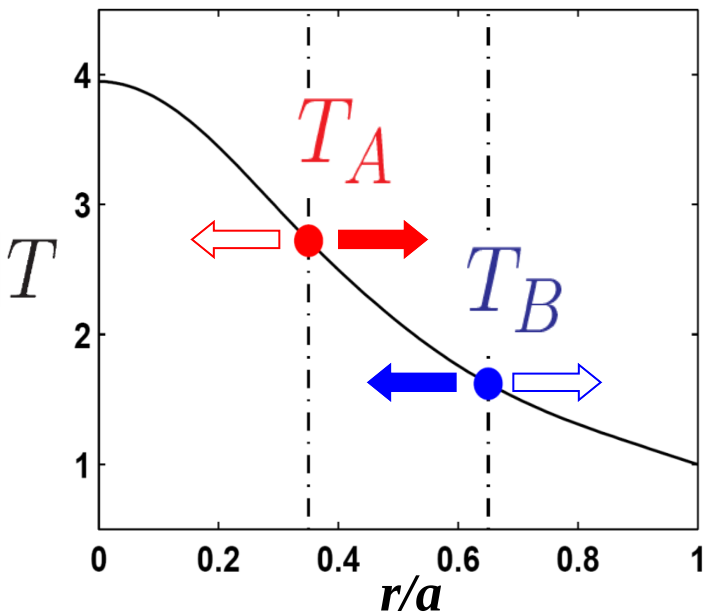


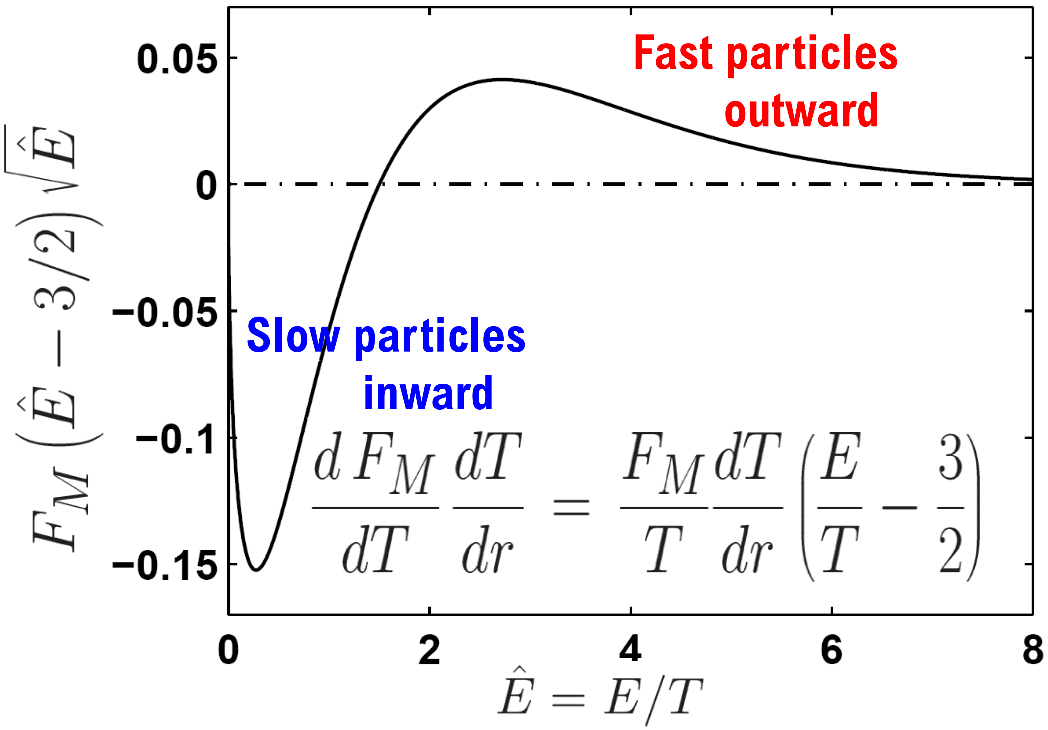
- ➢ **This general description also includes result that electron heating increases density peaking in ITG turbulence, in contrast to observations in TEM turbulence**

[Angioni et al NF 04, PoP 05]

Off-diagonal terms can reverse direction depending on instability, thermo-diffusion

- Temperature profile, temperature gradient



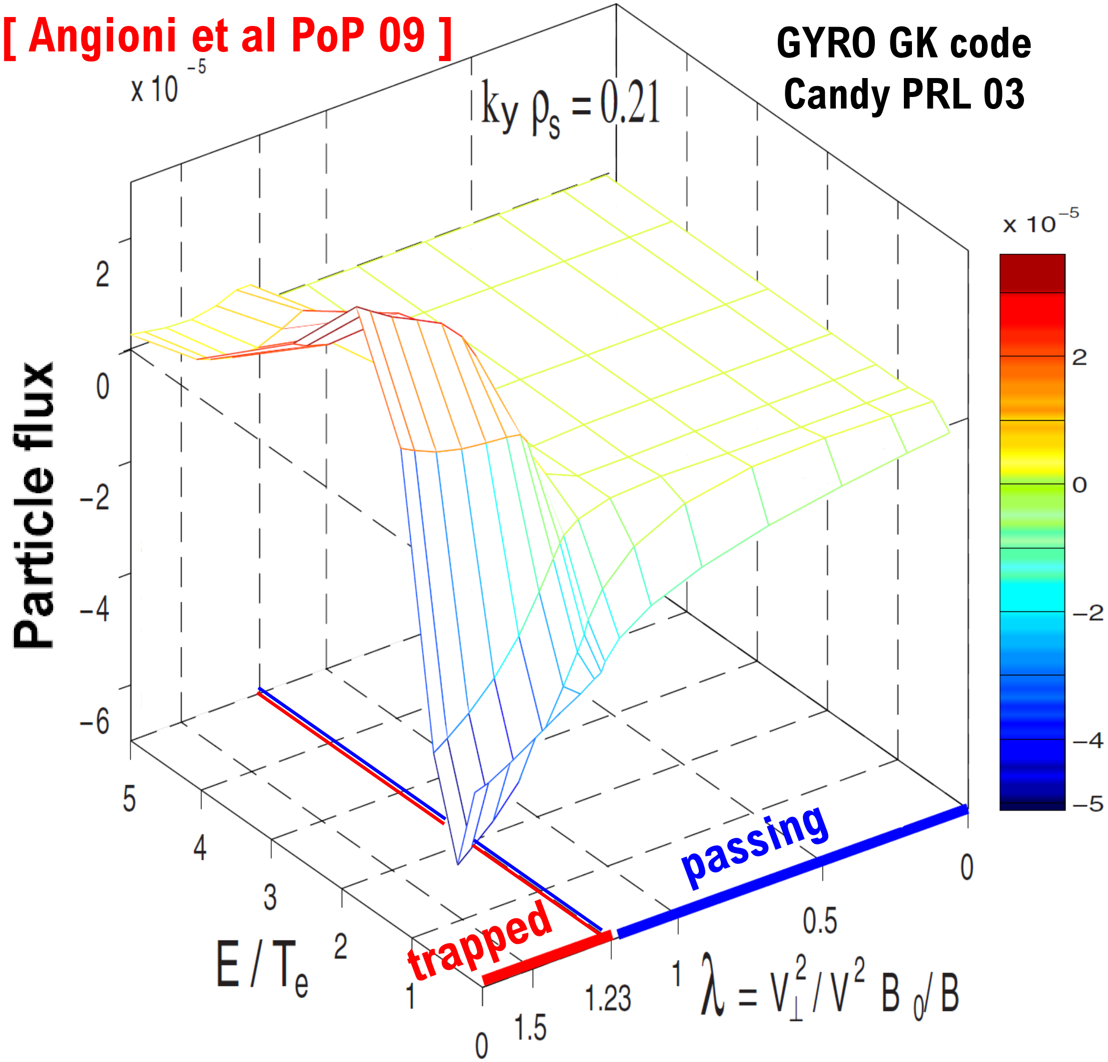


- No perturbation, no net flux
- Energy dependent perturbation: net flux is produced
- Inward if resonance at low energy (ITG)
- Outward if resonance at sufficiently large energy (TEM)

[Angioni et al NF 12]

Slow trapped electrons move radially inward, fast trapped electrons move outward

[Angioni et al PoP 09]

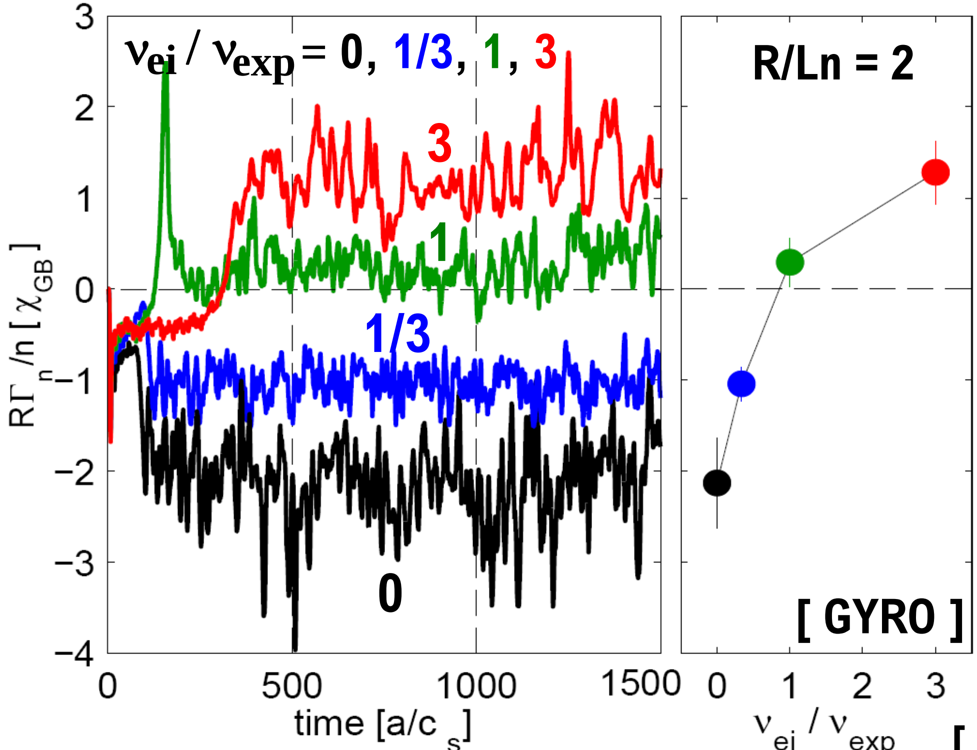


- Slow and fast particles behave differently (factor (E-3/2) in the gradient of Maxwellian)
- Turbulence simultaneously sustains large heat fluxes and particle fluxes close to zero
- Collisions detrap particles: stronger effect on slow particles $\Rightarrow$ reduces inward component of the flux

Collisions reduce the particle pinch in ITG turbulence

- Nonlinear gyrokinetic turbulence simulations (ASDEX Upgrade plasma, ITER ν_e)
- Null of the flux at the exp. value of ν_e , providing local R/Ln = 2 at mid-radius

[Angioni et al PoP 09]



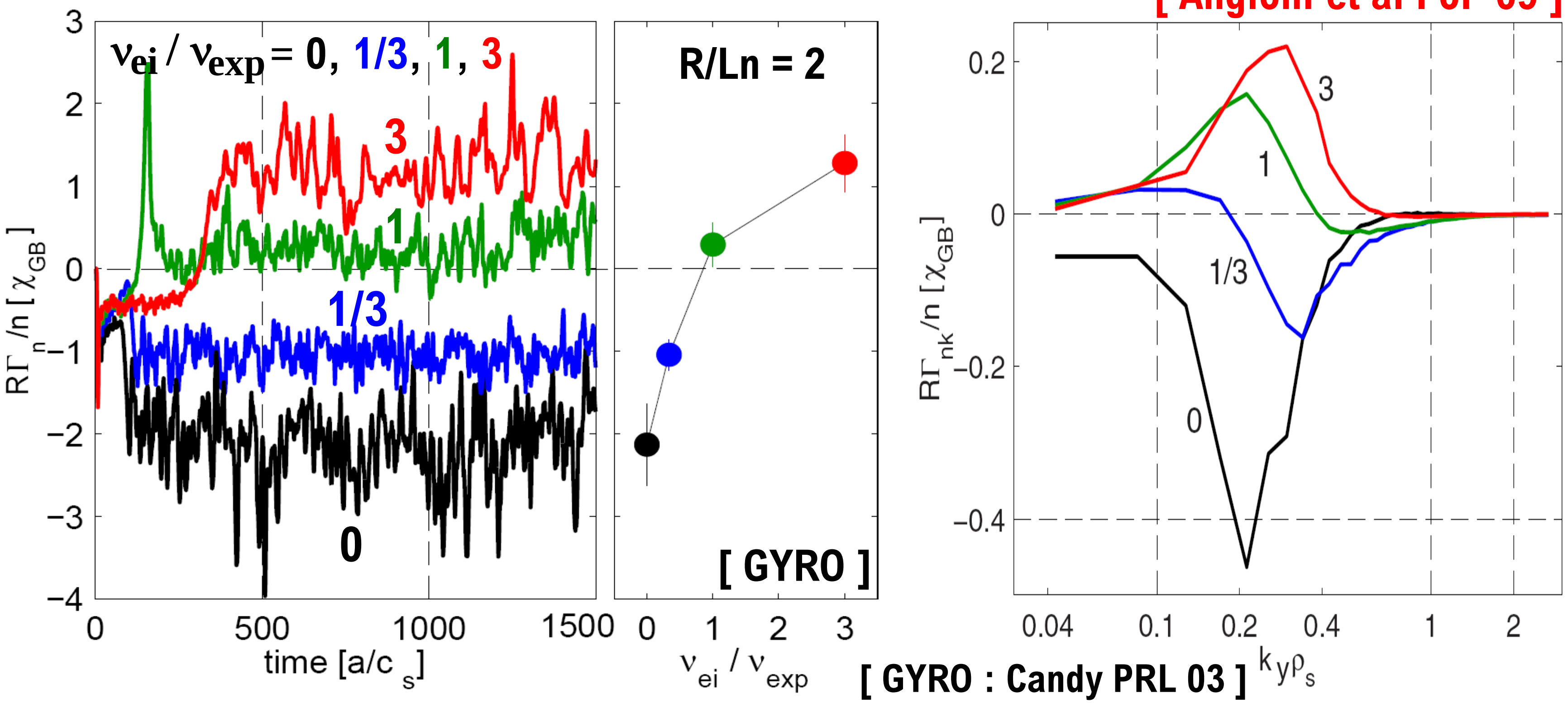
[GYRO : Candy PRL 03]

Collisions reduces the particle pinch in ITG turbulence

- Nonlinear gyrokinetic turbulence simulations (ASDEX Upgrade plasma, ITER ν_e)
- Null of the flux at the exp. value of ν_e , providing local R/Ln = 2 at mid-radius

- Condition of zero flux met by balance between outward and inward components at different scales

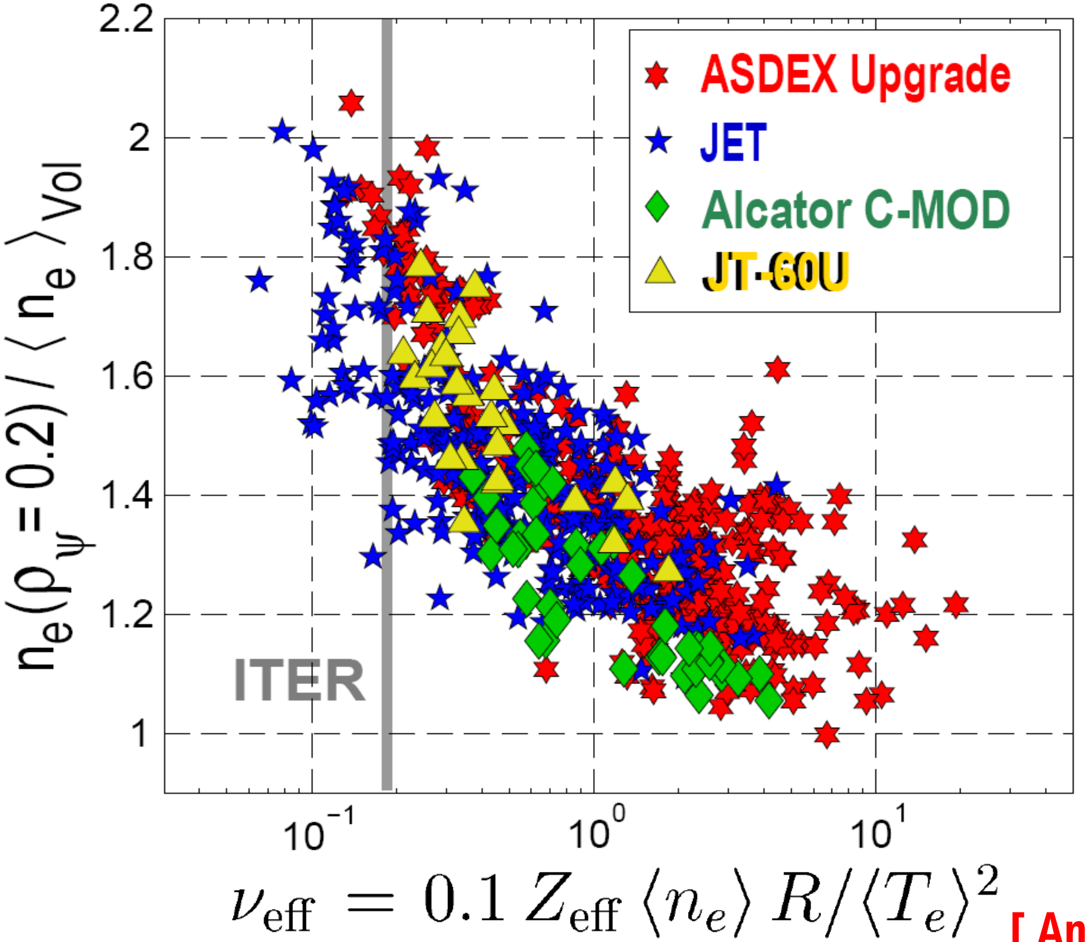
[Angioni et al PoP 09]



[GYRO : Candy PRL 03]

Density peaking observed to increase with decreasing collisionality in H-mode plasmas

- Consistent observations in AUG [Angioni et al PRL 03], JET [Weisen et al NF 05], C-Mod [Greenwald et al NF 07] and JT-60U [Takenaga et al NF 08]



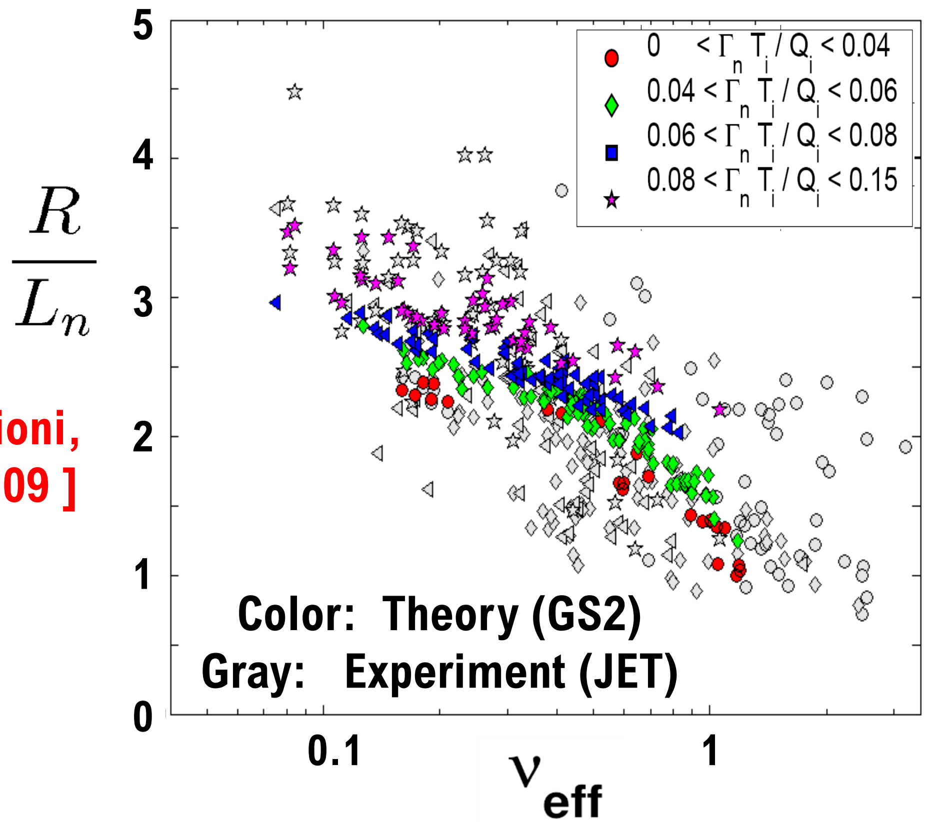
[Angioni et al PPCF 09]

Theory quantitatively reproduces the experimentally observed dependence

- **Agreement between linear and nonlinear simulations** [Angioni et al PoP 09] $\Rightarrow$ **apply linear model to large dataset**

- **Quantitative agreement with JET data**

[Maslov, Angioni, Weisen, NF 09]



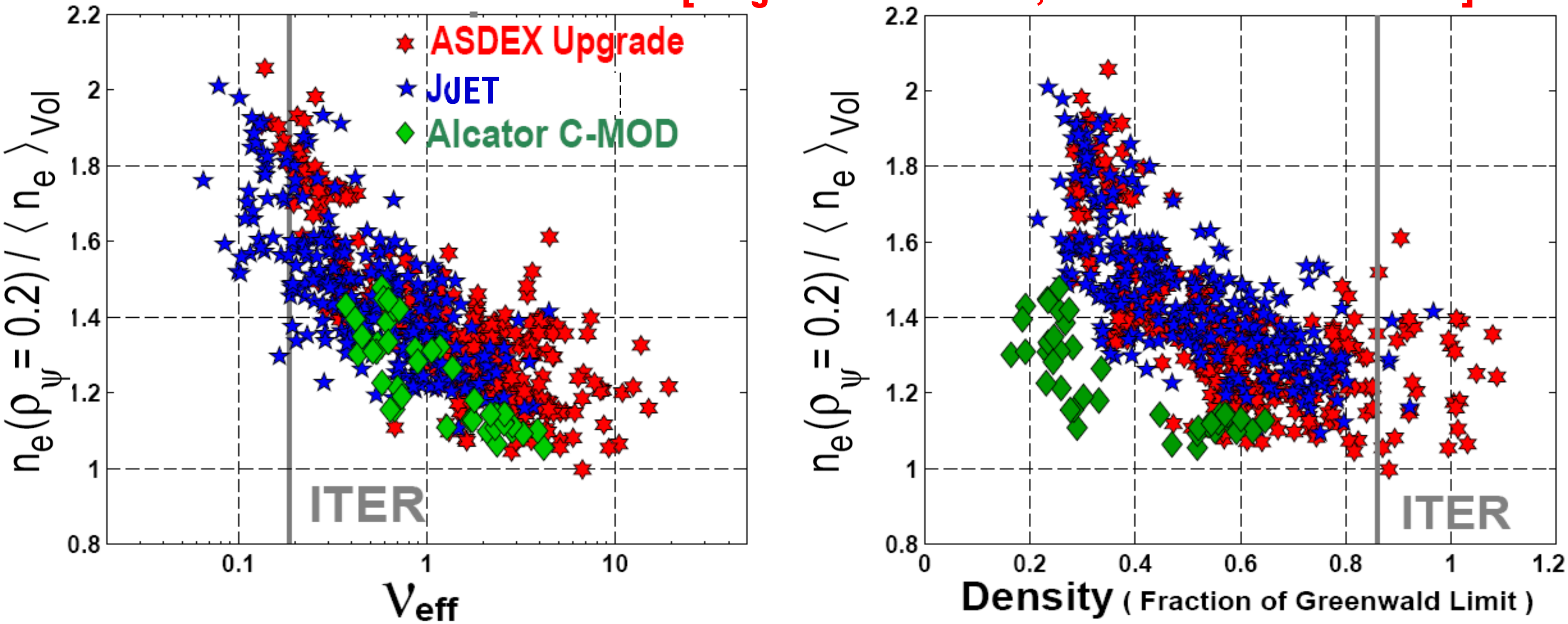
[GS2: Dorland PRL 00]

$$\nu_{\rm eff} = 0.1\, Z_{\rm eff}\, \langle n_e \rangle\, R / \langle T_e \rangle^2$$

Consistent with empirical approach: Collisionality is the right extrapolation parameter

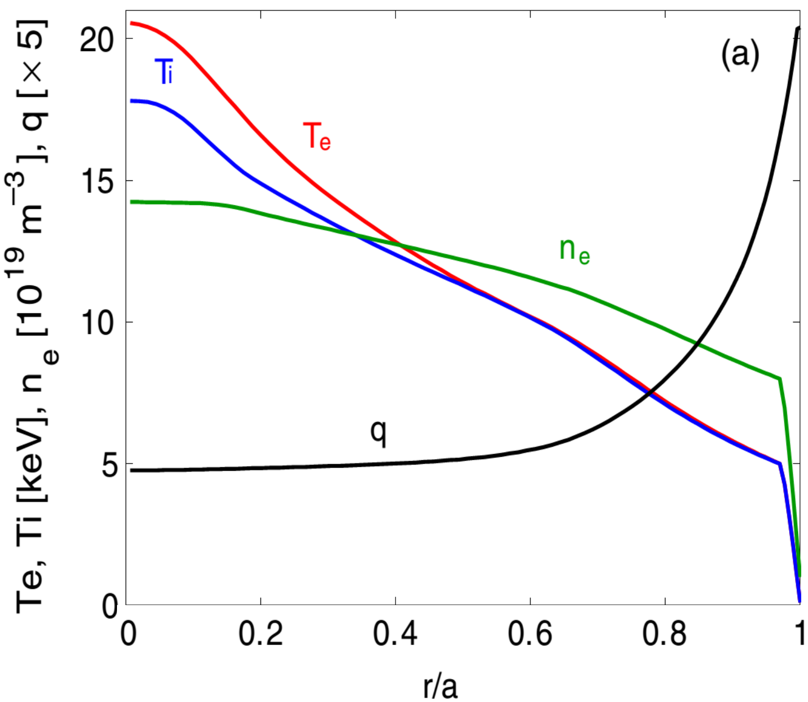
- Correlation with density (Greenwald fraction) resolved by observations in C-Mod
- Density peaking in ITER standard scenario $n_{e0}/\langle n_e \rangle_{\mathrm{Vol}} = 1.4 - 1.6$
- Favorable effects: operation at higher averaged density and potential of increased fusion power by about 20-30% w.r.t. flat profile with same averaged density

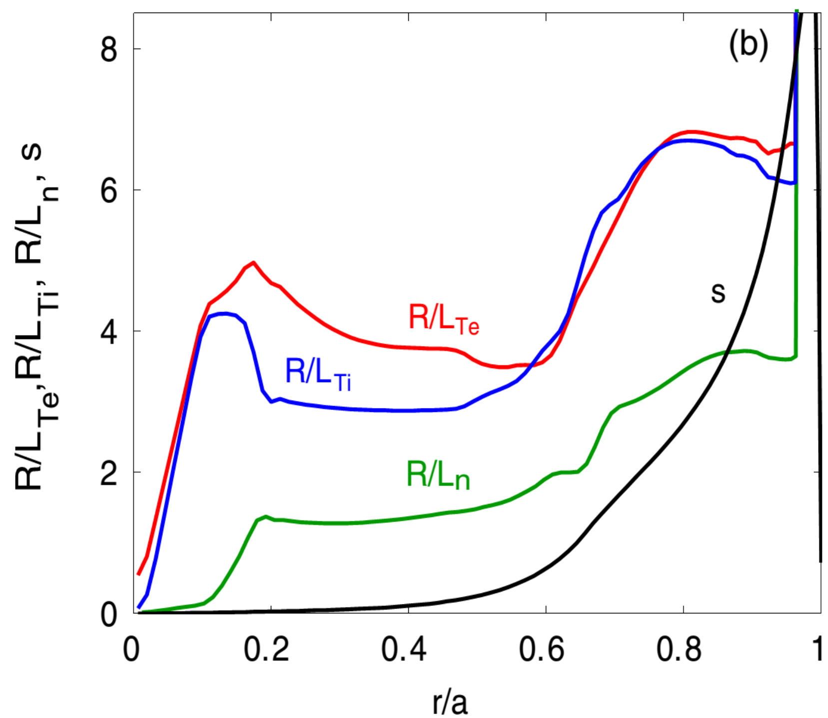
[Angioni et al NF 07; Greenwald et al NF 07]



Theory-based models predict centrally peaked density profiles in ITER

- At ITER standard scenario (H-mode) parameters, low collisionality leads to the development of a significant particle pinch, which produces a moderate peaking of the electron density profile.





[ASTRA with GLF23, Pereverzev NF 05, Angioni NF 2009]

Heavy Impurity transport is important for ITER, as it will have a W (Z = 72, A = 184) divertor

- **Transport of heavy impurities is produced by a combination of neoclassical transport and turbulent transport**
- **In tokamaks, neoclassical transport of impurities has this general expression**

$$R\langle \Gamma_z^{\mathrm{neo}} \cdot \nabla r\rangle \propto n_i T_i \nu_{ii} Z \left[\boxed{P_A} \left(\underbrace{\boxed{-\frac{R}{L_{n_i}}}}_{\text{Pinch}} + \underbrace{\boxed{\alpha_T \frac{R}{L_{T_i}}}}_{\text{Screening}} + \underbrace{\boxed{\frac{1}{Z}\frac{R}{L_{n_z}}}}_{\text{Diffusion}} \right) - 0.33 \boxed{P_B} f_c \boxed{\frac{R}{L_{T_i}}} \right]$$

radial transport

Poloidal asymmetry factors

[Hirshman and Sigmar, Nucl. Fusion 1981; Fulop and Helander Phys. Plasmas 1999]

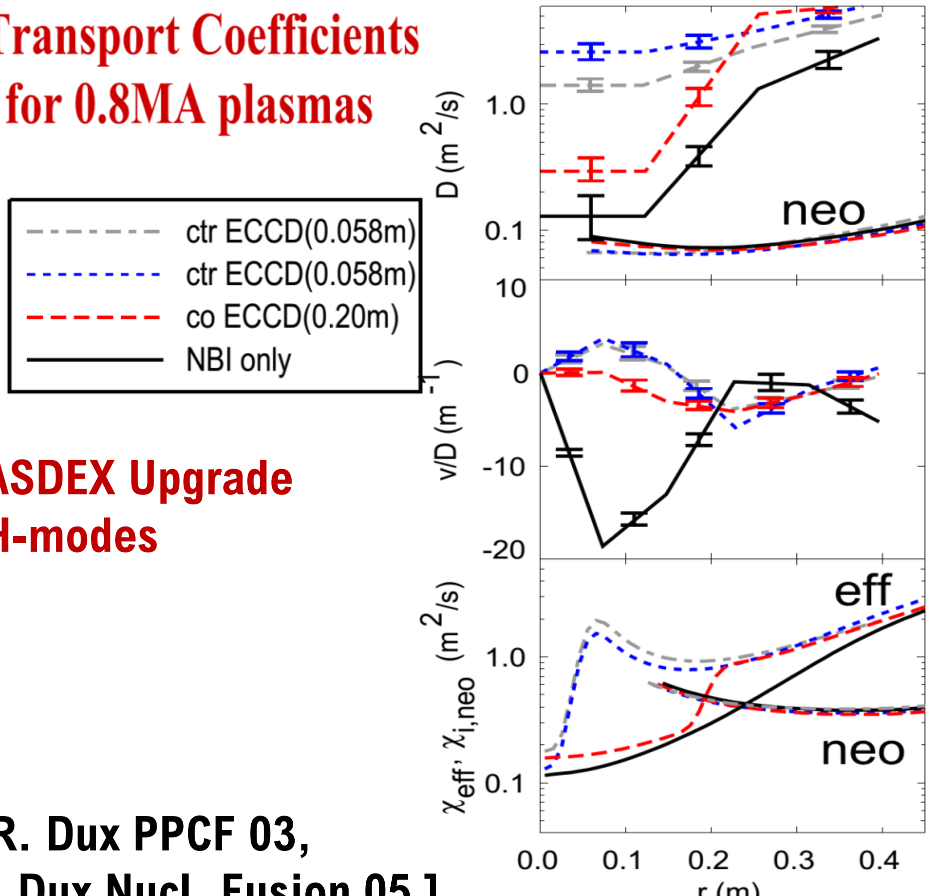
- **The peaking of the main plasma density profile produces an inward convection (pinch) of impurities, proportional to the impurity charge (very unfavorable)**
- **This can be at least partly mitigated by an outward convection proportional to the main ion temperature gradient (temperature screening)**

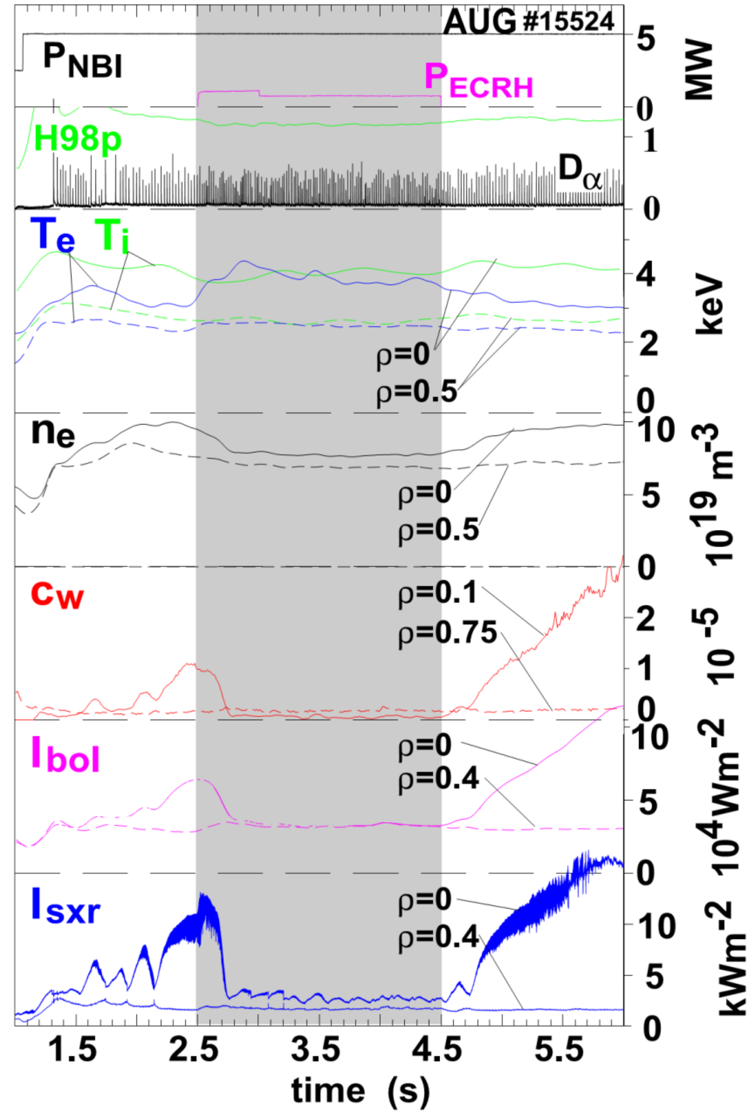
- **Turbulent transport does not have any component which strongly increases with increasing charge (or mass) (favorable, can offset neoclassical pinch)**

Experimentally, heavy impurity central accumulation can be avoided by application of central heating

- Examples from ASDEX Upgrade, dedicated experiments with Si ablation, and control of W accumulation with central ECH

Transport Coefficients for 0.8MA plasmas





ASDEX Upgrade H-modes

[R. Dux PPCF 03, R. Dux Nucl. Fusion 05]

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