

91<sup>st</sup> Meeting of the GOR Working Group

Praxis der Mathematischen Optimierung  
("Real World Mathematical Optimization")

## **Deterministic Global Optimization**

12.-13. December, 2013

Bad Honnef, Germany

(<https://gor.uni-paderborn.de/index.php?id=224>)

Organization

Josef Kallrath & Steffen Rebennack  
GOR AG „Praxis der mathematischen Optimierung“

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## Deterministic Global Optimization

This symposium is about real world problems which are solved using Global Optimization Techniques. Global Optimization addresses the computation and characterization of global optima (i.e., minima and maxima) of nonconvex functions over nonconvex feasible regions. Global Optimization has ubiquitous applications across many branches of engineering, applied sciences and mathematical optimization.

This two-day event will attempt to give an overview of the current state of the art of Deterministic Global Optimization techniques.

In talks, each approx. 40 to 50 minutes experts from practice, research institutions or software companies, will present selected problems and the corresponding solutions. Confirmations for their talks have been obtained from the following speakers:

Prof. Dr. Dr. h.c. mult. Panos M. Pardalos (University of Florida, Florida, USA) – confirmed  
*On Optimality Conditions in Non-convex Optimization and Related Issues*

Dr. Pietro Belotti (FICO, Birmingham, UK) – confirmed  
*Linear Cuts for Polynomial Optimization Problems*

Dr. Michael Bussieck (GAMS GmbH, Braunschweig, Germany) – confirmed  
*Open-source Quality Assurance and Performance Analysis Tools*

Prof. Dr. Josef Kallrath (Weisenheim am Berg, Germany) – confirmed  
*Global Optimization with GAMS using SOS-2 Variables & Global Optimization in Cutting Stock Problems*

Dr. Leo Liberti (IBM Research, USA) – confirmed  
*On feasibility-based bounds tightening*

Prof. Dr. Steffen Rebennack (Colorado School of Mines, Golden, CO, USA) – confirmed  
*Mixed AC-DC Distribution Systems Design for Commercial Buildings through Generalized Benders Decomposition*

Dr. Stefan Vigerske (GAMS GmbH, Braunschweig, Germany) – confirmed  
*Global Optimization Solver Technology & Towards MINLPLib 2.0*

We expect an interesting overview on the field and exciting discussions. Part of the official program is a visit and a guided tour through the private house of the first German chancellor, Konrad Adenauer.

91. Meeting of the GOR Working Group  
„Real World Mathematical Optimization“

## Deterministic Global Optimization

Physikzentrum, Bad Honnef, December 12 & 13, 2013

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Thursday, Dec. 12 - 2013: 10:30 – 22:00

- 10:30-10:45 **Opening and Welcome Session** (J. Kallrath & S. Rebennack)
- 10:45-11:55 **Prof. Dr. Dr. h.c. mult. Panos Pardalos** (University of Florida, USA)  
*On Optimality Conditions in Non-convex Optimization and Related Issues*
- 12:00-13:30 ----- Lunch Break -----
- 13:30-14:20 **Prof. Dr. Josef Kallrath** (GOR Arbeitsgruppe, Weisenheim am Berg)  
*Global Optimization in Cutting Stock Problems*
- 14:20-15:10 **Dr. Stefan Vigerske** (GAMS GmbH, Braunschweig, Germany)  
*Global Optimization Solver Technology*
- 15:15-15:45 ----- Coffee Break -----
- 15:45-17:15 ----- Visit & Guided Tour: Stiftung Bundeskanzler-Adenauer-Haus -----
- 17:15-18:05 **Prof. Dr. Josef Kallrath** (GOR Arbeitsgruppe, Weisenheim am Berg)  
*Global Optimization with GAMS using SOS-2 Variables*
- 18:10-18:20 **Internal Meeting** of the Working Group
- 18:30 - **Conference Dinner** – Buffet; get-together in the wine-cellar  
*Celebrating the 91st meeting of our GOR Working Group*
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## Friday, Dec. 13 - 2013: 09:30 – 16:30

09:30-10:40 **Dr. Leo Liberti** (IBM Research, USA)  
*On feasibility-based bounds tightening*

10:45-11:15 ----- Coffee Break -----

11:15-12:05 **Dr. Michael Bussieck** (GAMS GmbH, Braunschweig, Germany)  
*Open-source Quality Assurance and Performance Analysis Tools*

12:15-13:45 ----- Lunch Break -----

13:45-14:35 **Dr. Stefan Vigerske** (GAMS GmbH, Braunschweig, Germany)  
*Towards MINLPLib 2.0*

14:35-15:25 **Dr. Pietro Belotti** (FICO, Birmingham, UK)  
*Linear Cuts for Polynomial Optimization Problems*

15:25-16:15 **Prof. Dr. Steffen Rebennack** (Colorado School of Mines, Golden, USA)  
*Mixed AC-DC Distribution Systems Design for Commercial Buildings through Generalized Benders Decomposition*

16:15-16:30 **Final Discussion – End of the Workshop – Coffee Break**

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## The Speakers

**Pietro Belotti** is a developer in the Xpress team at FICO. He worked at several universities in the US prior to joining FICO: Carnegie Mellon University, Lehigh University, and Clemson University. His main research interests are in mixed integer nonlinear programming (MINLP) and global optimization. He is the main developer and maintainer of Couenne, an open-source software for MINLP that is available in the COIN-OR repository.

**Michael R. Bussieck** is a Senior Research Analyst at GAMS Software GmbH. From 1999 to 2004 he worked at the GAMS Development headquarters in Washington DC, USA. He received his Ph.D. from Technical University Braunschweig, Germany.

**Josef Kallrath** obtained his PhD in astrophysics from Bonn University (Germany) in 1989. He is a professor at the University of (Gainesville, FL, [www.astro.ufl.edu/~kallrath](http://www.astro.ufl.edu/~kallrath)), and solves real-world problems in industry using a broad spectrum of methods in scientific computing, from modeling physical systems to supporting decisions processes by mathematical optimization. He has written review articles on the subject, about 70 research papers in astronomy and applied mathematics, and several books on mixed integer optimization, as well as one on eclipsing binary stars.

He leads the Real World Optimization Working Group of the German Operations Research Society. His current research interests are polyhedral modeling and solution approaches to solve large-scale or difficult optimization problems, for instance, by decomposition techniques such as column generation, or hybrid methods.

**Leo Liberti** obtained his Ph.D. from Imperial College London in 2004, works at the IBM “T.J. Watson” Research Center in Yorktown Heights, and is a professor of Computer Science at Ecole Polytechnique in France. His main research fields are MINLP and Distance Geometry.

**Panos M. Pardalos** serves as Distinguished Professor of Industrial and Systems Engineering at the University of Florida. He is also an affiliated faculty member of the Computer and Information Science Department, the Hellenic Studies Center, and the Biomedical Engineering Program. He is also the Director of the Center for Applied Optimization. Dr. Pardalos is a world leading expert in global and combinatorial optimization. His recent research interests include network design problems, optimization in telecommunications, e-commerce, data mining, biomedical applications, and massive computing.

Professor Pardalos is a Fellow of AAAS and INFORMS, member of several Academies of Sciences and holds several Honorary professorships and Phd degrees. Panos Pardalos, has been awarded the 2013 EURO Gold Medal Prize, bestowed by the Association for European Operational Research Societies, and the 2013 Constantin Carathodory Prize of the International Society of Global Optimization.

**Steffen Rebennack** is Assistant Professor at the Colorado School of Mines, USA. He obtained his PhD at the University of Florida. His research interests are in dimension-reduction techniques for large-scale optimization problems, particularly with applications in power systems, stochastic optimization and global optimization. He is the vice-president of the Real World Optimization Working Group of the German Operations Research Society.

**Stefan Vigerske** is a Senior Optimization Analyst at GAMS Software GmbH. From 2005 to 2012, he held a position as research associate at Humboldt-University Berlin, Germany, where he worked as one of the main developers of the numerical optimization framework SCIP. Closely collaborating with Zuse Institute Berlin, his main task was the extension of the mixed-integer linear optimizer in SCIP to a solver for mixed-integer nonlinear programs. He received his Ph.D. from Humboldt-University Berlin, Germany.

# Linear Cuts for Polynomial Optimization Problems

Pietro Belotti

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We propose linear inequalities to be used in branch-and-bound methods for solving polynomial optimization (PO) problems, which consist of minimizing a polynomial subject to nonlinear constraints also identified by polynomials. These inequalities use a tensor representation of a set of monomials, described by Burer and Dong and Qi et al..

These inequalities, which are more useful for dense polynomials (i.e., polynomials where most monomials have a nonzero coefficients) extend the SDP-based cuts proposed in the context of quadratically constrained quadratic programming (QCQP), a special case of PO where objective function and constraints are quadratic. This extension brings a more general class of inequalities, although at the price of a hard separation problem. Therefore, we describe a heuristic for separation and report on some preliminary computational results.

# Open-source Quality Assurance and Performance Analysis Tools

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Until recently, much of the math programming community has focused primarily on performance testing and benchmarking, while the general commercial environment has emphasized reliability over performance. Around 10 years ago we introduced the PAVER platform (Performance Analysis and Visualization for Efficient Reproducibility) to aid in both QA and performance analysis of solver software. We will present new and enhanced QA and performance tools implemented in the second generation of the PAVER platform.



# Global Optimization in Cutting Stock Problems

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A set of circles, rectangles, convex polygons or ellipses is to be cut from rectangular design plates to be produced or from a set of stocked rectangles of known geometric dimensions. The objective is to minimize the area of the design rectangles subject to lower and upper bounds of their widths and lengths. The objects are free of any orientation restrictions.

If all nested objects fit into one design or stocked plate the problem is formulated and solved as a nonconvex nonlinear programming problem. If the number of objects cannot be cut from one plate, additional integer variables are needed to represent the allocation problem leading to a nonconvex mixed integer nonlinear optimization problem.

We present exact mathematical programming solutions to both the design and allocation problem and for small number of objects to be cut we compute globally optimal solutions. Circles and arbitrary convex polygons are treated simultaneously in this context. The formulation for ellipses and their non-overlap condition exploits coordinate transformations techniques.

One key idea in the developed NLP and MINLP models is to use separating planes to ensure that rectangles, polygons or ellipses do not overlap with each other or with one of the circles.

Another important idea for considering with several resource rectangles is to develop a model formulation which connects the binary variables only to the variables representing the center of the circles or the vertices of the polytopes but not to the non-overlap or shape constraints.

We support the solution process by symmetry breaking constraints. In addition we compute lower bounds, which are constructed by a relaxed model in which each polygon is replaced by the largest circle fitting into that polygon.

We have successfully applied several solution techniques to solve this problem among them the Branch&Reduce Optimization Navigator (**BARON**) and the **LindoGlobal** solver called from **GAMS**, and a column enumeration approach with columns representing the assignments.

Good feasible solutions are computed within seconds or minutes usually during preprocessing. In most cases they turn out to be globally optimal. For up to 10 circles, we prove global optimality up to a gap of the order of  $10^{-8}$  in short time. Cases with a modest number of objects, for instance, 6 circles and 3 rectangles, are also solved in short time to global optimality. For test instances involving non-rectangular polygons it is difficult to obtain small gaps. In such cases we are content to obtain gaps of the order of 10 percent.

**Keywords:** Global Optimization, mixed integer programming, cutting stock, packing, shape constraints, non-overlap constraints, design problem, assignment

# Global Optimization with GAMS using SOS-2 Variables

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This tutorial discusses algorithmic techniques and state-of-the-art software implementations for the global optimization of nonlinear and mixed-integer nonlinear optimization problems (NLP and MINLP). For hands-on examples, we will use the algebraic modeling system GAMS.

# On feasibility-based bounds tightening

Leo Liberti

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Mathematical programming problems involving nonconvexities are usually solved to optimality using a spatial Branch-and-Bound (sBB) algorithm. Algorithmic efficiency depends on many factors, among which the widths of the bounding box for the problem variables at each Branch-and-Bound node naturally plays a critical role. The practically fastest box-tightening algorithm is known as FBBT (Feasibility-Based Bounds Tightening): an iterative procedure to tighten the variable ranges. Depending on the instance, FBBT may not converge finitely to its limit ranges, even in the case of linear constraints. Tolerance-based termination criteria yield finite termination, but not in worst-case polynomial time. We model FBBT by using fixed-point equations in terms of the variable bounding box, and we treat these equations as constraints of an auxiliary mathematical program, which turns out to be a linear program. We demonstrate the usefulness of our approach by improving the open-source sBB solver Couenne.

# On Optimality Conditions in Non-convex Optimization and Related Issues

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In this talk we are going to present recent results regarding global optimality conditions for general non-convex optimization problems. First we are going to discuss complexity issues regarding the existence of points satisfying optimality conditions and the connection to complementarity problems. In addition, we are going to discuss surprising connections between optimality conditions and continuous formulations of discrete optimization problems.

In the second part of the talk we are going to discuss our recent result regarding optimality conditions of locally Lipschitz functions. Namely, we show how the necessary conditions for a local minimum can be used to obtain a sufficient optimality condition of first order for a global minimum of a locally Lipschitz function on a closed convex set in a Banach space.

# Mixed AC-DC Distribution Systems Design for Commercial Buildings through Generalized Benders Decomposition

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With the advent of inexpensive computing and efficient power electronics, the load mix in commercial buildings has experienced a fundamental shift away from almost exclusively traditional alternating current (AC) loads toward primarily direct current (DC) loads devices which use DC electricity either for end-use or as a power conditioning stage. Simultaneously, installations of DC distributed generation sources for commercial buildings, such as rooftop photovoltaic arrays, are accelerating. Despite this proliferation of DC devices, the basic design of building electrical distribution systems has changed very little in the past century: AC distribution remains the industry standard. The AC-DC electricity conversions required to connect DC sources and loads to the AC electric grid result in wasted energy. Partial replacement of AC distribution with DC distribution can improve overall building electrical energy efficiency; the result is a mixed AC-DC electrical distribution system.

In this talk, we develop a mixed-integer nonlinear nonconvex mathematical programming problem which determine maximally energy efficient designs for mixed AC-DC building electrical distribution systems. The model precisely quantifies building electrical energy efficiency at a systems level, not simply the level of individual devices. We develop a tailored global optimization algorithm, based in nonconvex Benders decomposition. The results of two case studies validate the power of the optimization algorithm and demonstrate that well designed mixed AC-DC building electrical distribution systems can achieve higher efficiency than either AC or DC distribution used alone.

# Global Optimization Solver Technology

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This presentation gives a short overview on algorithmic techniques that are employed in state-of-the-art global solver for nonlinear optimization problems (NLP), in particular convexification and bound tightening approaches.

# Towards MINLPLib 2.0

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The MINLPLib is a collection of Mixed Integer Nonlinear Programming models with the purpose to provide algorithm developers with a large and varied set of both theoretical and practical test instances. In this talk, we report on ongoing work to extend, update, and categorize MINLPLib and related libraries.