DOPPLERS

Problem 1

Physics in sports

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Background Physics is everywhere in the world around us. Therefore it is not surprising that researchers also have been describing everyday-life activities like sports from a physics point of view. This task deals with two not-so-common sports, namely rock climbing and diving from high board.

Climbing Falling is one of the most usual casualties in rock climbing. Hence, every climber will secure herself or himself with carabiners. We assume a situation in which two climbers, leader and follower, climb a rock and they are connected to each other with a rope. In case one of the climbers falls the other can keep hold of the rock and secure the other. Let us first consider a very simple model. The rope of the climbers is assumed as a linear spring.

a) [*2 points*] For a given height h of the leader above the last carabiner, a given L as rope length between leader and follower, and given M as rope elasticity [1] , compute the maximal elongation of the rope. First, draw a sketch which includes both climbers and the given lenghts.

b) [*0.5 points*] The solution of (a) contains the term $\frac{2h}{L} := f$ which is called *fall factor. Which values are can f take and which situations do the limits correspond to?*

c) [*0.5 points*] *What is the maximal force acting upon the climber when he is caught by the rope after falling?*

d) [*1 point*] With the result of (c) and typical values for the mass of the climber, a worst case fall factor f, and the maximal acceleration a human can withstand of $5g$ the corresponding maximal rope stretch of y_{max}/L is about 133%. A rope would need to be a rubber to be able to stretch that much. In fact, real climbing ropes only stretch up to 40%, being limited by the International Climbing and Mountaineering Federation (UIAA). This is achieved by strong damping within the rope.

This can be described with the Maxwell model that includes a dashpot [Dämpfer] and a spring in a chain (see Fig. 1). The resulting force of the dashpot is proportional to the velocity, but in an opposite direction.

What are the two equations of motion? The solution is only valid if, additionally to the formulae, the reasoning is given.

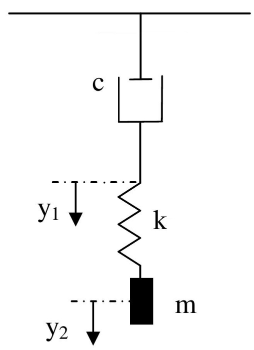

Figure 1 Maxwell model: a dashpot and a spring in chain

e) [*3 points*] *Solve the equations of motion in order to calculate the maximum force that acts on the falling climber.* For the moment when the climber starts to fall into the rope (so when the rope starts to elongate) it is a good approximation to set the gravitational acceleration to $g = 0$. This is a good approximation for larger downfalls when the initial friction is much larger than the gravitational acceleration.

Keep in mind what the initial conditions of the fall are. You can use an initial velocity of the falling climber of $v_0 := \sqrt{2gh}$.

If convenient, feel free to use the definitions of $\Omega = \frac{k}{m} - \frac{k^2}{4c^2}$ and $\kappa = \frac{k}{2c}$.

[1]The elasticity M is given by the product of Young modulus E and cross sectional area A. The modulus is defined as $E = (F/A)/(\Delta l/L)$ with F being the force needed to be applied to a body with cross section area A and length L to stretch it by Δl.

You might want to use the relation $\sin(\text{atan}(x)) = \frac{x}{\sqrt{x^2+1}}$.

Just if you are interested: The exact solution of (e) can be expanded and with an error of the order $\mathcal{O}\left(\frac{km}{c^2}\right)$ it holds

$$F_{\text{max}} \approx mg \left(1 + \sqrt{1 + 2\frac{M}{mg} f e^{-\frac{\pi}{2Q}}}\right) \tag{1.1}$$

Here, a quality factor $Q = \frac{\omega}{2\kappa}$ is introduced. It is defined as the relation of total energy and energy loss per oscillation period. This is why $1/Q$ indicates the impact of damping. The internal damping in this model decreases exponentially the maximum force in comparison with the non-damping case discussed first in this task. More precise methods show that the damping of the here-discussed Maxwell model is giving a bit too high energy absorption values compared with the reality.

Springboard diving In a second part we want to have a look at springboard divers (Turmspringer). A lot of physics is happening in a jump of a diver or a trampolinist performing a somersault (Salto) or a twist (Spirale).

f) [*3 points*] *Do springboard divers violate angular momentum conservation? Is it possible for a diver to perform a somersault or twist with zero angular momentum at all times during the stunt or is it necessary to take a net momentum into the rotation from the diving board?* Make use of explanatory examples and/or figures as needed.

Note: This task does not require specific calculations. Instead, a logic line of argumentation with physical arguments is required.